The Pont du Gard, in southern France, was built by the Romans nearly 2000 years ago, and was built to last.

CHAPTER 9

BODIES IN EQUILIBRIUM; ELASTICITY AND FRACTURE

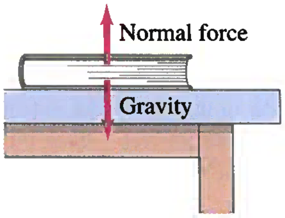
In this chapter, we will study a special case of motion—when the net force and the net torque on an object, or system of objects, are both zero. In this case the object or system is either at rest, or its center of mass is moving at constant velocity. We will be concerned mainly with the first case, when the object or objects are all at rest. Now you may think that the study of objects at rest is not very interesting since the objects will have neither velocity nor acceleration; and the net force and the net torque will be zero. But this does not imply that no forces at all act on the objects. In fact it is virtually impossible to find a body on which no forces act at all. Sometimes the forces may be so great that the object is seriously deformed, or it may even fracture (break)—and avoiding such problems gives this field of **statics** great importance.

This chapter deals with forces within objects at rest

9–1 Statics—The Study of Forces in Equilibrium

All objects within our experience have at least one force acting on them (gravity), and if they are at rest then there must be other forces acting on them as well so that the net force is zero. An object at rest on a table, for example, has two forces acting on it, the downward force of gravity and the normal force the table exerts upward on it (Fig. 9–1). Since the net

FIGURE 9–1 The book is in equilibrium; the net force on it is zero.



force is zero, the upward force exerted by the table must be equal in nitude to the force of gravity acting downward. (Do not confuse these forces with the equal and opposite forces of Newton's third law which on different bodies; here both forces act on the same body.) Such a bo said to be in **equilibrium** (Latin for "equal forces" or "balance") under action of these two forces.

The subject of statics is concerned with the calculation of the fo acting on and within structures that are in equilibrium. Determinatio these forces, which occupies us in the first part of this chapter, then all a determination of whether the structures can sustain the forces wit significant deformation or fracture, subjects we discuss later in this ch ter. These techniques can be applied in a wide range of fields. Archit and engineers must be able to calculate the forces on the structural c ponents of buildings, bridges, machines, vehicles, and other structu since any material will break or buckle if too much force is app (Fig. 9–2). In the human body, a knowledge of the forces in muscles joints is of great value for medicine and physical therapy, and is also v able for the study of athletic activity.

FIGURE 9–2 Elevated walkway collapse in a Kansas City hotel in 1981. How a simple physics calculation could have prevented the tragic loss of 100 lives is considered in Example 9–14.

Let us take a simple Example of the addition of forces applied orthodonture.

PHYSICS APPLIED

Orthodonture

EXAMPLE 9–1 **Straightening teeth.** The wire band shown in Fig. 9 has a tension F_T of 2.0 N along it. It therefore exerts forces of 2.0 N the tooth (to which it is attached) in the two directions shown. Calcu the resultant force on the tooth due to the wire, F_W.

SOLUTION Since the two forces are equal, their sum will be direc along the line that bisects the angle between them, which we have beled the y axis. The x components of the two forces add up to zero. y component of each force is $(2.0\,\mathrm{N})(\cos 70°) = 0.68\,\mathrm{N}$; adding the together we get a total force F_W of 1.36 N as shown in Fig. 9–3b. N that if the wire is firmly attached to the tooth, the tension to the ri say, can be made larger than that to the left, and the resultant fo would correspondingly be directed more toward the right.

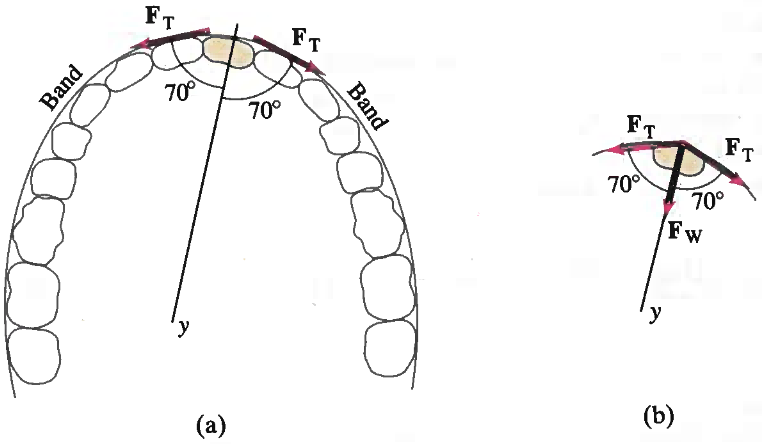


FIGURE 9–3 Forces on a tooth. Example 9–1.

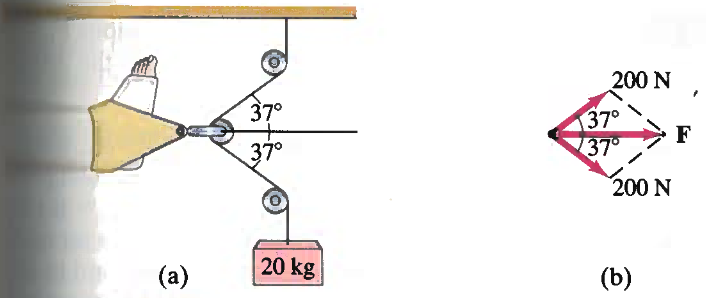


FIGURE 9–4 Traction apparatus exerts force on a leg.

XAMPLE 9–2 **Traction.** Calculate the force exerted on the leg by the traction apparatus shown in Fig. 9–4. Assume the pulleys are frictionless.

SOLUTION There is a tension of 20 kg × 9.8 m/s^2 = 200 N all along the cord. Thus there are two 200-N forces acting at 37° angles on the central pulley and on the leg, Fig. 9–4b. So the resultant force on the leg is $F = 2(200\,\text{N})\cos 37° = 320\,\text{N}$ acting to the right. (The leg is in equilibrium, so there must be another 320-N force acting on the leg to keep it at rest. What exerts this force?)

9–2 The Conditions for Equilibrium

For a body to be at rest, the sum of the forces acting on it must add up to zero. Since force is a vector, the components of the net force must each be zero. Hence, a condition for equilibrium is that

First condition for equilibrium: the sum of all forces is zero

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma F_z = 0. \tag{9–1}$$

We will mainly be dealing with forces that act in a plane, so we usually need only the x and y components. We must remember that if a particular force component points along the negative x or y axis, it must have a negative sign. Equation 9–1 is called the **first condition for equilibrium.**

XAMPLE 9–3 **Pull-ups on a scale.** A 90-kg weakling cannot do even one pull-up. By standing on a scale (Fig. 9–5), he can determine how close he gets. His best effort results in a scale reading of 23 kg. What force is he exerting?

SOLUTION There are three forces acting on our nonathlete, as shown in Fig. 9–5: gravity, $mg = (90\,\text{kg})(9.8\,\text{m/s}^2)$ downward, and two upward forces which are (1) the force the bar pulls upward on him, F_B (equal and opposite to the force he exerts on the bar), and (2) the force the scale exerts on his feet, F_S. At best, $F_S = (23\,\text{kg})(g)$. The person doesn't move, so the sum of these forces is zero:

$$\Sigma F_y = 0$$

$$F_B + F_S - mg = 0.$$

We solve for F_B:

$$F_B = mg - F_S$$

$$= (90\,\text{kg} - 23\,\text{kg})(g) = (67\,\text{kg})(9.8\,\text{m/s}^2) = 660\,\text{N}.$$

That is, he could lift himself if his mass were only 67 kg.

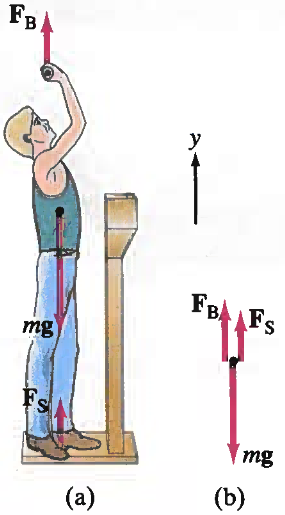


FIGURE 9–5 Example 9–3: (a) A person trying to do a pull-up while standing on a scale. (b) Simple free-body diagram.

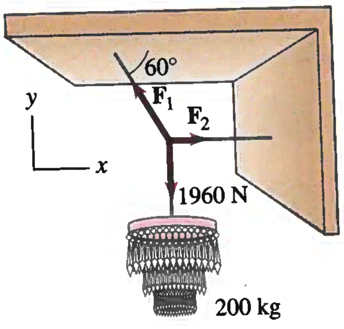


FIGURE 9–6 Example 9–4.

EXAMPLE 9–4 **Chandelier cord tension.** Calculate the tensions **F** **F**$_2$ in the two cords that are connected to the cord supporting the 20 chandelier in Fig. 9–6.

SOLUTION The three forces, **F**$_1$, **F**$_2$, and the weight of the 200-kg ch lier, act at the point where the three cords join. We choose this junction (it could be a knot) as the object for which we write $\Sigma F_x = 0$, $\Sigma F_y = 0$, don't bother considering the chandelier itself since only two forces act gravity downward and the equal but opposite force exerted upward by cord, both of which equal $mg = (200\,\text{kg})(9.8\,\text{m/s}^2) = 1960\,\text{N}$.) There are unknowns ($F_1$ and F_2)[†] and we can solve for them using Eqs. 9–1. We resolve **F**$_1$ into its horizontal (x) and vertical (y) components. Altho we don't know the value of F_1, we can write $F_{1x} = -F_1 \cos 60°$ $F_{1y} = F_1 \sin 60°$. **F**$_2$ has only an x component. In the vertical direction have only the weight of the chandelier $= (200\,\text{kg})(g)$ acting down and the vertical component of **F**$_1$ upward. Since $\Sigma F_y = 0$, we have

$$\Sigma F_y = F_1 \sin 60° - (200\,\text{kg})(g) = 0$$

so

$$F_1 = \frac{(200\,\text{kg})g}{\sin 60°} = \frac{(200\,\text{kg})g}{0.866} = (231\,\text{kg})g = 2260\,\text{N}.$$

In the horizontal direction,

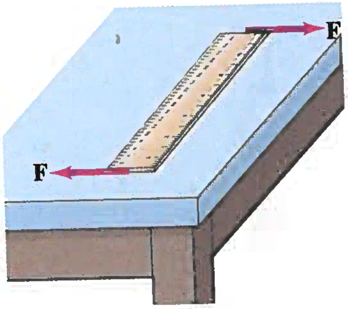
$$\Sigma F_x = F_2 - F_1 \cos 60° = 0.$$

Thus

$$F_2 = F_1 \cos 60° = (231\,\text{kg})(g)(0.500) = (115\,\text{kg})g = 1130$$

The magnitudes of **F**$_1$ and **F**$_2$ determine the strength of cord or wire must be used. In this case, the wire must be able to hold at least 23 Note in this example that we didn't insert the value of g, the acceler due to gravity, until the end. In this way we found the magnitude of force in terms of the number of kilograms (which may be a more fa iar quantity than newtons) times g.

FIGURE 9–7 Although the net force on it is zero, the ruler will move (rotate). A pair of equal forces acting in opposite directions but at different points on a body (as shown here) is referred to as a *couple*.



Second condition for equilibrium: the sum of all torques is zero

Although Eqs. 9–1 must be true if an object is to be in equilibr they are not a sufficient condition. Figure 9–7 shows an object on wh the net force is zero. Although the two forces labeled **F** add up to zero net force on the object, they do give rise to a net torque that will tate the object. Referring to Eq. 8–14, $\Sigma\tau = I\alpha$, we see that if a body remain at rest, the net torque applied to it (calculated about *any* must be zero. Thus we have the **second condition for equilibrium**: that sum of the torques acting on a body must be zero:

$$\Sigma\tau = 0. \qquad (9$$

This will assure that the angular acceleration, α, about any axis will be If the body is not rotating initially ($\omega = 0$), it will not start rotating. Eq tions 9–1 and 9–2 are the only requirements for a body to be in equilibr

We will consider cases in which the forces all act in a plane (we cal the xy plane). In this case the torque is calculated about an axis that is

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[†] The directions of **F**$_1$ and **F**$_2$ are known, since tension in a rope can only be along the ro any other direction would cause the rope to bend, as already pointed out in Chapter 4. our unknowns are the magnitudes F_1 and F_2.

perpendicular to the xy plane. *The choice of this axis is arbitrary.* If the body is at rest, then $\Sigma\tau = 0$ about any axis whatever. Therefore we can choose any axis that makes our calculation easier.

CONCEPTUAL EXAMPLE 9–5 **A lever.** The bar in Fig. 9–8 is being used as a lever to pry up a large rock. The small rock acts as a fulcrum. The force F_P required at the long end of the bar can be quite a bit smaller than the rock's weight mg, since it is the *torques* that balance in the rotation about the fulcrum. If, however, the leverage isn't quite good enough, and the rock isn't budged, what are two ways to increase the leverage?

RESPONSE One way is to increase the lever arm of the force F_P by slipping a pipe over the end of the bar and thereby pushing with a longer lever arm. A second, perhaps handier, way is to move the fulcrum closer to the large rock. This may change the long lever arm R only a little, but it changes the short lever arm r by a good fraction and therefore changes the ratio of R/r dramatically. In order to pry the rock, the torque due to F_P must at least balance the torque due to mg, so $mgr = F_P R$ and

$$\frac{r}{R} = \frac{F_P}{mg}.$$

With r smaller, the weight mg can be balanced with less force F_P.

PHYSICS APPLIED

The lever

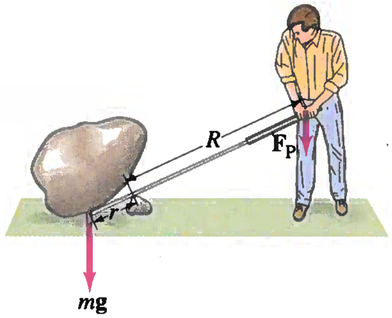


FIGURE 9–8 Example 9–5

9–3 Solving Statics Problems

The subject of statics is important because it allows us to calculate certain forces on (or within) a structure when some of the forces on it are already known. There is no single technique for attacking such statics problems, but the following procedure may be helpful:

PROBLEM SOLVING Statics

1. Choose one body at a time for consideration, and make a careful *free-body diagram* for it by showing all the forces acting on that body and the points at which these forces act.
2. Choose a convenient coordinate system, and resolve the forces into their components.
3. Using letters to represent unknowns, write down equations for
$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \text{and} \qquad \Sigma\tau = 0.$$
4. For the $\Sigma\tau = 0$ equation, choose any axis perpendicular to the xy plane that you like. (For example, you can reduce the number of unknowns in the resulting equation by choosing the axis so that one of the unknown forces passes through the axis; then this force will have zero lever arm and produce zero torque and so won't appear in the equation.) Pay careful attention to determining the lever arm for each force correctly. Give each torque a + or − sign. If torques that would tend to rotate the object counterclockwise are given a + sign, then torques that would tend to rotate it clockwise are negative.
5. Solve these equations for the unknowns. Three equations allow a maximum of three unknowns to be solved for; they can be forces, distances, even angles. (If an unknown force comes out negative in your solution, it means the direction you originally chose for that force is actually the opposite.)

Probably the hardest step is (1): *all* the forces *on* the body must be included, but the forces exerted *by* this body on other objects must *not* be included.

It is useful to refer back to Example 9–4 to see how this procedure followed. Notice that we considered the forces on the chandelier briefly (gravity downward and the rope pulling upward with an equal nitude force). We wanted to know the tension in the two cords, so we c the junction point (or knot) of the cords as our object. Since the obj essentially a point, there would be no torques, so the torque equation not used. Of course, you do not have to use all three equations if they not needed. In many situations we must make use of the torque equat as we shall soon see. Just as the components of a force are positive or ative depending on their direction, so too are torques. If a torque that t to rotate the object counterclockwise is considered positive, then a tor that tends to rotate it clockwise must be considered negative.

