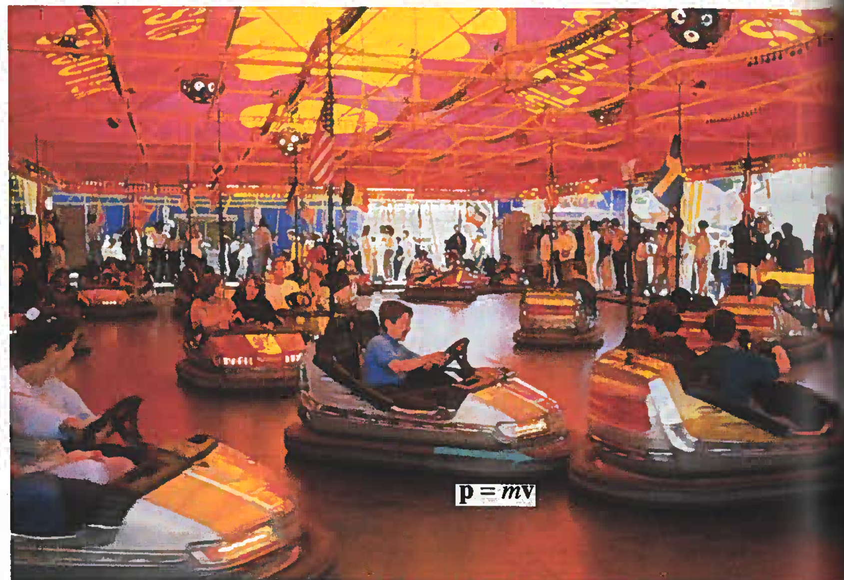


The faster a bumper car goes, and the more mass it has, the more momentum it has. When two bumper cars collide, the collision may be nearly elastic: that is, momentum and kinetic energy are conserved.

CHAPTER 7

LINEAR MOMENTUM

The law of conservation of energy, which we discussed in the previous chapter, is one of several great conservation laws in physics. Among the other quantities found to be conserved are linear momentum, angular momentum, and electric charge. We will eventually discuss all of these because the conservation laws are among the most important in all of science. In this chapter, we discuss linear momentum and its conservation. We then make use of the laws of conservation of linear momentum and of energy to analyze collisions. Indeed, the law of conservation of momentum is particularly useful when dealing with two or more bodies that interact with each other, as in collisions.

Our focus up to now has been mainly on the motion of a single object, often thought of as a "particle" in the sense that we have ignored any rotation or internal motion. In this chapter we will deal with systems of two or more particles (and with extended bodies which can be considered as collections of particles). An important concept for this study is that of center of mass, which we discuss later in the chapter.

7–1 Momentum and Its Relation to Force

The **linear momentum** (or "momentum" for short) of a body is defined as the product of its mass and its velocity. Momentum (plural is momenta) is usually represented by the symbol $\mathbf{p}$. If we let m represent the mass of a

body and $\mathbf{v}$ represent its velocity, then its momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}. \tag{7–1}$$

Linear momentum

Since velocity is a vector, momentum is a vector. The direction of the momentum is the direction of the velocity, and the magnitude of the momentum is $p = mv$. Since $\mathbf{v}$ depends on the reference frame, this frame must be specified. The unit of momentum is simply that of mass × velocity, which in SI units is kg·m/s. There is no special name for this unit.

Everyday usage of the term ***momentum*** is in accord with the definition above. For according to Eq. 7–1, a fast-moving car has more momentum than a slow-moving car of the same mass, and a heavy truck has more momentum than a small car moving with the same speed. The more momentum an object has, the harder it is to stop it, and the greater effect it will have if it is brought to rest by impact or collision. A football player is more likely to be stunned if tackled by a heavy opponent running at top speed than by a lighter or slower-moving tackler. A heavy, fast-moving truck can do more damage than a slow-moving motorcycle.

A force is required to change the momentum of an object, whether it is to increase the momentum, to decrease it (such as to bring a moving object to rest), or to change its direction. Newton originally stated his second law in terms of momentum (although he called the product mv the "quantity of motion"). Newton's statement of the **second law of motion**, translated into modern language, is as follows:

The rate of change of momentum of a body is equal to the net force applied to it.

We can write this as an equation,

$$\Sigma\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}, \tag{7–2}$$

NEWTON'S SECOND LAW

where $\Sigma\mathbf{F}$ is the net force applied to the object (the vector sum of all forces acting on it) and $\Delta\mathbf{p}$ is the resulting momentum change that occurs during the time interval† Δt. We can readily derive the familiar form of the second law, $\Sigma\mathbf{F} = m\mathbf{a}$, from Eq. 7–2 for the case of constant mass. If $\mathbf{v}_0$ is the initial velocity of an object and $\mathbf{v}$ is its velocity after a time Δt has elapsed, then

$$\Sigma\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{m\mathbf{v} - m\mathbf{v}_0}{\Delta t} = \frac{m(\mathbf{v} - \mathbf{v}_0)}{\Delta t}$$

$$= m\frac{\Delta\mathbf{v}}{\Delta t}$$

$$= m\mathbf{a} \qquad \text{[constant mass]}$$

because, by definition, $\mathbf{a} = \Delta\mathbf{v}/\Delta t$. Newton's statement, Eq. 7–2, is actually more general than the more familiar one because it includes the situation in which the mass may change. This is important in certain circumstances, such as for rockets which lose mass as they burn fuel.