Alternate form of $\Sigma\tau = 0, \Sigma\mathbf{F} = 0$

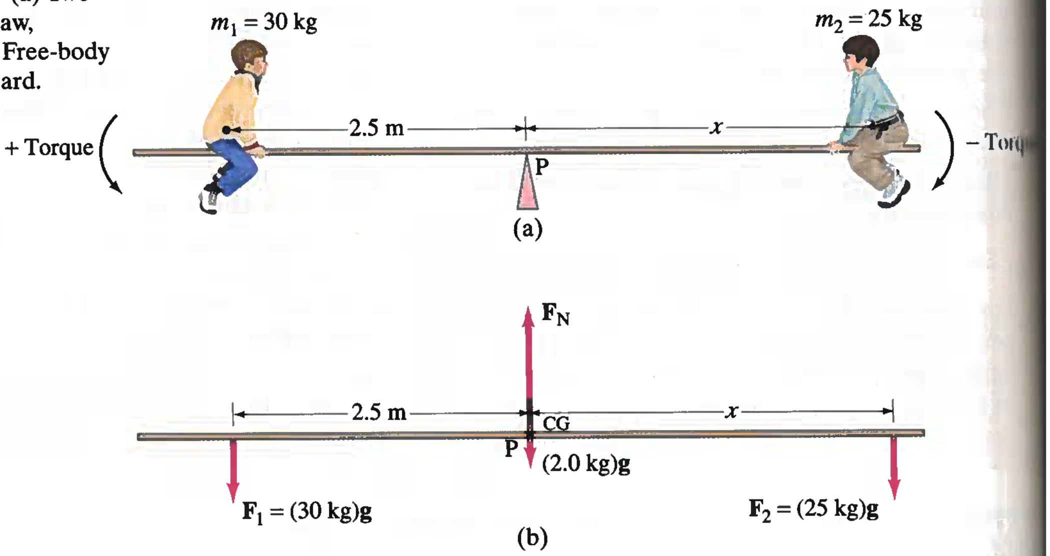
Another way of stating the torque equilibrium condition is that sum of all clockwise torques is equal to the sum of all counterclock torques. And for the forces, the sum of the upward forces is equal to sum of the downward forces, and the sum of horizontal forces to the le equal to the sum of horizontal forces to the right.

One of the forces that acts on bodies is the force of gravity. Our an sis in this chapter is greatly simplified if we use the concept of cent gravity (CG) or center of mass (CM), which for practical purposes are same point. As we discussed in Section 7–8, we can consider the force gravity on the body as a whole as acting at its CG. For uniform symme cally shaped bodies, the CG is at the geometric center. For more compli ed bodies, the CG can be determined as discussed in Section 7–8.

EXAMPLE 9–6 **Balancing a seesaw.** A 2.0-kg board serves as a sec for two children, as shown in Fig. 9–9a. One child has a mass of 30 kg sits 2.5 m from the pivot point (i.e., his CG is 2.5 m from the pivot). At w distance, x, from the pivot must a 25-kg child place herself to balance seesaw? Assume the board is uniform and centered over the pivot.

SOLUTION The free-body diagram for the board is as shown Fig. 9–9b. The forces acting on the board are the forces exerted dow ward on it by each child, $\mathbf{F}_1$ and $\mathbf{F}_2$, the upward force exerted by

FIGURE 9–9 (a) Two children on a seesaw, Example 9–6. (b) Free-body diagram of the board.



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pivot, $\mathbf{F}_N$, and the force of gravity (its weight), which acts at the center of the uniform board. Let us calculate the torque about the pivot point, P: the lever arms for F_N and for the weight of the board are then zero, and they will not appear in the torque equation. Thus the torque equation will involve only the forces $\mathbf{F}_1$ and $\mathbf{F}_2$, which are equal to the weights of the children. The torque exerted by each child will be mg times the appropriate lever arm, which here is the distance of each child from the pivot point. Hence the torque equation is

$$\Sigma\tau = (30\text{ kg})(g)(2.5\text{ m}) - (25\text{ kg})(g)(x) = 0,$$

where a torque tending to rotate the board counterclockwise has been chosen to be a positive torque, and a torque tending to rotate it clockwise to be negative. The acceleration of gravity, g, appears in both terms and cancels out. We solve for x and find:

$$x = \frac{(30\text{ kg})(2.5\text{ m})}{25\text{ kg}} = 3.0\text{ m}.$$

To balance the seesaw, the second child must sit so that her CG is 3.0 m from the pivot. This makes sense: since she is lighter, she must sit farther from the pivot.

EXAMPLE 9–7 **Forces on a beam and supports.** A uniform 1500-kg beam, 20.0 m long, supports a 15,000-kg printing press 5.0 m from the right support column (Fig. 9–10). Calculate the force on each of the vertical support columns.

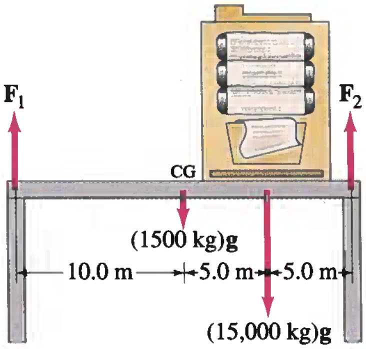


FIGURE 9–10 A 1500-kg beam supports a 15,000-kg machine. Example 9–7.

SOLUTION We analyze the forces on the beam, since the force the beam exerts on the columns is equal and opposite to the forces exerted by the columns on the beam. We call the latter $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 9–10. The weight of the beam itself acts at its center of gravity, 10.0 m from either end. Since it doesn't matter which point we choose as the axis for writing the torque equation, we can choose one that is convenient. If we calculate the torques about the point of application of $\mathbf{F}_1$, then $\mathbf{F}_1$ will not enter the equation (its lever arm will be zero) and we will have an equation in only one unknown, F_2. We choose the counterclockwise direction as positive, and $\Sigma\tau = 0$ gives

$$\Sigma\tau = -(10.0\text{ m})(1500\text{ kg})g - (15.0\text{ m})(15{,}000\text{ kg})g + (20.0\text{ m})F_2 = 0.$$

Solving for F_2 we find $F_2 = (12{,}000\text{ kg})g = 118{,}000\text{ N}$. To find F_1, we use $\Sigma F_y = 0$:

$$\Sigma F_y = F_1 - (1500\text{ kg})g - (15{,}000\text{ kg})g + F_2 = 0.$$

Putting in $F_2 = (12{,}000\text{ kg})g$ we find that $F_1 = (4500\text{ kg})g = 44{,}100\text{ N}$.

To confirm that the results don't depend on where the axis is chosen, let us choose a different axis, say, at the other end of the beam, where F_2 acts. In this case, the torque equation is

$$\Sigma\tau = -(20.0\text{ m})F_1 + (10.0\text{ m})(1500\text{ kg})g + (5.0\text{ m})(15{,}000\text{ kg})g = 0.$$

We solve this for F_1 and find $F_1 = (4500\text{ kg})g$, just as before. The ΣF_y equation is the same as before and gives $F_2 = (12{,}000\text{ kg})g$. So our results are the same, no matter which axis we choose to calculate the torques about.

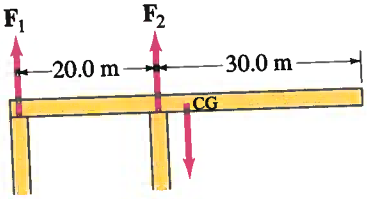


FIGURE 9–11 A cantilever.

Figure 9–11 shows a beam that extends beyond its support like a di board. Such a beam is called a *cantilever*. The forces acting on the bea this figure are those due to the supports, $\mathbf{F}_1$ and $\mathbf{F}_2$, and the force of gra which acts at the CG, 5.0 m to the right of the right-hand support. If you low the procedure of the last Example to calculate F_1 and F_2, assuming t point upward as shown in the figure, you will find that F_1 comes out nega If the beam has a mass of 1200 kg, then $F_2 = 15{,}000\,\mathrm{N}$ and $F_1 = -300$ (see Problem 15). Whenever an unknown force comes out negative, it m ly means that the force actually points in the opposite direction from wh you assumed. Thus in Fig. 9–11, $\mathbf{F}_1$ actually points downward. With a little flection it should become obvious that the left-hand support must ind pull downward on the beam (by means of bolts, screws, fasteners and/ glue) if the beam is to be in equilibrium; otherwise the sum of the torq about the CG (or about the point where $\mathbf{F}_2$ acts) could not be zero.

PROBLEM SOLVING

If a force comes out negative

Our next Example involves a beam attached to a wall by a hinge a supported by a guy wire (Fig. 9–12). It is important to remember th flexible cord or wire can support a force only along its length. (If th were a component of force perpendicular to the cord or wire, it wo bend because it is flexible.) For a rigid device, such as the hinge Fig. 9–12, the force can be in any direction and we can know the directi only after solving the problem.

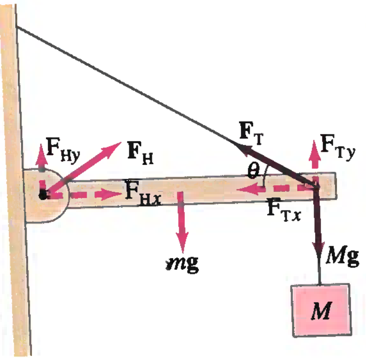


FIGURE 9–12 Example 9–8.

EXAMPLE 9–8 **Beam and wire.** A uniform beam, 2.20 m long with m $m = 25.0\,\mathrm{kg}$, is mounted by a hinge on a wall as shown in Fig. 9–12. T beam is held in a horizontal position by a wire that makes an angle $\theta = 30$ as shown. The beam supports a mass $M = 280\,\mathrm{kg}$ suspended from its e Determine the components of the force $\mathbf{F}_{\mathrm{H}}$ that the hinge exerts on t beam, and the components of the tension $\mathbf{F}_{\mathrm{T}}$ in the supporting wire.

SOLUTION Figure 9–12 is the free-body diagram for the beam, sho ing all the forces acting on the beam; it also shows the components of and F_{T}. We have three unknowns, $F_{\mathrm{H}x}$, $F_{\mathrm{H}y}$, and F_{T} (we are given θ), we will need all three equations, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma\tau = 0$. The sum the forces in the vertical (y) direction is

$$\Sigma F_y = F_{\mathrm{H}y} + F_{\mathrm{T}y} - mg - Mg = 0. \quad (i$$

In the horizontal (x) direction, the sum of the forces is

$$\Sigma F_x = F_{\mathrm{H}x} - F_{\mathrm{T}x} = 0. \quad (ii$$

For the torque equation, we choose the axis at the point where $\mathbf{F}_{\mathrm{T}}$ a Mg act (so our equation then contains only one unknown, $F_{\mathrm{H}y}$, and we c solve it more quickly); we choose torques that tend to rotate the bea counterclockwise as positive. The weight mg of the (uniform) beam ac at its center, so we have:

$$\Sigma\tau = -(F_{\mathrm{H}y})(2.20\,\mathrm{m}) + (mg)(1.10\,\mathrm{m}) = 0$$

or

$$F_{\mathrm{H}y} = \tfrac{1}{2}mg = \tfrac{1}{2}(25.0\,\mathrm{kg})(9.80\,\mathrm{m/s}) = 123\,\mathrm{N}.$$

Next, since the tension $\mathbf{F}_{\mathrm{T}}$ in the wire acts along the wire ($\theta = 30°$),

$$F_{\mathrm{T}y} = F_{\mathrm{T}x}\tan\theta = 0.577\,F_{\mathrm{T}x}. \quad (ii$$

From Eqs. i, ii, and iii we get

$$F_{Ty} = (m + M)g - F_{Hy} = (305\text{ kg})(9.80\text{ m/s}^2) - 123\text{ N} = 2870\text{ N}$$

$$F_{Tx} = F_{Ty}/0.577 = 4970\text{ N}$$

$$F_{Hx} = F_{Tx} = 4970\text{ N}.$$

The components of $\mathbf{F}_H$ are $F_{Hy} = 123\text{ N}$ and $F_{Hx} = 4970\text{ N}$. The tension in the wire is $F_T = \sqrt{F_{Tx}^2 + F_{Ty}^2} = 5740\text{ N}$.

EXAMPLE 9–9 **Ladder.** A 5.0-m-long ladder leans against a wall at a point 4.0 m above the ground as shown in Fig. 9–13. The ladder is uniform and has mass 12.0 kg. Assuming the wall is frictionless (but the ground is not), determine the forces exerted on the ladder by the ground and the wall.

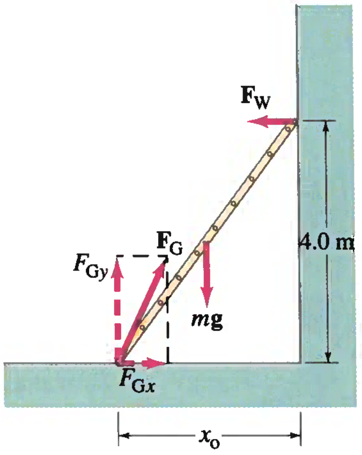


FIGURE 9–13 A ladder leaning against a wall. Example 9–9.

SOLUTION Figure 9–13 shows the free-body diagram for the ladder, showing all the forces acting on the ladder. The wall, since it is frictionless, can exert a force only perpendicular to the wall, and we label that force $\mathbf{F}_W$. The ground, however, can exert both horizontal, F_{Gx}, and vertical, F_{Gy}, force components, the former being frictional and the latter the normal force. Finally, gravity exerts a force $mg = (12.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$ on the ladder at its midpoint, since the ladder is uniform. The y component of the force equation is

$$\Sigma F_y = F_{Gy} - mg = 0,$$

so immediately we have $F_{Gy} = mg = 118\text{ N}$. The x component of the force equation is

$$\Sigma F_x = F_{Gx} - F_W = 0.$$

To determine F_{Gx} and F_W, which are both unknowns, we need another equation, namely a torque equation, which we calculate about the point where the ladder touches the ground. This point is a distance $x_0 = \sqrt{(5.0\text{ m})^2 - (4.0\text{ m})^2} = 3.0\text{ m}$ from the wall. The lever arm for mg is half this, or 1.5 m, and the lever arm for F_W is 4.0 m. Since $\mathbf{F}_G$ acts at the axis, its lever arm is zero and so doesn't enter the equation (we planned it like that), and we get

$$\Sigma\tau = (4.0\text{ m})F_W - (1.5\text{ m})mg = 0.$$

Thus

$$F_W = \frac{(1.5\text{ m})(12.0\text{ kg})(9.8\text{ m/s}^2)}{4.0\text{ m}} = 44\text{ N}.$$

Then, from the x component of the force equation,

$$F_{Gx} = F_W = 44\text{ N}.$$

Since the components of $\mathbf{F}_G$ are $F_{Gx} = 44\text{ N}$ and $F_{Gy} = 118\text{ N}$, then

$$F_G = \sqrt{(44\text{ N})^2 + (118\text{ N})^2} = 126\text{ N} \approx 130\text{ N}$$

(rounded off to two significant figures) and it acts at an angle

$$\theta = \tan^{-1}(118\text{ N}/44\text{ N}) = 70°$$

to the ground. Note the force $\mathbf{F}_G$ does *not* have to act along the ladder's direction because the ladder is rigid and not flexible like a cord or cable.

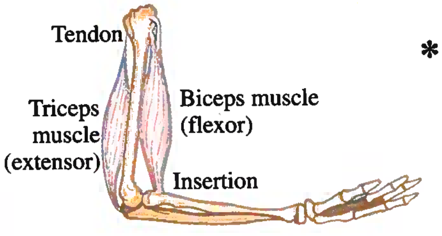


FIGURE 9–14 Diagram showing the biceps (flexor) and triceps (extensor) muscles in the human arm.

PHYSICS APPLIED

Forces in muscles and joints

* 9–4 Applications to Muscles and Joints

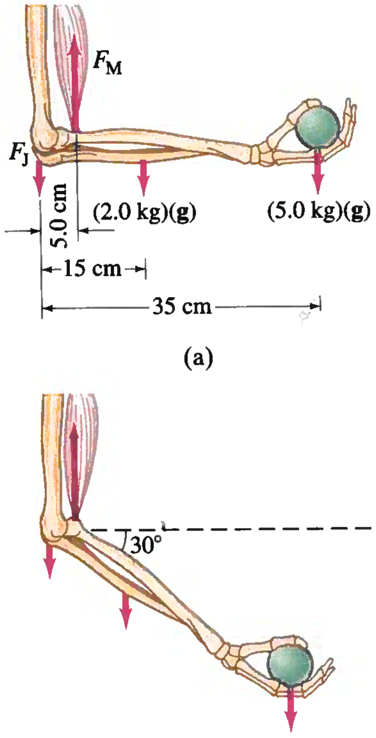
The techniques we have been discussing for calculating forces on bodie equilibrium can readily be applied to the human (or animal) body. Thi of great use in studying the forces on muscles, bones, and joints for org isms in motion or at rest. Generally a muscle is attached, via tendons, two different bones (see Fig. 9–14). The points of attachment are called *sertions*. The two bones are flexibly connected at a *joint*, such as those the elbow, knee, and hip. A muscle exerts a pull when its fibers contr under stimulation by a nerve, but it cannot exert a push. Muscles that te to bring two limbs closer together, such as the biceps muscle in the upp arm (Fig. 9–14) are called *flexor* muscles; those that act to extend a lim outward, such as the triceps, are called *extensor* muscles. The flexor mus in the upper arm is used when lifting an object in your hand; the exten muscle is used when throwing a ball.

EXAMPLE 9–10 Force exerted by biceps muscle. How much fo must the biceps muscle exert when a 5.0-kg mass is held in the ha (*a*) with the arm horizontal as in Fig. 9–15a, and (*b*) when the arm is at 30° angle as in Fig. 9–15b? Assume that the mass of forearm and han together is 2.0 kg and their CG is as shown.

SOLUTION (*a*) The forces acting on the forearm are shown Fig. 9–15a and include the upward force F_M exerted by the muscle and force F_J exerted at the joint by the bone in the upper arm (both assum to act vertically). We wish to find F_M, which is done most easily by usi the torque equation and by choosing our axis through the joint so that does not enter:

$$\Sigma\tau = (0.050\,\mathrm{m})(F_M) - (0.15\,\mathrm{m})(2.0\,\mathrm{kg})(g) - (0.35\,\mathrm{m})(5.0\,\mathrm{kg})(g)$$

We solve this for F_M and find $F_M = (41\,\mathrm{kg})(g) = 400\,\mathrm{N}$.
(*b*) The lever arm, as calculated about the joint, is reduced by the fact cos 30° for all three forces. So our torque equation will look like the o just above, except that each term will have a "cos 30°." The latter will ca cel out so the same result will be obtained, $F_M = 400\,\mathrm{N}$.



(b)

FIGURE 9–15 Example 9–10.

PHYSICS APPLIED

Muscle insertion and lever arm

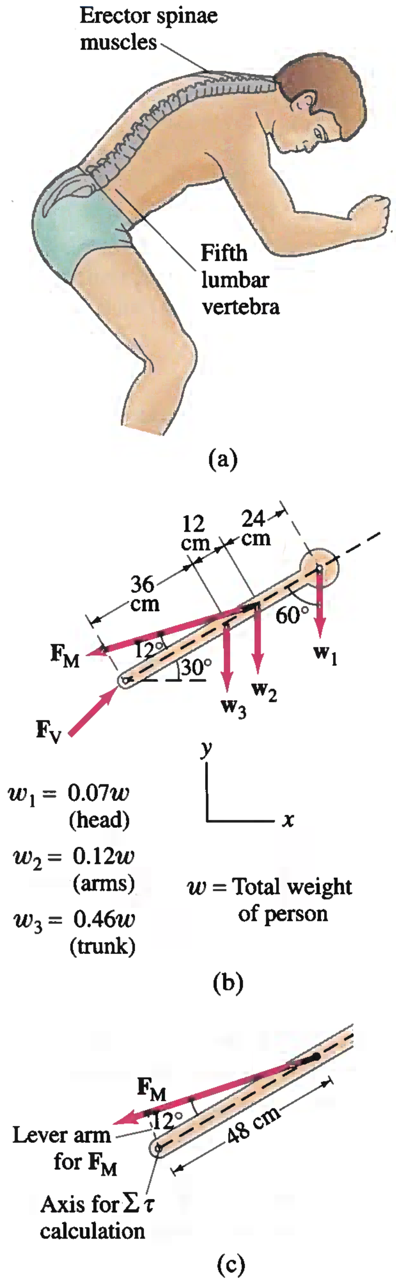
Note in this Example that the force required of the muscle (400 N) quite large compared to the weight of the object lifted (49 N). Indeed, t muscles and joints of the body are generally subjected to quite large forc

The point of insertion of a muscle varies from person to person. slight increase in the point of insertion of the biceps muscle from 5.0 cm 5.5 cm can be a considerable advantage for lifting and other exertions. deed, champion athletes are often found to have muscle insertions farth from the joint than the average person, and if this applies to one muscle usually applies to all.

As another example of the large forces acting within the human bod we consider the muscles used to support the trunk when a person ben forward (Fig. 9–16a). The lowest vertebra on the spinal column (fifth lumbar vertebra) acts as a fulcrum for this bending position. The "erector spinae" muscles in the back that support the trunk act at an effective angle of about 12° to the axis of the spine. Figure 9–16b is a simplified schematic drawing showing the forces on the upper body. We assume the trunk makes an angle of 30° with the horizontal. The force exerted by the back muscles is represented by $\mathbf{F}_M$, the force exerted on the base of the spine at the lowest vertebra is $\mathbf{F}_V$, and $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$ represent the weights of the head, freely hanging arms, and trunk, respectively. The values shown are approximations taken from Table 7–1. The distances (in cm) refer to a person 180 cm tall, but are approximately in the same ratio of 1:2:3 for an average person of any height, and the result in the following Example is then independent of the height of the person.

PHYSICS APPLIED

Forces on the spine, and back pain



(a)

(b)

(c)

FIGURE 9–16 (a) A person bending over. (b) Forces on the back exerted by the back muscles ($\mathbf{F}_M$) and by the vertebrae ($\mathbf{F}_V$) when a person bends over.

EXAMPLE 9–11 **Forces on your back.** Calculate the magnitude and direction of the force $\mathbf{F}_V$ acting on the fifth lumbar vertebra for the example shown in Fig. 9–16b.

SOLUTION First we calculate F_M using the torque equation, taking the axis at the base of the spine. To figure the lever arms, we need to use trigonometric functions. For $\mathbf{F}_M$, the lever arm (perpendicular distance from axis to line of action of the force) will be the real distance to where the force acts (48 cm) times sin 12° as shown in Fig. 9–16c. The lever arms for $\mathbf{w}_1$, $\mathbf{w}_2$, and $\mathbf{w}_3$ can be seen from Fig. 9–16b to be their respective distances times sin 60°. Thus $\Sigma\tau = 0$ gives

$$(0.48\text{ m})(\sin 12^\circ)(F_M) - (0.72\text{ m})(\sin 60^\circ)(w_1)$$
$$- (0.48\text{ m})(\sin 60^\circ)(w_2) - (0.36\text{ m})(\sin 60^\circ)(w_3) = 0,$$

where we chose the positive sign for counterclockwise torque. Putting in the values for w_1, w_2, w_3 given in the figure, we find $F_M = 2.2w$, where w is the total weight of the body. To get the components of $\mathbf{F}_V$ we use the x and y components of the force equation (noting that $30^\circ - 12^\circ = 18^\circ$):

$$\Sigma F_y = F_{Vy} - F_M \sin 18^\circ - w_1 - w_2 - w_3 = 0$$
$$F_{Vy} = 1.3w$$
$$\Sigma F_x = F_{Vx} - F_M \cos 18^\circ = 0$$
$$F_{Vx} = 2.1w.$$

Then

$$F_V = \sqrt{F_{Vx}^2 + F_{Vy}^2} = 2.5w.$$

The angle θ that F_V makes with the horizontal is given by $\tan\theta = F_{Vy}/F_{Vx} = 0.62$, so $\theta = 32^\circ$.

The force on the lowest vertebra is thus $2\frac{1}{2}$ times the body weight! This force is transmitted from the "sacral" bone at the base of the spine, through the fluid-filled and somewhat flexible *intervertebral disc*. The discs at the base of the spine are clearly being compressed under very large forces the body was less bent over (say the 30° angle in Fig. 9–16b becomes 60 70°) then the stress on the lower back will be less (see Problem 40).]

If the person in Fig. 9–16 has a mass of 90 kg, and is holding 20 kg in hands (this increases w_2 to $0.33w$), then F_V is increased to nearly five ti the person's weight ($5w$)! (For this 200-lb person, the force on the would be 1000 lb!) With such strong forces acting, it is little wonder that many people suffer from low back pain at one time or another in their li

* 9–5 Stability and Balance

Stable and unstable equilibrium

A body in static equilibrium, if left undisturbed, will undergo no tran tional or rotational acceleration since the sum of all the forces and the of all the torques acting on it are zero. However, if the object is displa slightly, three different outcomes are possible: (1) the object returns to original position, in which case it is said to be in **stable equilibrium**; (2) t object moves even farther from its original position, in which case it is to be in **unstable equilibrium**; or (3) the object remains in its new positi in which case it is said to be in **neutral equilibrium**.

Consider the following examples. A ball suspended freely from a str is in stable equilibrium, for if it is displaced to one side, it will quick return to its original position (Fig. 9–17a). On the other hand, a pen standing on its point is in unstable equilibrium. If its CG is directly over tip (Fig. 9–17b), the net force and net torque on it will be zero. But if it displaced ever so slightly—say by a slight vibration or tiny air current there will be a torque on it, and it will continue to fall in the direction the original displacement. Finally, an example of an object in neutral equ librium is a sphere resting on a horizontal tabletop. If it is placed sligh to one side, it will remain in its new position.

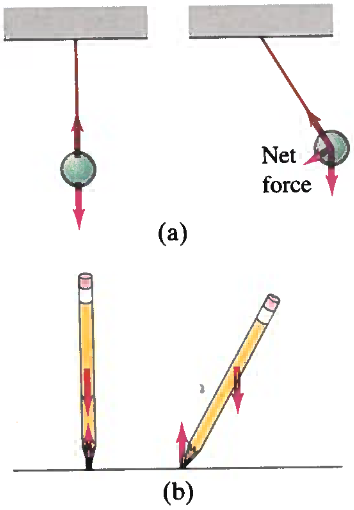
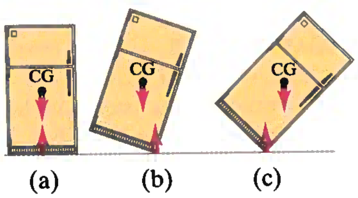


FIGURE 9–17 (a) Stable equilibrium, and (b) unstable equilibrium.

FIGURE 9–18 Equilibrium of a refrigerator resting on a surface.



In most situations, such as in the design of structures and in worki with the human body, we are interested in maintaining stable equilibri or *balance*, as we sometimes say. In general, an object whose CG is bel its point of support, such as a ball on a string, will be in stable equilib um. If the CG is above the base of support, we have a more complica situation. Consider a standing refrigerator (Fig. 9–18a). If it is tipp slightly, it will return to its original position due to the torque on it shown in Fig. 9–18b. But if it is tipped too far, Fig. 9–18c, it will fall ov The critical point is reached when the CG is no longer above the base support. In general, *a body whose CG is above its base of support will stable if a vertical line projected downward from the CG falls within base of support*. This is because the normal force upward on the obj (which balances out gravity) can be exerted only within the area of co tact, so that if the force of gravity acts beyond this area, a net torque w act to topple the object. Stability, then, can be relative. A brick lying on widest face is more stable than a brick standing on its end, for it will ta more of an effort to tip it over. In the extreme case of the pencil Fig. 9–17b, the base is practically a point and the slightest disturbance w topple it. In general, the larger the base and the lower the CG, the mo stable the object.

In this sense, humans are much less stable than four-legged mammals, which not only have a larger base of support because of their four legs, but also have a lower center of gravity. The human species has had to develop special apparatus, such as certain very strong muscles, in order to deal with the problem of keeping a person upright and at the same time stable. Because of their upright position, humans suffer from numerous ailments such as low back pain due to the large forces involved, as we saw in Example 9–11. When walking and performing other kinds of movement, a person continually shifts the body so that its CG is over the feet, although in the normal adult this requires no conscious thought. Even as simple a movement as bending over requires moving the hips backward so that the CG remains over the feet, and this repositioning is done without thinking about it. To see this, position yourself with your heels and back to a wall and try to touch your toes. You won't be able to do it without falling. Persons carrying heavy loads automatically adjust their posture so that the CG of the total mass is over their feet, Fig. 9–19.

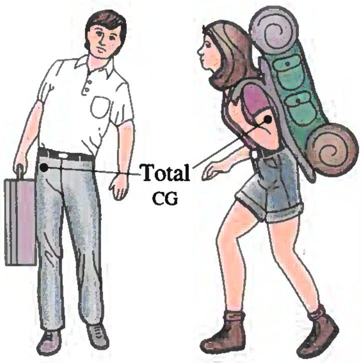


FIGURE 9–19 Humans adjust their posture to achieve stability when carrying loads.

9–6 Elasticity; Stress and Strain

In the first part of this chapter we studied how to calculate the forces on objects in equilibrium. In this Section we study the effects of these forces: any object changes shape under the action of applied forces. In Section 9–7 we will see that if the forces are great enough, the object will break or *fracture*.

If a force is exerted on an object, such as the vertically suspended metal rod shown in Fig. 9–20, the length of the object changes. If the amount of elongation, ΔL, is small compared to the length of the object, experiment shows that ΔL is proportional to the weight or force exerted on the object. This proportionality, as we saw in Section 6–4, can be written as an equation:

$$F = k\,\Delta L. \tag{9–3}$$

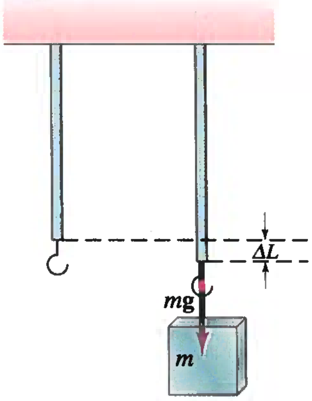
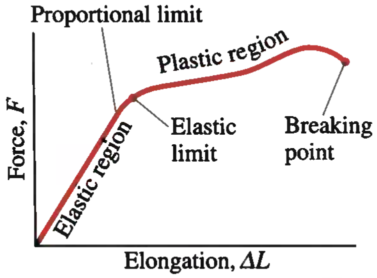


FIGURE 9–20 Hooke's law: $\Delta L \propto$ applied force.

Hooke's law (again)

Here F represents the force (or weight) pulling on the object, ΔL is the change in length, and k is a proportionality constant. Equation 9–3, which is sometimes called **Hooke's law**[†], after Robert Hooke (1635–1703) who first noted it, is found to be valid for almost any solid material from iron to bone, but it is valid only up to a point. For if the force is too great, the object stretches excessively and eventually breaks. Figure 9–21 shows a typical graph of elongation versus applied force. Up to a point called the **proportional limit**, Eq. 9–3 is a good approximation for many common materials, and the curve is a straight line. Beyond this point, the graph deviates from a straight line, and no simple relationship exists between F and ΔL. Nonetheless, up to a point farther along the curve called the **elastic limit**, the object will return to its original length if the applied force is removed. The region from the origin to the elastic limit is called the *elastic region*. If the object is stretched beyond the elastic limit, it enters the *plastic region*: it does not return to the original length upon removal of the external force, but remains permanently deformed (like bending a paper clip). The maxi-

FIGURE 9–21 Applied force vs. elongation for a typical metal under tension.



[†] The term "law" applied to this relation is not really appropriate, since first of all, it is only an approximation, and secondly, it refers only to a limited set of phenomena. Most physicists prefer to reserve the word "law" for those relations that are deeper and more encompassing and precise, such as Newton's laws of motion or the law of conservation of energy.

mum elongation is reached at the *breaking point*. The maximum force th can be applied without breaking is called the **ultimate strength** of the ma rial (actually force per unit area as discussed in Section 9–7).

The amount of elongation of an object, such as the rod shown i Fig. 9–20, depends not only on the force applied to it, but also on the ma rial from which it is made and on its dimensions. That is, the constant k i Eq. 9–3 can be written in terms of these factors. If we compare rods made the same material but of different lengths and cross-sectional areas, it i found that for the same applied force, the amount of stretch (again assum small compared to the total length) is proportional to the original leng and inversely proportional to the cross-sectional area. That is, the longer th object, the more it elongates for a given force; and the thicker it is, the less elongates. These findings can be combined with Eq. 9–3 to yield

$$\Delta L = \frac{1}{E}\frac{F}{A}L_0, \qquad (9\text{–}4$$

Young's modulus

where L_0 is the original length of the object, A is the cross-sectional are and ΔL is the change in length due to the applied force F. E is a const of proportionality[†] known as the **elastic modulus**, or **Young's modulus**, an its value depends only on the material. The value of Young's modulus fo various materials is given in Table 9–1 (note: shear and bulk modulus i

TABLE 9–1 Elastic Moduli

Material	Elastic Modulus, E (N/m²)	Shear Modulus, G (N/m²)	Bulk Modul B (N/m²)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine)			
(parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	5×10^9		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases[†]			
Air, H_2, He, CO_2			1.01×10^5

[†] At normal atmospheric pressure; no variation in temperature during process.

[†] The fact that E is in the denominator, so that $1/E$ is the actual proportionality constant, is merely a convention. When we rewrite Eq. 9–3 to get Eq. 9–5, E is found in the numerator.

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80×10^9
70×10^9
70×10^9
45×10^9
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1.0×10^9
2.5×10^9
1.01×10^5

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this Table are discussed later in this section). Because E is a property only of the material and is independent of the object's size or shape, Eq. 9–4 is far more useful for practical calculation than Eq. 9–3.

From Eq. 9–4, we see that the change in length of an object is directly proportional to the product of the object's length L_0 and the force per unit area F/A applied to it. It is general practice to define the force per unit area as the **stress**:

Stress

$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A},$$

which has units of N/m^2. Also, the **strain** is defined to be the ratio of the change in length to the original length:

Strain

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta L}{L_0},$$

and is dimensionless (no units). Strain is thus the fractional change in length of the object, and is a measure of how much the bar has been deformed. Stress is applied to the material by external agents, whereas strain is the material's response to the stress. Equation 9–4 can be rewritten as

$$\frac{F}{A} = E\frac{\Delta L}{L_0} \tag{9–5}$$

or

Young's modulus (again)

$$E = \frac{F/A}{\Delta L/L_0} = \frac{\text{stress}}{\text{strain}}.$$

Thus we see that the strain is directly proportional to the stress, in the linear (elastic) region of Fig. 9–21.

EXAMPLE 9–12 **Tension in piano wire.** A 1.60-m-long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.30 cm when tightened?

SOLUTION We solve for F in Eq. 9–5 and note that the area $A = \pi r^2 = (3.14)(0.0010\,\text{m})^2 = 3.1 \times 10^{-6}\,\text{m}^2$. Then

$$\begin{aligned} F &= E\frac{\Delta L}{L_0}A \\ &= (2.0 \times 10^{11}\,\text{N/m}^2)\left(\frac{0.0030\,\text{m}}{1.60\,\text{m}}\right)(3.1 \times 10^{-6}\,\text{m}^2) = 1200\,\text{N}, \end{aligned}$$

where we obtained the value for E from Table 9–1. The strong tension in all the wires in a piano must be supported by a strong frame.

Tension

The rod shown in Fig. 9–20 is said to be under *tension* or **tensile stress**. For not only is there a force pulling down on the rod at its lower end, but since the rod is in equilibrium we know that the support at the top is exerting an equal[†] upward force on the rod at its upper end,

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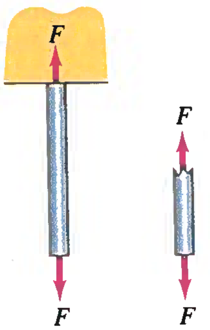
[†] If we ignore the weight of the bar.



(a) (b)

FIGURE 9–22 Stress exists *within* the material.

FIGURE 9–23 This Greek temple, in Agrigento, Sicily, built 2500 years ago, shows th post-and-beam construction.

Fig. 9–22a. In fact, this tensile stress exists throughout the material. (Co sider, for example, the lower half of a suspended rod as shown Fig. 9–22b. This lower half is in equilibrium, so there must be an upw force on it to balance the downward force at its lower end. What exe this upward force? It must be the upper part of the rod. Thus we see th external forces applied to an object give rise to internal forces, or str within the material itself. (Recall also the discussion of tension in a co page 92.)

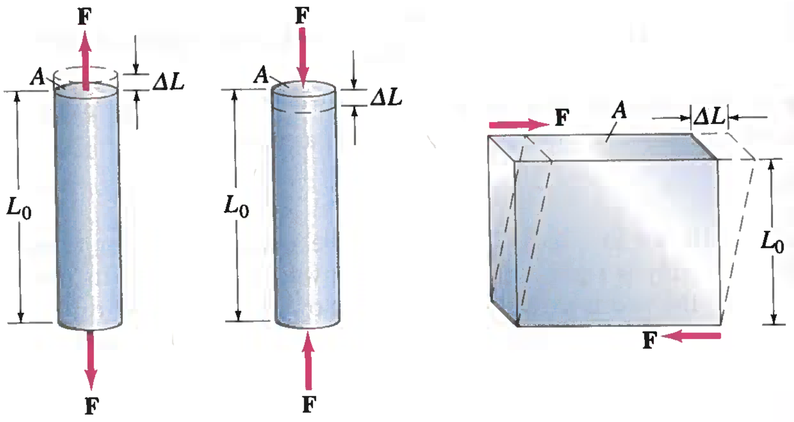
Strain or deformation due to tensile stress is but one type of stress which materials can be subjected. There are two other common types stress: compressive and shear. **Compressive stress** is the exact opposite tensile stress. Instead of being stretched, the material is compressed: th forces act inwardly on the body. Columns that support a weight, such the columns of a Greek temple (Fig. 9–23), or those that support the be in Fig. 9–10, are subjected to compressive stress. Equations 9–4 and 9 apply equally well to compression and tension, and the values for the el tic modulus E are usually the same.

Compression

Figure 9–24 compares tensile and compressive stresses as well as th third type, shear stress. An object under **shear stress** has equal and opp site forces applied *across* its opposite faces. An example is a book or bri firmly attached to a tabletop, on which a force is exerted parallel to th

Shear

FIGURE 9–24 The three types of stress for rigid bodies.



Tension (a) Compression (b) Shear (c)

top surface. The table exerts an equal and opposite force along the bottom surface. Although the dimensions of the object do not change significantly, the shape of the object does change as shown in the figure. An equation similar to Eq. 9–4 can be applied to calculate shear strain:

$$\Delta L = \frac{1}{G}\frac{F}{A}L_0, \qquad \textbf{(9–6)}$$

but ΔL, L_0, and A must be reinterpreted as indicated in Fig. 9–24c. Note that A is the area of the surface *parallel* to the applied force (and not perpendicular as for tension and compression), and ΔL is *perpendicular* to L_0. The constant of proportionality, G, is called the **shear modulus** and is generally one half to one third the value of the elastic modulus, E (see Table 9–1). Figure 9–25 illustrates why $\Delta L \propto L_0$: the fatter book shifts more for the same shearing force.

Shear modulus

The rectangular object undergoing shear in Fig. 9–24c would not actually be in equilibrium under the forces shown, for a net torque would exist. If the object is in fact in equilibrium, there must be two more forces acting on it which balance out this torque. One acts vertically upward on the right, and the other acts vertically downward on the left, as shown in Fig. 9–26. This is generally true of shear forces. If the object is a brick or book lying on a table, these two additional forces can be exerted by the table and by whatever exerts the other horizontal force (such as a hand pushing across the top of a book).

If an object is subjected to inward forces from all sides, its volume will decrease. A common situation is a body submerged in a fluid; for in this case, the fluid exerts a pressure on the object in all directions, as we shall see in Chapter 10. Pressure is defined as force per area, and thus is the equivalent of stress. For this situation the change in volume, ΔV, is found to be proportional to the original volume, V_0, and to the increase in the pressure, ΔP. We thus obtain a relation of the same form as Eq. 9–4 but with a proportionality constant called the **bulk modulus**, B:

$$\frac{\Delta V}{V_0} = -\frac{1}{B}\Delta P \qquad \textbf{(9–7)}$$

or

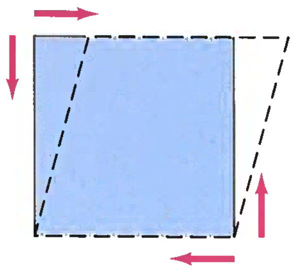
$$B = -\frac{\Delta P}{\Delta V/V_0}.$$

The minus sign is included to indicate that the volume *decreases* with an

FIGURE 9–25 The fatter book (a) shifts more than the thinner book (b) with the same applied shear force.

(a) (b)

FIGURE 9–26 Balance of forces and torques for shear stress.



increase in pressure. Values for the bulk modulus are given in Table 9–1. Since liquids and gases do not have a fixed shape, only the bulk modulus applies to them. ,

9–7 Fracture

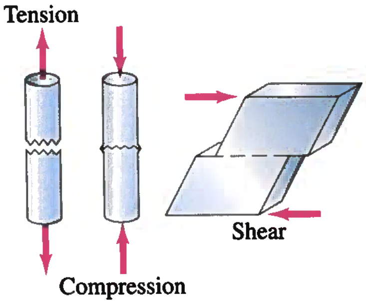


FIGURE 9–27 Fracture as a result of the three types of stress.

If the stress on a solid object is too great, the object fractures or breaks (Fig. 9–27). Table 9–2 lists the ultimate tensile strength, compressive strength, and shear strength for a variety of materials. These values give the maximum force per unit area that an object can withstand under each of these three types of stress. They are, however, representative values only, and the actual value for a given specimen can differ considerably. It is therefore necessary to maintain a "safety factor" of from 3 to perhaps 10 or more—that is, the actual stresses on a structure should not exceed one tenth to one third of the values given in the table. You may encounter tables of the "allowable stresses" in which appropriate safety factors have already been included.

TABLE 9–2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast	170×10^6	550×10^6	170×10^6
Steel	500×10^6	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Brick		35×10^6	
Marble		80×10^6	
Granite		170×10^6	
Wood (pine)			
(parallel to grain)	40×10^6	35×10^6	5×10^6
(perpendicular to grain)		10×10^6	
Nylon	500×10^6		
Bone (limb)	130×10^6	170×10^6	

EXAMPLE 9–13 Size and compression of support columns. (*a*) What minimum cross-sectional area should the two columns have to support the beam of Example 9–7 (Fig. 9–10) assuming the columns are made of concrete and a safety factor of 6 is required? We saw in Example 9–7 that the column on the left supports 4.4×10^4 N and that on the right supports 1.2×10^5 N. (*b*) How much will the chosen supports compress under the given load?

SOLUTION (*a*) The right-hand column receives the larger force, $1.2 \times 10^5\,\text{N}$. It is clearly under compression, and from Table 9–2, we see that the ultimate compressive strength of concrete is $2.0 \times 10^7\,\text{N/m}^2$. Using a safety factor of 6, the maximum allowable stress is $\frac{1}{6}(2.0 \times 10^7\,\text{N/m}^2) = 3.3 \times 10^6\,\text{N/m}^2$, which equals F/A. Since $F = 1.2 \times 10^5\,\text{N}$, we can solve for A, and we find:

$$A = \frac{1.2 \times 10^5\,\text{N}}{3.3 \times 10^6\,\text{N/m}^2} = 3.6 \times 10^{-2}\,\text{m}^2, \text{or } 360\,\text{cm}^2.$$

A support 18 cm × 20 cm will be adequate.
(*b*) We solve for

$$\frac{\Delta L}{L_0} = \frac{1}{E}\frac{F}{A} = \left(\frac{1}{2.0 \times 10^{10}\,\text{N/m}^2}\right)(3.3 \times 10^6\,\text{N/m}^2) = 1.7 \times 10^{-4}.$$

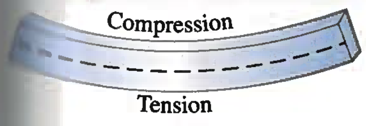
Thus, if the support has a length $L_0 = 5.0\,\text{m}$, $\Delta L = 0.85 \times 10^{-3}\,\text{m}$, or about 1 mm. This calculation was for the right-hand support. If the left-hand support is made of the same cross-sectional area, it will compress less and this should be taken into account.

PHYSICS APPLIED

Reinforced concrete and prestressed concrete

As can be seen in Table 9–2, concrete (like stone and brick) is reasonably strong under compression but extremely weak under tension. Thus concrete can be used as vertical columns placed under compression but is of little value as a beam since it cannot withstand the tensile forces that arise (see Fig. 9–28). *Reinforced concrete*, in which iron rods are embedded in the concrete, is much stronger (Fig. 9–29). But the concrete on the lower edge of a loaded beam still tends to crack because of its weakness under tension. This problem is solved with **prestressed concrete**, which also contains iron rods or a wire mesh, but during the pouring of the concrete, the rods or wire are held under tension. After the concrete dries, the tension on the iron is released, putting the concrete under compression. The amount of compressive stress is carefully predetermined so that when the design loads are applied to the beam, they reduce the compression on the lower edge but never put the concrete into tension.

FIGURE 9–28 A beam sags, at least a little (but is exaggerated here), even under its own weight. The beam thus changes shape so that the upper portion is compressed, and the lower portion is under tension (elongated). Shearing stress also occurs within the beam.



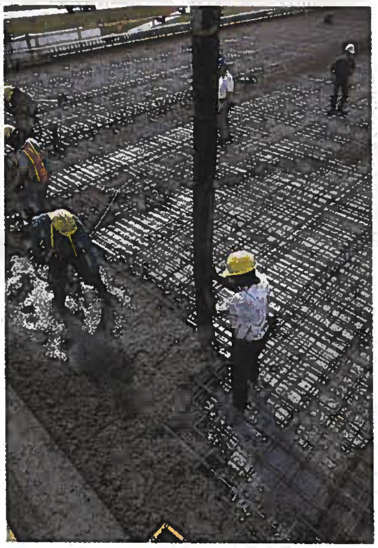


FIGURE 9–29 Steel rods waiting for concrete to be poured around them to form a new highway.

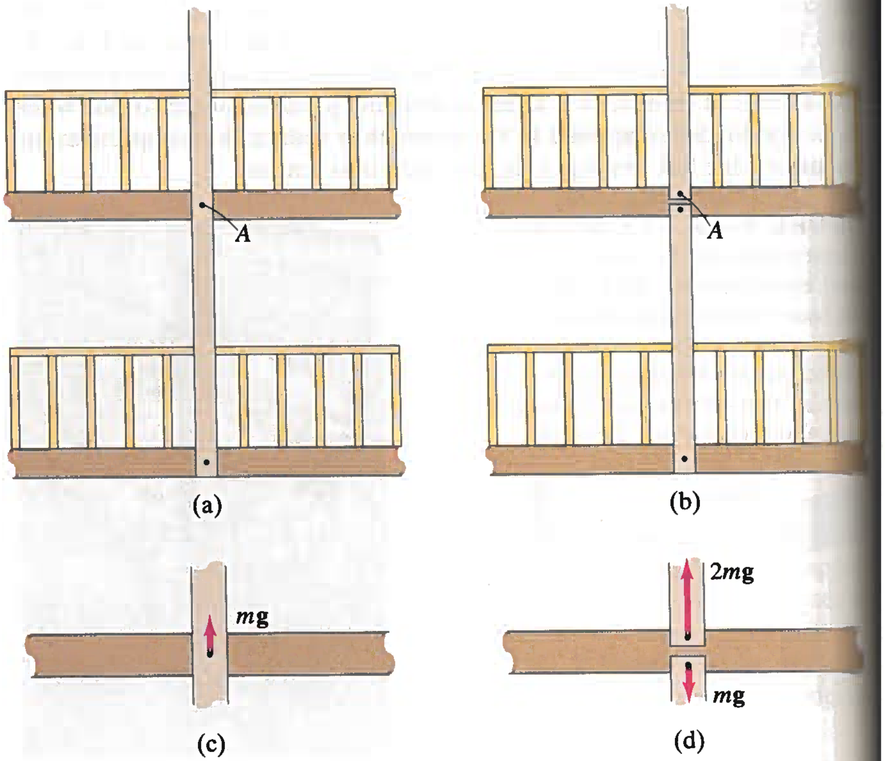
Let's consider one more instructive Example.

PHYSICS APPLIED

A tragic collapse

CONCEPTUAL EXAMPLE 9–14 **A tragic substitution.** Two walkwa one above the other, are suspended from vertical rods attached to th ceiling of a high hotel lobby, Fig. 9–30a (only one rod is shown). Th original design called for single rods 14 m long, but when such long ro proved to be unwieldy to install, it was decided to replace each long ro with two shorter ones as shown in Fig. 9–30b. Determine the net fo exerted by the rods on the supporting pin A (assumed the same size) f each design. Assume each vertical rod supports a mass m of each brid

RESPONSE The single long vertical rod in Fig. 9–30a exerts an upwa force equal to mg on pin A to support the mass m of the upper brid Why? Because the pin is in equilibrium, and the other force that balan this is the downward force mg exerted on it by the upper bridge. (There thus a shear stress on the pin.) See Fig. 9–30c. The situation when tw shorter rods support the bridges (Fig. 9–30b) is shown in Fig. 9–30d i which only the connections at the upper bridge are shown. The lower ro exerts a force of mg downward on the lower of the two pins because i supports the lower bridge. The upper rod exerts a force of $2mg$ on th upper pin (pin A) because the upper rod supports both bridges. Thus w see that when the builders substituted two shorter rods for each sing long one, the stress in the supporting pin A was *doubled*. What perha seemed like a simple substitution did, in fact, lead to a tragic collapse i 1981 with a loss of life of over 100 people (see Fig. 9–2). Having a feel fo physics, and being able to make simple calculations based on physics, ca have a great effect, literally, on people's lives.



(a) (b)

(c) (d)

FIGURE 9–30 Example 9–14.

9–8 Spanning a Space: Arches and Domes

There are a great many areas where the arts and humanities overlap the sciences, and this is especially clear in architecture, where the forces in the materials that make up a structure need to be understood to avoid excessive deformation and collapse. Many of the features we admire in the architecture of the past were introduced not simply for their decorative effect, but often for technical reasons. One example is the development of methods to span a space, from the simple beam to arches and domes.

It might be said that the first important architectural invention was the post-and-beam (or post-and-lintel) construction, in which two upright posts support a horizontal beam. Before steel was introduced in the nineteenth century, the length of a beam was quite limited because the strongest building materials were then stone and brick. Hence the width of a span was limited by the size of available stones. Equally important, stone and brick, though strong under compression—are very weak under tension and shear; all three types of stress occur in a beam, as shown in Fig. 9–28. The minimal space that could be spanned using stone is shown by the closely spaced columns of the great Greek temples (Fig. 9–23).

The introduction of the semicircular **arch** by the Romans (Fig. 9–31), aside from its aesthetic appeal, was a tremendous technological innovation. It had been preceded by the so-called "triangle arch" and the "corbeled arch," but these were relatively small improvements over the post-and-beam (see Fig. 9–32). The advantage of the "true" or semicircular arch is that, if well designed, its wedge-shaped stones experience stress which is mainly compressive even when supporting a large load such as the wall and roof of a cathedral. Because the stones are forced to squeeze against one another, they are mainly under compression (see Fig. 9–33). Note, however, that the arch transfers horizontal as well as vertical forces to the supports. A round arch consisting of many well-shaped stones could span a very wide space. However, considerable buttressing on the sides was needed to support the horizontal components of the forces.

PHYSICS APPLIED

Architecture: Beams, arches and domes

FIGURE 9–31 Round arches in the Roman Forum. The one in the background is the Arch of Titus.

FIGURE 9–33 Stones in a round (or "true") arch are mainly under compression.

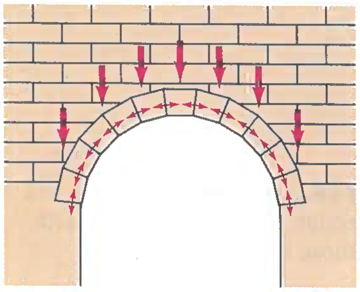


FIGURE 9–32 Various means to span a space.

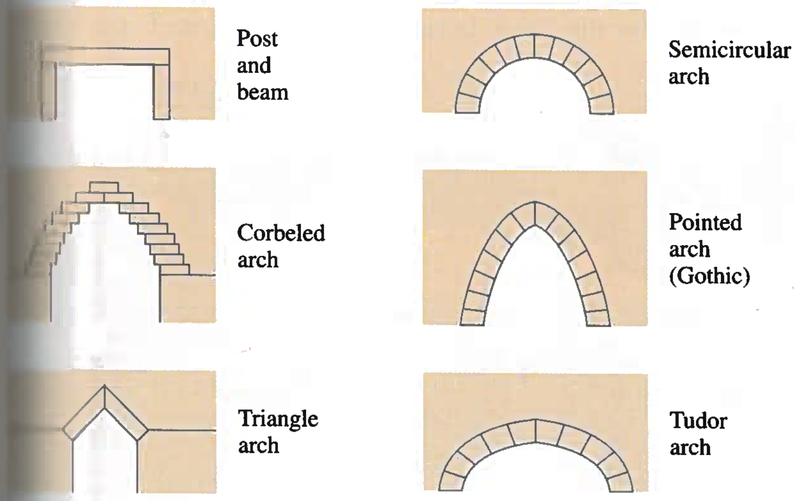


FIGURE 9–34 Flying buttresses (on the cathedral of Notre Dame, in Paris).

The pointed arch came into use about A.D. 1100 and became the mark of the great Gothic cathedrals. It too was an important technic novation, and was first used to support heavy loads such as the tower cathedral, and as the central arch. Apparently the builders realized th because of the steepness of the pointed arch, the forces due to the we above could be brought down more nearly vertically, so less horizo buttressing would be needed. The pointed arch reduced the load on walls, so there could be more openness and light. The smaller buttre needed was provided on the outside by graceful flying buttre (Fig. 9–34).

The technical innovation of the pointed arch was achieved not thro calculation but through experience and intuition, for it was not until m later that detailed calculations, such as those presented earlier in this ch ter, came into use. To make an accurate analysis of a stone arch is quite ficult in practice. But if we make some simplifying assumptions, we show why the horizontal component of the force at the base is less f pointed arch than for a round one. Figure 9–35 shows a round arch an pointed arch, each with an 8.0-m span. The height of the round arch is 4.0 m, whereas that of the pointed arch is larger and has been chosen t 8.0 m. Each arch supports a weight of $12.0 \times 10^4\,\text{N}$ (= 12,000 kg which, for simplicity, we have divided into two parts (each $6.0 \times 10^4\,\text{N}$) ing on the two halves of each arch as shown. To be in equilibrium, each the supports must exert an upward force of $6.0 \times 10^4\,\text{N}$. Each support exerts a horizontal force, F_H, at the base of the arch, and it is this we to calculate. We focus only on the right half of each arch. We set equ zero the total torque calculated about the apex of the arch due to forces exerted on that half arch, as if there were a hinge at the apex. the round arch, the torque equation is

$$(4.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (2.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (4.0\,\text{m})(F_H)$$

Thus $F_H = 3.0 \times 10^4\,\text{N}$. For the pointed arch, the torque equation is

$$(4.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (2.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (8.0\,\text{m})(F_H)$$

Solving, we find that $F_H = 1.5 \times 10^4\,\text{N}$—only half as much! From this culation we can see that the horizontal buttressing force required fo pointed arch is less because the arch is higher, and there is therefore longer lever arm for this force. Indeed, the steeper the arch, the less

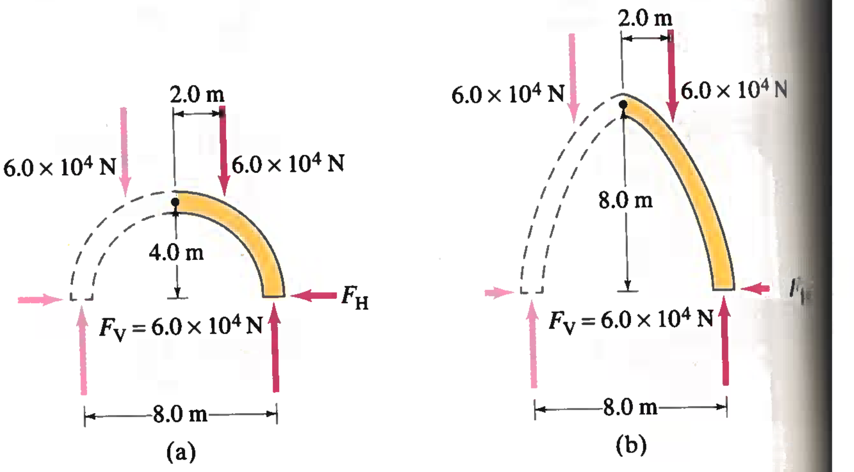


FIGURE 9–35 Forces in a round arch (a), compared with those in a pointed arch (b).

horizontal component of the force needs to be, and hence the more nearly vertical is the force exerted at the base of the arch.

The further development of the arch was one of decline. For the subsequent flattened arches, such as the Tudor arch (Fig. 9–32), were structurally weaker than the simple pointed arch. However, with the coming of advanced methods of calculation in the nineteenth and twentieth centuries, it became possible to calculate the best shape of arch for a given load condition. For example, if the load is uniform across the span, it can be shown that the stresses within the arch will be purely compressive if the arch has a parabolic shape.

Whereas an arch spans a two-dimensional space, a **dome**—which is essentially an arch rotated about a vertical axis—spans a three-dimensional space. The Romans built the first large domes. Their shape was hemispherical and some still stand, such as that of the Pantheon in Rome (Fig. 9–36). By the time of the Renaissance, the technique for constructing large domes seems to have been lost. Indeed, the dome of the Pantheon was a source of wonder to Renaissance architects. The problem came to the fore in fifteenth-century Florence with the designing of a new cathedral that was to have a dome 43 m in diameter to rival that of the Pantheon. In 1418, after the cathedral was finished except for the dome, a competition for the design of the dome was held and was won by Filippo Brunelleschi (1377–1446). One problem that had to be dealt with was that the dome was to rest on a "drum" that had been completed with no external abutments; and there was no place to put any. Hence the dome needed to exert a minimum of horizontal force. Brunelleschi solved this by designing a pointed dome (Fig. 9–37), since a pointed dome, like a pointed arch, exerts a smaller side thrust against its base.

The other major problem was how to support the dome during construction. A dome, like an arch, is not stable until all the stones are in place. It had been the custom to support a dome during construction with a wooden framework. But no trees big enough or strong enough could be found to span the 43-m space required for the cathedral in Florence. Instead of using a wooden framework, Brunelleschi built the dome in horizontal layers. Each

FIGURE 9–36 Interior of the Pantheon in Rome, built in the first century. This view, showing the great dome and its central opening for light, was painted about 1740 by Panini. Photographs do not capture its grandeur as well as this painting does.

FIGURE 9–37 The skyline of Florence, showing Brunelleschi's famous dome on the cathedral.

FIGURE 9–38 The dome of the Small Sports Palace in Rome, built for the 1960 Olympics.

layer was bonded to the previous one, which held it in place until the stone of the circle was placed, and then that circle was stable just as a c pleted arch is stable. Each closed ring was then strong enough to sup the next layer. It was an amazing feat.

To end this Section we will consider the forces necessary to suppo modern dome, that of the Small Sports Palace in Rome (Fig. 9–38) dome, like an arch, is statically most stable when under compression. 36 buttresses which support the 1.2×10^6-kg dome are positioned at angle and connect smoothly with the dome.

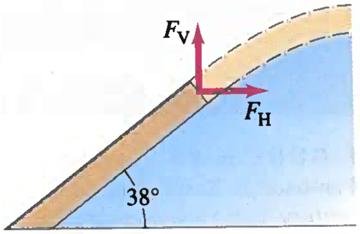


FIGURE 9–39 Example 9–15.

EXAMPLE 9–15 **A modern dome.** Calculate the components of force, F_V and F_H, that each buttress exerts on the dome so that the fo acts compressively—that is, at a 38° angle (Fig. 9–39).

SOLUTION The vertical load on *each* buttress is $\frac{1}{36}$ of the total wei Thus

$$F_V = \frac{(1.2 \times 10^6\,\text{kg})(9.8\,\text{m/s}^2)}{36} = 3.4 \times 10^5\,\text{N}.$$

The force must act at a 38° angle at the base of the dome in order to purely compressive. Thus

$$\tan 38^\circ = \frac{F_V}{F_H} = \frac{340{,}000\,\text{N}}{F_H}$$

$$F_H = \frac{340{,}000\,\text{N}}{\tan 38^\circ} = 430{,}000\ \ \text{N}.$$

In order that each of the buttresses be able to exert this 430,000-N ho zontal force, a prestressed-concrete tension ring surrounds the base the buttresses beneath the ground (see Problem 63 and Fig. 9–71).

SUMMARY

A body at rest [or one in uniform motion at constant velocity] is said to be in **equilibrium**. The subject concerned with the determination of the forces within a structure at rest is called **statics**.

The two necessary conditions for a body to be in equilibrium are (1) the vector sum of all the forces on it must be zero, and (2) the sum of all the torques (calculated about any arbitrary axis) must also be zero:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma\tau = 0.$$

It is important when doing statics problems to apply the equilibrium conditions to only one body at a time.

A body in static equilibrium is said to be in (*a*) **stable**, (*b*) **unstable**, or (*c*) **neutral equilibrium**, depending on whether a slight displacement leads to (*a*) a return to the original position, (*b*) further movement away from the original position, or (*c*) rest in the new position. An object in st equilibrium is also said to be in **balance**.

Hooke's law applies to many elastic solids, states that the change in length of an object is p portional to the applied force:

$$F = k\,\Delta L.$$

If the force is too great, the object will exceed **elastic limit**, which means it will no longer ret to its original shape when the distorting force is moved. If the force is even greater, the **ulti strength** of the material can be exceeded and object fractures.

The force per unit area acting on a body called the **stress**, and the resulting fractional cha in length is called the **strain**.

The stress on a body is present within the and can be of three types: **compression**, **tens** and **shear**.

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The ratio of stress to strain is called the **elastic modulus** of the material. **Young's modulus** applies for compression and tension, and the **shear modulus** for shear; **bulk modulus** applies to an object whose volume changes as a result of pressure on all sides. All three moduli are constants for a given material when distorted within the elastic region.

QUESTIONS

1. Describe several situations where a body is not in equilibrium, even though the net force on it is zero.
2. A bungee jumper momentarily comes to rest at the bottom of the dive before he springs back upward. At that moment, is the bungee jumper in equilibrium? Explain.
3. You can find the center of gravity of a meter stick by resting it horizontally on your index fingers, and then slowly drawing your fingers together. First the meter stick will slip on one finger, and then on the other, but eventually the fingers meet at the CG. Why does this work?
4. Your doctor's scale has arms on which weights slide to counter your weight, Fig. 9–40. These weights are obviously much lighter than you are. How does this work?

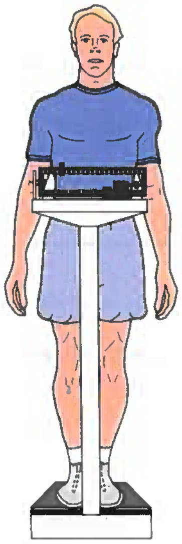


FIGURE 9–40 Question 4.

5. A bear sling, Fig. 9–41, is used in some national parks for placing backpackers' food out of the reach of bears. Explain why the force needed to pull the backpack up increases as the backpack gets higher and higher. Is it possible to pull the rope hard enough so that it doesn't sag at all?
6. A ladder, leaning against a wall, makes a 60° angle with the ground. When is it more likely to slip: when a person stands near the top or near the bottom? Explain.
7. Explain why touching the toes while seated on the floor with outstretched legs produces less stress on the lower spinal column than when touching the toes from a standing position. Use a diagram.

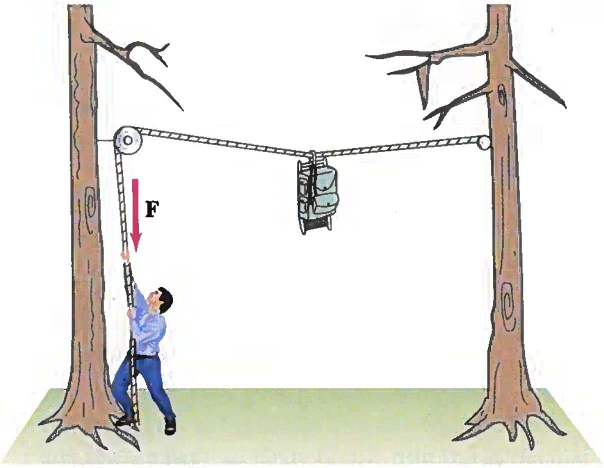
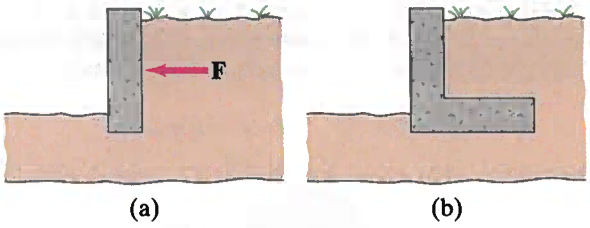


FIGURE 9–41 Question 5, Problems 24 and 29.

8. An earthen retaining wall is shown in Fig. 9–42a. The earth, particularly when wet, can exert a significant force F on the wall. (*a*) What force produces the torque to keep the wall upright? (*b*) Explain why the retaining wall in Fig. 9–42b would be much less likely to overturn.



(a) (b)

FIGURE 9–42 Question 8.

9. Figure 9–43 shows a cone. Draw how to lay it on a flat table so that it is in (*a*) stable equilibrium, (*b*) unstable equilibrium, (*c*) neutral equilibrium.

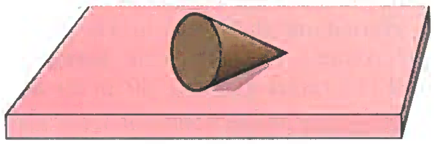


FIGURE 9–43 Question 9.

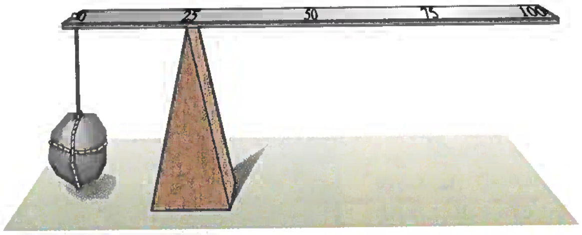
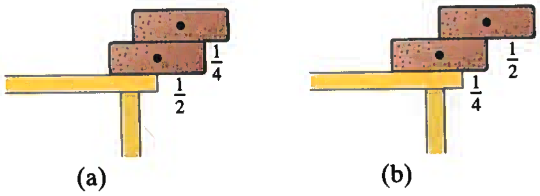


FIGURE 9–44 Question 10.

10. A uniform meter stick supported at the 25-cm mark is in equilibrium when a 1-kg rock is suspended at the 0-cm end (as shown in Fig. 9–44). Is the mass of the meter stick greater than, equal to, or less than the mass of the rock? Explain your reasoning.

11. Which of the configurations of brick, (a) or (b) of Fig. 9–45, is the more likely to be stable? Why?



(a) (b)

FIGURE 9–45 The dots indicate the CG of each brick. The fractions $\frac{1}{4}$ and $\frac{1}{2}$ indicate what portion of each brick is hanging beyond its support. Question 11.

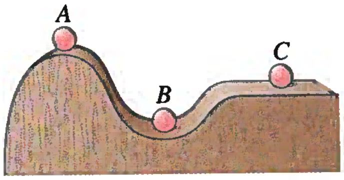


FIGURE 9–46 Question 12.

12. Name the type of equilibrium for each posit the ball in Fig. 9–46.

13. Why do you tend to lean backward when carry heavy load in your arms?

14. Place yourself facing the edge of an open door. tion your feet astride the door with your nose abdomen touching the door's edge. Try to rise your tiptoes. Why can't this be done?

15. Why is it not possible to sit upright in a chair rise to one's feet without first leaning forward?

16. Why is it more difficult to do sit-ups when your k are bent than when your legs are stretched out?

17. Examine how a pair of scissors or shears cuts thro a piece of cardboard. Is the name "shears" justifi

18. Materials such as ordinary concrete and stone very weak under tension or shear. Would it be wi use such a material for either of the supports of cantilever shown in Fig. 9–11? If so, which one(s)

PROBLEMS

SECTIONS 9–1 TO 9–3

1. (I) Three forces are applied to a tree sapling, as shown in Fig. 9–47, to stabilize it. If $\mathbf{F}_1 = 282\,\mathrm{N}$ and $\mathbf{F}_2 = 355\,\mathrm{N}$, find $\mathbf{F}_3$ in magnitude and direction.

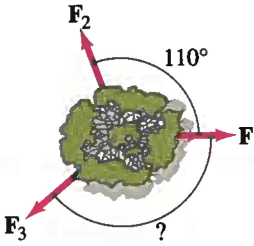


FIGURE 9–47 Problem 1.

2. (I) What should be the tension in the wire if the net force exerted on the tooth in Fig. 9–3 is to be 0.75 N? Assume that the angle between the two forces is 155° rather than the 140° in the figure.

3. (I) Calculate the torque about the front support of a diving board, on the right in Fig. 9–48, exerted by a 60-kg person 3.0 m from that support.

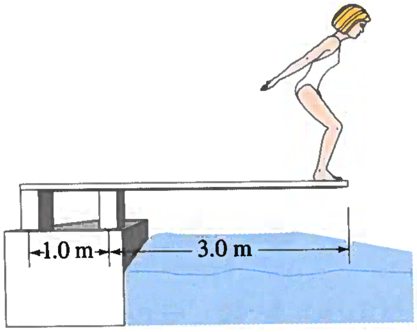


FIGURE 9–48 Problems 3, 4, 19, and 20.

4. (I) How far out on the diving board (Fig. 9–48) w a 60-kg diver have to be to exert a torque of 1000 on the board, relative to the left support post?

5. (I) Two cords support a chandelier in the man shown in Fig. 9–6 except that the upper wire make angle of 45° with the ceiling. If the cords can sust force of 1300 N without breaking, what is the m mum chandelier weight that can be supported?

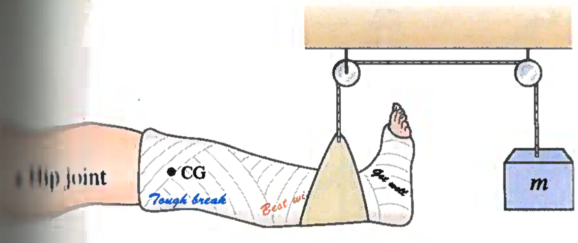


FIGURE 9–49 Problems 6 and 21.

6. (I) Calculate the mass m needed in order to suspend the leg shown in Fig. 9–49. Assume the leg (with cast) has a mass of 15.0 kg, and its CG is 35.0 cm from the hip joint; the sling is 80.5 cm from the hip joint.

7. (II) A 160-kg horizontal beam is supported at each end. A 300-kg piano rests a quarter of the way from one end. What is the vertical force on each of the supports?

8. (II) A uniform steel beam has a mass of 1000 kg. On it is resting half of an identical beam, as shown in Fig. 9–50. What is the vertical support force at each end?

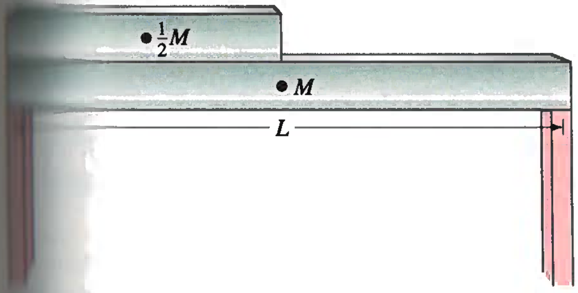


FIGURE 9–50 Problem 8.

9. (II) Suppose that the net force as calculated in Example 9–1 is 10° to the left of where it should point if the tooth is to move correctly. If the tension to the left is 2.0 N, what should the tension to the right be to make the net force act in the correct direction?

10. (II) A 70-kg adult sits at one end of a 10-m board, on the other end of which sits his 30-kg child. Where should the pivot be placed so the board (ignore its mass) is balanced?

11. (II) Repeat Problem 10 taking into account the board's 15-kg mass.

12. (II) Find the tension in the two cords shown in Fig. 9–51. Neglect the mass of the cords, and assume that the angle θ is 30° and the mass m is 200 kg.

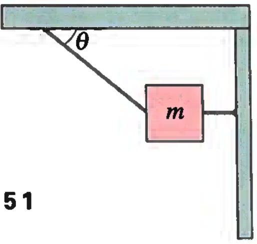


FIGURE 9–51 Problem 12.

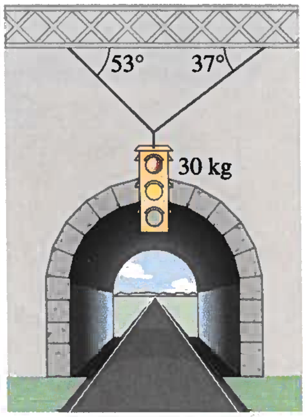


FIGURE 9–52 Problems 13 and 48.

13. (II) Find the tension in the two wires supporting the traffic light shown in Fig. 9–52.

14. (II) Determine the force F_N that the pivot exerts on the seesaw board in Fig. 9–9.

15. (II) Calculate F_1 and F_2 for the uniform cantilever shown in Fig. 9–11 whose mass is 1200 kg.

16. (II) A 0.60 kg sheet hangs from a massless clothesline as shown in Fig. 9–53. The line on either side of the sheet makes an angle of 3.5° with the horizontal. Calculate the tension in the clothesline on either side of the sheet. Why is the tension so much greater than the weight of the sheet?

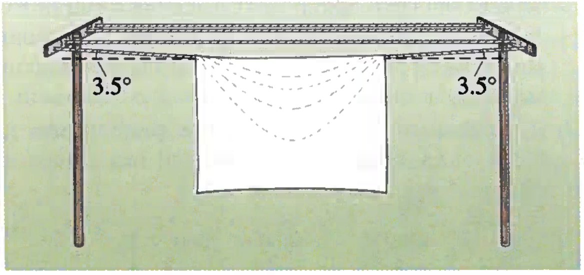
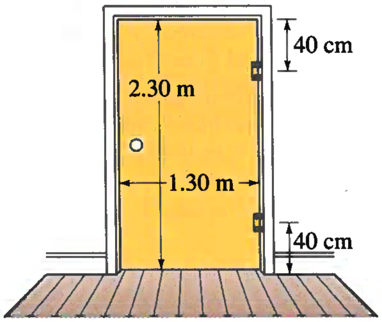


FIGURE 9–53 Problem 16.

17. (II) A door, 2.30 m high and 1.30 m wide, has a mass of 13.0 kg. A hinge 0.40 m from the top and another hinge 0.40 m from the bottom each support half the door's weight (Fig. 9–54). Assume that the center of gravity is at the geometrical center of the door, and determine the horizontal and vertical force components exerted by each hinge on the door.

FIGURE 9–54 Problem 17.



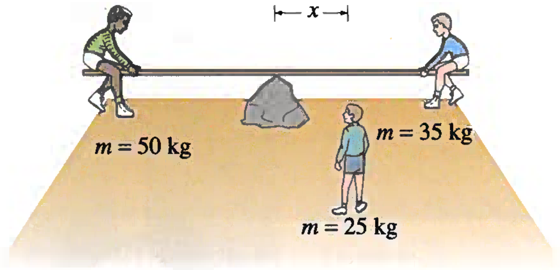


FIGURE 9–55 Problem 18.

18. (II) Three boys are trying to balance on a seesaw, which consists of a fulcrum rock, acting as a pivot at the center, and a very light board 3.6 m long (Fig. 9–55). Two boys are already on either end. One has a mass of 50 kg, and the other a mass of 35 kg. Where should the third boy, whose mass is 25 kg, place himself so as to balance the seesaw?

19. (II) Calculate the forces F_1 and F_2 that the supports exert on the diving board of Fig. 9–48 when a 60-kg person stands at its tip. Ignore the weight of the board.

20. (II) Repeat the last problem, taking into account the board's mass of 35 kg. Assume the board's CG is at its center.

21. (II) Calculate the mass m required in Fig. 9–49 to support the leg (without cast), using the result of Example 7–12 and the values given in Table 7–1, assuming a 60.0-kg person 160 cm tall. The leg pivots about the hip joint and the support acts at the ankle joint.

22. (II) Calculate F_1 and F_2 for the beam shown in Fig. 9–56. Assume it is uniform and has a mass of 250 kg.

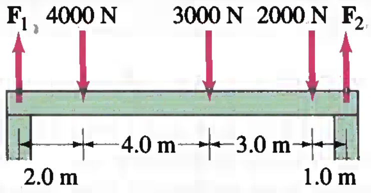


FIGURE 9–56 Problem 22.

23. (II) Calculate the tension F_T in the wire that supports the 30-kg beam shown in Fig. 9–57, and the force $\mathbf{F}_W$ exerted by the wall on the beam (give magnitude and direction).

24. (II) The two trees in Fig. 9–41 are 7.6 m apart. Calculate the magnitude of the force **F** a backpacker must exert to hold a 16-kg backpack so that the rope sags at its midpoint by (*a*) 1.5 m, (*b*) 0.15 m.

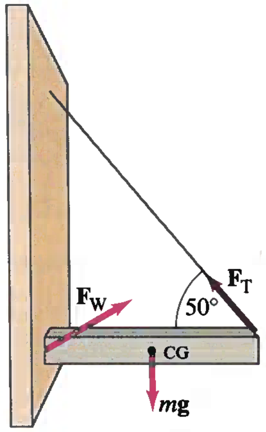


FIGURE 9–57 Problem 23.

25. (II) A 170-cm-tall person lies on a light (m
board which is supported by two scales, one und
feet and one beneath the top of the head (Fig. 9
The two scales read, respectively, 31.6 and 3
Where is the center of gravity of this person?

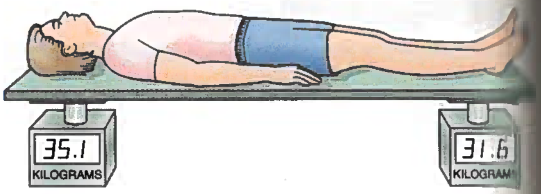


FIGURE 9–58 Problem 25.

26. (II) A shop sign weighing 215 N is supported
uniform 135-N beam as shown in Fig. 9–59. Find
tension in the guy wire and the horizontal and v
cal forces exerted by the hinge on the beam.

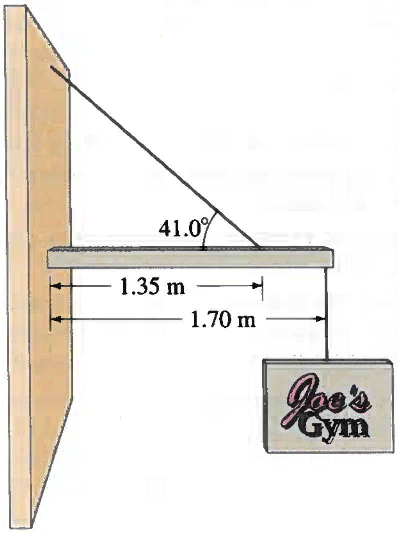


FIGURE 9–59 Problem 26.

23.

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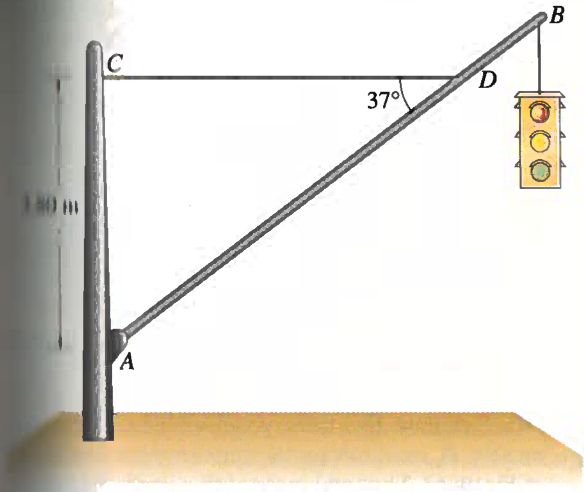

FIGURE 9–60 Problem 27.

(II) A traffic light hangs from a structure as shown in Fig. 9–60. The uniform aluminum pole AB is 7.5 m long and has a mass of 8.0 kg. The mass of the traffic light is 12.0 kg. Determine the tension in the horizontal massless cable CD, and the vertical and horizontal components of the force exerted by the pivot A on the aluminum pole.

(II) A uniform ladder of mass m and length L leans at an angle θ against a frictionless wall, Fig. 9–61. If the coefficient of static friction between the ladder and the ground is μ, what is the minimum angle at which the ladder will not slip?

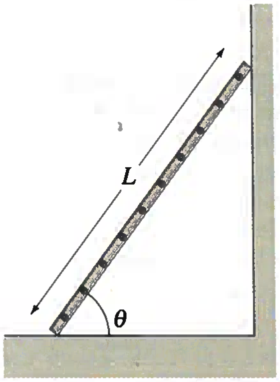

FIGURE 9–61 Problems 28, 76, and 77.

(III) A 23.0 kg backpack is suspended midway between two trees by a light cord as in Fig. 9–41. A bear grabs the backpack and pulls vertically downward with a constant force, so that each section of cord makes an angle of 30° below the horizontal. Initially, without the bear pulling, the angle was 15°; the tension in the cord with the bear pulling is double what it was when he was not. Calculate the force the bear is exerting on the backpack.

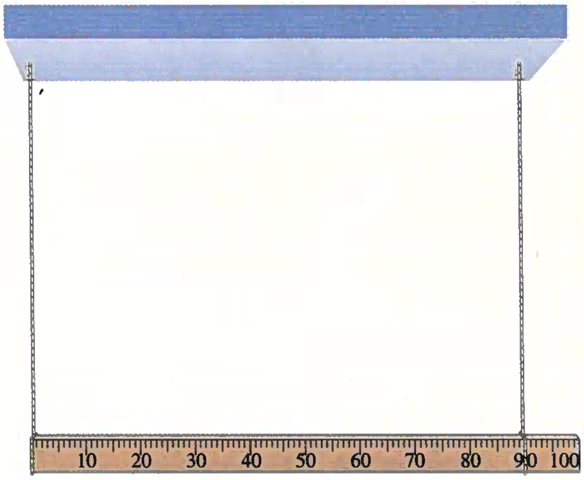

FIGURE 9–62 Problem 30.

30. (III) A meter stick with a mass of 230 g is supported horizontally by two vertical strings, one at the 0-cm mark and the other at the 90-cm mark (Fig. 9–62). (*a*) What is the tension in the string at 0 cm? (*b*) What is the tension in the string at 90 cm?

31. (III) Consider again the ladder of Example 9–9 but with a painter climbing up. If the mass of the ladder is 12.0 kg, the mass of the painter is 60.0 kg, and the ladder begins to slip at its base when she is 70 percent of the way up the length of the ladder, what is the coefficient of static friction between the ladder and the floor? Again assume the wall is frictionless. A free-body diagram is shown in Fig. 9–63.

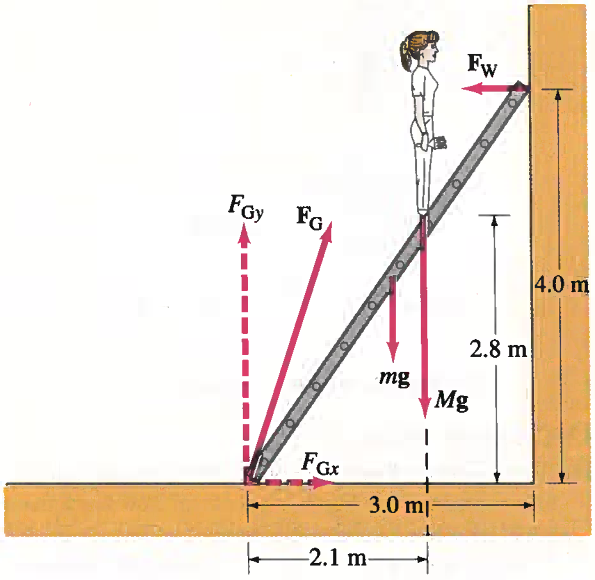

FIGURE 9–63 Problem 31.

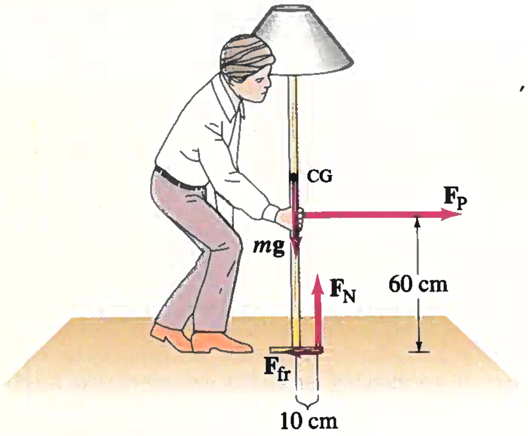

FIGURE 9–64 Problem 32.

32. (III) A person wants to push a lamp (mass 7.2 kg) across the floor. (*a*) Assuming the person pushes at a height of 60 cm above the ground and the coefficient of friction is 0.20, determine whether the lamp will slide or tip over (Fig. 9–64). (*b*) Calculate the maximum height above the floor at which the person can push the lamp so that it slides rather than tips.

33. (III) Two guy wires run from the top of a pole 2.6 m tall that supports a volleyball net. The two wires are anchored to the ground 2.0 m apart and each is 2.0 m from the pole (Fig. 9–65). The tension in each wire is 95 N. What is the tension in the net, assumed horizontal and attached at the top of the pole?

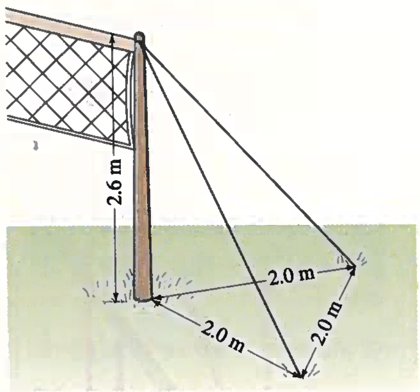

FIGURE 9–65 Problem 33.

*SECTION 9–4

* 34. (I) If the point of insertion of the biceps muscle into the lower arm shown in Fig. 9–15a is 6.0 cm, how much mass could the person hold with a muscle exertion of 400 N?

* 35. (I) Approximately what force, F_M, must the extensor muscle in the upper arm exert on the lower arm to hold a 7.3-kg shot put (Fig. 9–66)? Assume the lower arm has a mass of 2.8 kg and its CG is 12 cm from the pivot point.

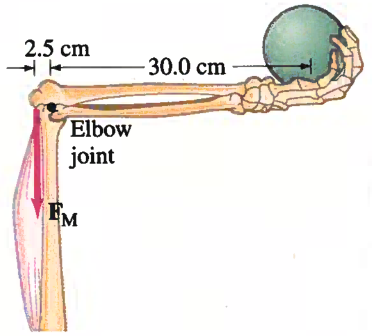

FIGURE 9–66 Problem 35.

* 36. (II) Calculate the force required of the "del muscle, F_M, to hold up the outstretched arm sh in Fig. 9–67. The total mass of the arm is 3.3 kg.

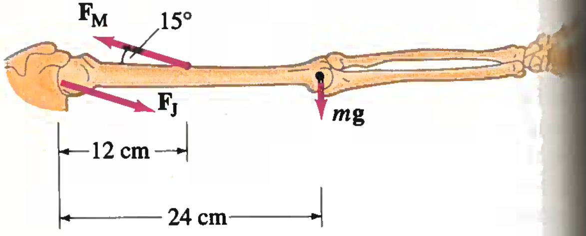

FIGURE 9–67 Problems 36, 37, and 38.

* 37. (II) Suppose the hand in the last problem holds a 1 mass. What force, F_M, is required of the deltoid mu assuming the mass is 52 cm from the shoulder joint

* 38. (II) Calculate the magnitude of the force F_J exe by the shoulder on the upper arm at the joint Problems 36 and 37.

* 39. (II) The Achilles tendon is attached to the rear of foot as shown in Fig. 9–68. When a person elev himself just barely off the floor on the "ball of foot," estimate the tension in the Achilles tend (pulling upward), and the (downward) force exe by the lower leg bone on the foot. Assume the pe has a mass of 70 kg, and that D is twice as long as

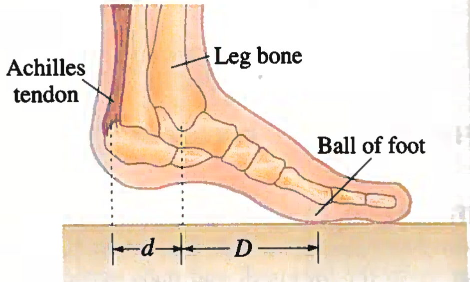

FIGURE 9–68 Problem 39.

40. (II) Redo Example 9–11 assuming now that the pe is less bent over so that the 30° in Fig. 9–16b is inst 60°. What will the magnitude of F_V on the vertebra

(III) Calculate the magnitude of the force at the base of the spine, $\mathbf{F}_V$ in Example 9–11 (Fig. 9–16b), if a mass of 20 kg is held in the person's hands, with arms hanging vertically. Assume the person's mass is 70 kg.

SECTION 9–5

(II) The Leaning Tower of Pisa is 55 m tall and about 7.0 m in diameter. The top is 4.5 m off center. Is the tower in stable equilibrium? If so, how much farther can it lean before it becomes unstable? Assume the tower is of uniform composition.

(III) Four bricks are to be stacked at the edge of a table, each brick overhanging the one below it, so that the top brick extends as far as possible beyond the edge of the table. (*a*) To achieve this, show that successive bricks must extend no more than (starting at the top) 1/2, 1/4, 1/6, and 1/8 of their length beyond the one below (Fig. 9–69). (*b*) Is the top brick completely beyond the base? (*c*) Determine a general formula for the maximum total distance spanned by *n* bricks if they are to remain stable. (*d*) A builder wants to construct a corbeled arch (Fig. 9–32) based on the principle of stability discussed in (*a*) and (*c*) above. What minimum number of bricks, each 0.30 m long, is needed if the arch is to span 1.0 m?

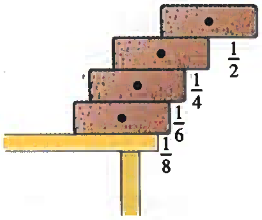

FIGURE 9–69 Problem 43.

SECTION 9–6

(I) A nylon tennis string on a racket is under a tension of 250 N. If its diameter is 1.00 mm, by how much is it lengthened from its untensioned length of 30.0 cm?

(I) A marble column of cross-sectional area 2.0 m^2 supports a mass of 25,000 kg. (*a*) What is the stress within the column? (*b*) What is the strain?

(I) By how much is the column in the previous problem shortened if it is 12 m high?

(I) A vertical steel girder with a cross-sectional area of 0.15 m^2 has a sign (mass 2000 kg) hanging from its end. (*a*) What is the stress within the girder? (*b*) What is the strain on the girder? (*c*) If the girder is 9.50 m long, how much is it lengthened? (Ignore the mass of the girder itself.)

(II) If the two wires in Fig. 9–52 (Problem 13) are made of steel wire 1.0 mm in diameter, what is the percentage stretch of each because of the load?

(II) One liter of alcohol (1000 cm^3) in a flexible container is carried to the bottom of the sea, where the pressure is 2.6×10^6 N/m^2. What will be its volume there?

50. (II) A 15-cm-long animal tendon was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm. Calculate the elastic modulus of this tendon.

51. (II) How much pressure is needed to compress the volume of an iron block by 0.10 percent? Express answer in N/m^2, and compare it to atmospheric pressure (1.0×10^5 N/m^2).

52. (II) At depths of 2000 m in the sea, the pressure is about 200 times atmospheric pressure (1.0×10^5 N/m^2). By what percentage does an iron bathysphere's volume change at this depth?

53. (III) A scallop forces open its shell with an elastic material called abductin, whose elastic modulus is about 2.0×10^6 N/m^2. If this piece of abductin is 3.0 mm thick and has a cross-sectional area of 0.50 cm^2, how much potential energy does it store when compressed 1.0 mm?

54. (III) A pole projects horizontally from the front wall of a shop. A 5.1-kg sign hangs from the pole at a point 2.2 m from the wall (Fig. 9–70). (*a*) What is the torque due to this sign calculated about the point where the pole meets the wall? (*b*) If the pole is not to fall off, there must be another torque exerted to balance it. What exerts this torque? Use a diagram to show how this torque must act. (*c*) Discuss whether compression, tension, and/or shear play a role in part (*b*).

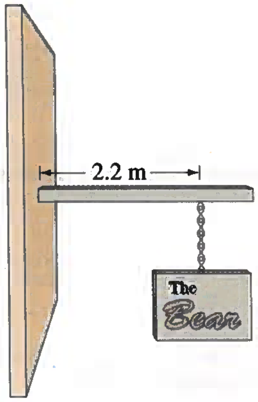

FIGURE 9–70 Problem 54.

SECTION 9–7

55. (I) The femur bone in the leg has a minimum effective cross section of about 3.0 cm^2 ($= 3.0 \times 10^{-4}$ m^2). How much compressive force can it withstand before breaking?

56. (II) What is the maximum tension possible in a 1.00-mm-diameter nylon tennis racket string? If you want tighter strings, what do you do to prevent breakage: go to thinner or thicker strings? Why? What causes strings to break when they are hit by the ball?

57. (II) If a compressive force of 3.6×10^4 N is exerted on the end of a 20-cm-long bone of cross-sectional area 3.6 cm^2, (*a*) will the bone break, and (*b*) if not, by how much does it shorten?

58. (II) (*a*) What is the minimum cross-sectional area required of a vertical steel cable from which is suspended a 320-kg chandelier? Assume a safety factor of 7.0. (*b*) If the cable is 7.5 m long, how much does it elongate?

59. (II) Assume the supports of the cantilever shown in Fig. 9–11 (mass = 2600 kg) are made of wood. Calculate the minimum cross-sectional area required of each, assuming a safety factor of 8.5.

60. (II) An iron bolt is used to connect two iron plates together. The bolt must withstand shear forces up to about 3200 N. Calculate the minimum diameter for the bolt, based on a safety factor of 6.0.

61. (III) A steel cable is to support an elevator whose total (loaded) mass is not to exceed 3100 kg. If the maximum acceleration of the elevator is 1.2 m/s^2, calculate the diameter of cable required. Assume a safety factor of 7.0.

*SECTION 9–8

* 62. (II) How high must a pointed arch be if it is
a space 8.0 m wide and exert one third the
tal force at its base that a round arch would?

* 63. (II) The subterranean tension ring that exer
balancing horizontal force on the abutments
dome in Fig. 9–38 is 36-sided, so each
makes a 10° angle with the adjacent one (Fig.
Calculate the tension F that must exist in
ment so that the required force of 4.3×10
be exerted at each corner (Example 9–15).

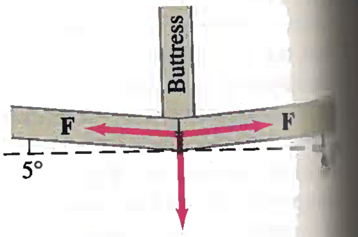

FIGURE 9–71 Problem 63.

GENERAL PROBLEMS

64. The mobile in Fig. 9–72 is in equilibrium. The object B has mass of 0.735 kg. Determine the masses of the objects A, C, and D. (Neglect the weights of the crossbars.)

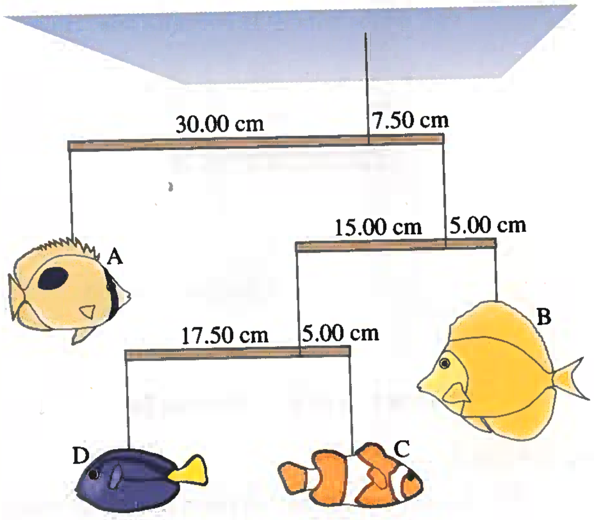

FIGURE 9–72 Problem 64.

65. A 50-story building is being planned. It is to be 200 m high with a base 40 m by 70 m. Its total mass will be about 1.8×10^7 kg and its weight therefore about 1.8×10^8 N. Suppose a 200-km/h wind exerts a force of 950 N/m^2 over the 70-m-wide face (Fig. 9–73). Calculate the torque about the potential pivot point, the rear edge of the building (whe
acts in Fig. 9–73), and determine whether the
ing will topple. Assume the total force of the
acts at the midpoint of the building's face, and
the building is not anchored in bedrock. [*Hint*
Fig. 9–73 represents the force that the Earth
on the building in the case where the building
beginning to tip.]

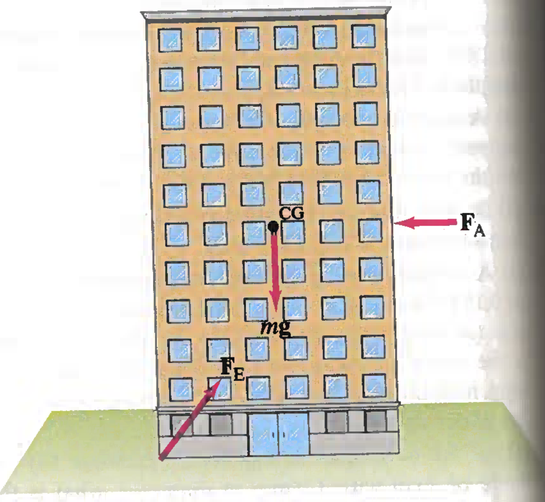

FIGURE 9–73 Force on a building subjected to wind ($\mathbf{F}_A$) and gravity (mg); $\mathbf{F}_E$ is the force on the building due to the Earth in the situation when the building is just about to tip. Problem 65.

A tightly stretched “high wire” is 46 m long. It sags 2.4 m when a 60-kg tightrope walker stands at its center. What is the tension in the wire? Is it possible to increase the tension in the wire so that there is no sag?

What minimum horizontal force F is needed to pull a wheel of radius R and mass M over a step of height h as shown in Fig. 9–74 ($R > h$)? (*a*) Assume the force is applied at the top edge as shown. (*b*) Assume the force is applied instead at the wheel’s center.

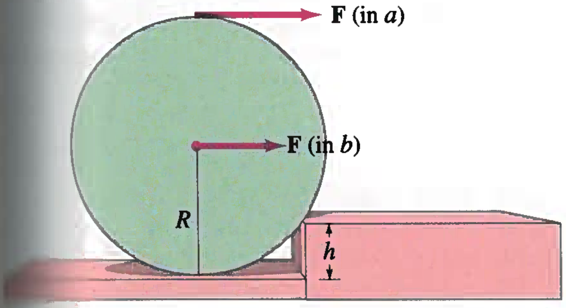

FIGURE 9–74 Problem 67.

The center of gravity of a loaded truck depends on how it is packed. If a truck is 4.0 m high and 2.4 m wide, and its CG is 2.2 m above the ground, how steep a slope can the truck be parked on without tipping over sideways (Fig. 9–75)?

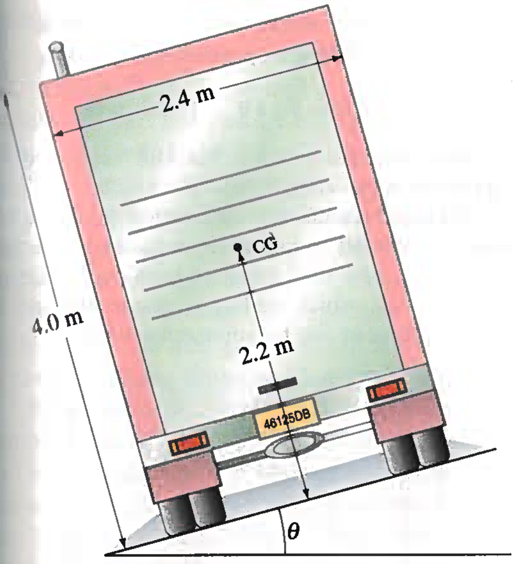

FIGURE 9–75 Problem 68.

In Example 7–5 in Chapter 7, we calculated the impulse and average force on the leg of a person who jumps 3.0 m down to the ground. If the legs are not bent upon landing, so that the body moves a distance d of only 1.0 cm during collision, determine (*a*) the stress in the tibia bone (area $= 3.0 \times 10^{-4}\,\mathrm{m}^2$), and (*b*) whether or not the bone will break. (*c*) Repeat for a bent-knees landing ($d = 50.0$ cm).

70. The roof of a 9.0 m × 10.0 m room in a school has a total mass of 12,600 kg. The roof is to be supported by “2 × 4s” (actually about 4.0 cm × 9.0 cm) along the 10.0-m sides. How many supports are required on each side and how far apart must they be? Consider only compression and assume a safety factor of 12.

71. In Fig. 9–76, consider the right-hand (northernmost) section of the Golden Gate Bridge, which has a length $d_1 = 343$ m. Assume the CG of this span is halfway between the tower and anchor. Determine F_{T1} and F_{T2} (which act on the northernmost cable) in terms of mg, the weight of the northernmost span, and calculate the tower height h needed for equilibrium. Assume the roadway is supported only by the suspension cables and neglect the mass of the cables. [*Hint*: F_{T3} does not act on this section.]

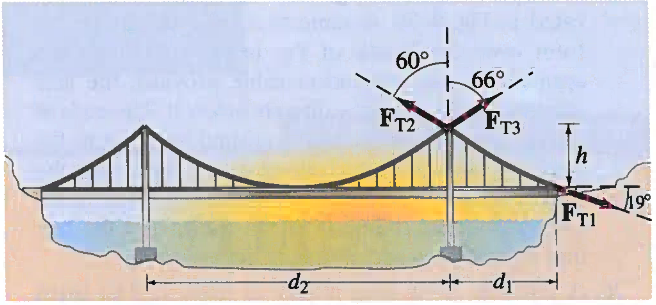

FIGURE 9–76 Problems 71 and 75.

72. A 20.0-m-long uniform beam weighing 600 N is supported on walls A and B, as shown in Fig. 9–77. (*a*) Find the maximum weight a person can be to walk to the extreme end D without tipping the beam. Find the forces that the walls A and B exert on the beam when the person is standing: (*b*) at D; (*c*) at a point 2.0 m to the right of B; (*d*) 2.0 m to the right of A.

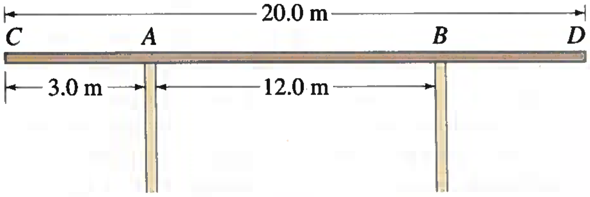

FIGURE 9–77 Problem 72.

73. A 36-kg round table is supported by three legs placed equal distances apart on the edge. What minimum mass, placed on the table’s edge, will cause the table to overturn?

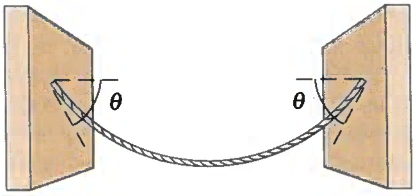

FIGURE 9–78 Problem 74.

74. A uniform flexible steel cable of weight mg is suspended between two equal elevation points as shown in Fig. 9–78, where $\theta = 60°$. Determine the tension in the cable (*a*) at its lowest point, and (*b*) at the points of attachment. (*c*) What is the direction of the tension force in each case?

75. Assume that a single-span suspension bridge such as the Golden Gate Bridge has the configuration indicated in Fig. 9–76. Assume that the roadway is uniform over the length of the bridge and that each segment of the suspension cable provides the sole support for the roadway directly below it. The ends of the cable are anchored to the ground only, not to the roadway. What must the ratio of d_2 to d_1 be so that the suspension cable exerts no net horizontal force on the towers? Neglect the mass of the cables and the fact that the roadway isn't precisely horizontal.

76. A uniform 7.0-m-long ladder of mass 15.0 kg leans against a smooth wall (so the force exerted by the wall, $\mathbf{F}_W$, is perpendicular to the wall). The ladder makes an angle $\theta = 20°$ with the vertical wall (see Fig. 9–61); and the ground is rough. (*a*) Calculate the components of the force exerted by the ground on the ladder at its base, and (*b*) determine what the coefficient of friction at the base of the ladder must be if the ladder is not to slip when a 70-kg person stands three fourths of the way up the ladder.

77. If the coefficient of friction between the ladder and the ground in the situation described in the preceding problem is 0.30, how far up the ladder can the person climb before the ladder starts to slip?

78. There is a maximum height of a uniform vertical column made of any material that can support itself without buckling, and it is independent of the cross-sectional area (why?). Calculate this height for (*a*) steel (density $7.8 \times 10^3\,\text{kg/m}^3$), and (*b*) granite (density $2.7 \times 10^3\,\text{kg/m}^3$).

79. From what minimum height must a 1.2-kg rectangular brick 15.0 cm × 6.0 cm × 4.0 cm be dropped above a rigid steel floor in order to break the brick? Assume the brick strikes the floor directly on its largest face, and that the compression of the brick is much greater than that of the steel (that is, ignore compression of the steel). State other simplifying assumptions that may be necessary.

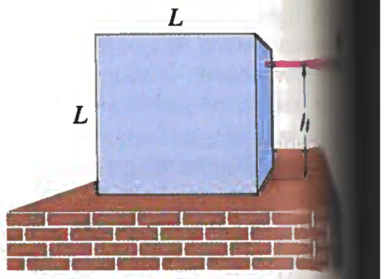

FIGURE 9–79
Problem 80.

80. A cube of side L rests on a rough floor. It is sul
ed to a steady horizontal pull, F, exerted a dist
above the floor as shown in Fig. 9–79. As F
creased, the block will either begin to slide, or
to tip over. (*a*) What must be the coefficient of
friction μ_s so that the block begins to slide
than tip? (*b*) What must be the coefficient of
friction so that the block begins to tip? [*Hint:* W
will the normal force on the block act if it tips?]

81. A man doing push-ups pauses in the position
in Fig. 9–80. His mass $m = 70\,\text{kg}$. Determine
normal force exerted by the floor (*a*) on each h
(*b*) on each foot.

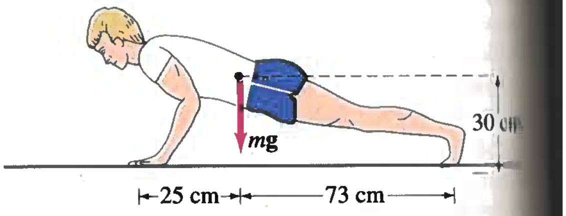

FIGURE 9–80 Problem 81.

82. A 60-kg painter is on a scaffold supported
above by ropes (Fig. 9–81). The scaffold has a
of 25 kg and is uniformly constructed. There is a 4.0
pail of paint off to one side, as shown in the fig
Can the painter walk safely to both ends of the
fold? If not, which end(s) is dangerous and
close to the end can he approach safely?

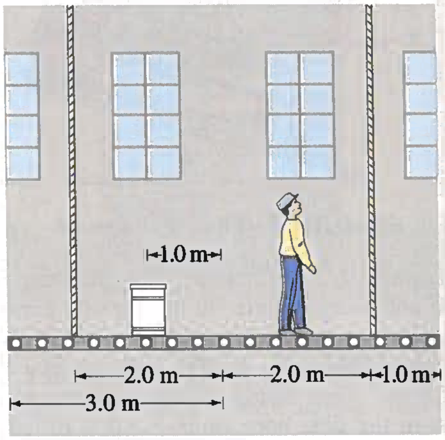

FIGURE 9–81 Problem 82.