† Normally we think of Δt as being a small time interval. If it is not small, then Eq. 7–2 is valid if $\Sigma\mathbf{F}$ is constant over that time interval, or if $\Sigma\mathbf{F}$ is the average net force over that time interval.

Here is a simple Example of momentum change.

EXAMPLE 7–1 **Washing a car: momentum change and force.** Water leaves a hose at a rate of 1.5 kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it, Fig. 7–1. (That is, we ignore any splashing back.) What is the force exerted by the water on the car?

SOLUTION We take the x direction positive to the right. In each second, water with a momentum of $p_x = mv_x = (1.5\,\text{kg})(20\,\text{m/s}) = 30\,\text{kg}\cdot\text{m/s}$ is brought to rest at the moment it hits the car. The magnitude of the force (assumed constant) that the car must exert to change the momentum of the water by this amount is

$$F = \frac{\Delta p}{\Delta t} = \frac{p_{\text{final}} - p_{\text{initial}}}{\Delta t} = \frac{0 - 30\,\text{kg}\cdot\text{m/s}}{1.0\,\text{s}} = -30\,\text{N}.$$

The minus sign indicates that the force on the water is opposite to the water's original velocity. The car exerts a force of 30 N to the left to stop the water, so by Newton's third law, the water exerts a force of 30 N on the car.

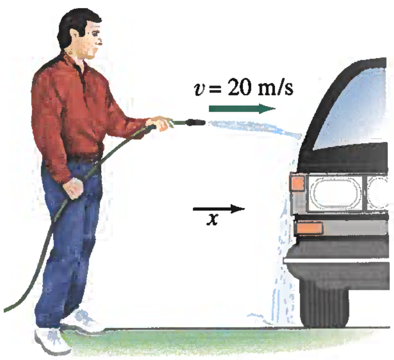


FIGURE 7–1 Example 7–1.

CONCEPTUAL EXAMPLE 7–2 **The water splashes back.** What if the water splashes back from the car in Example 7–1? Would the force on the car be greater or less?

RESPONSE If the water splashes back toward the hose, the change in momentum will be greater in magnitude, and so the force on the car will be greater in magnitude. Note that $\mathbf{p}_{\text{final}}$ will now point in the negative x direction, as shown in Fig. 7–2 (instead of being zero as in Example 7–1). So the result for F (see displayed equation in Example 7–1), will be minus something more than -30 N (i.e., -35 to -40 N, depending on the water's rebound speed). To put it simply, the car exerts not only a force to stop the water, but also an additional force to give it momentum in the opposite direction.

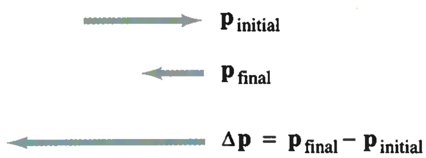
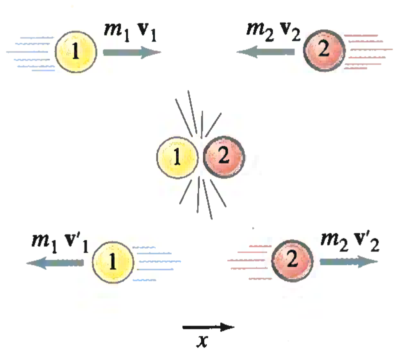


FIGURE 7–2 Conceptual Example 7–2. Momentum of water before and after splashing back, and $\Delta\mathbf{p}$.

7–2 Conservation of Momentum

The concept of momentum is particularly important because, under certain circumstances, momentum is a conserved quantity. In the mid-seventeenth century, shortly before Newton's time, it had been observed that the vector sum of the momenta of two colliding objects remains constant. Consider, for example, the head-on collision of two billiard balls, as shown in Fig. 7–3. We assume the net external force on this system of two balls is zero—that is, the only significant forces are those that each ball exerts on the other during the collision. Although the momentum of each of the two balls changes as a result of the collision, the *sum* of their momenta is found to be the same before as after the collision. If $m_1\mathbf{v}_1$ is the momentum of ball number 1 and $m_2\mathbf{v}_2$ the momentum of ball 2, both measured before the collision, then the total momentum of the two balls before the collision is $m_1\mathbf{v}_1 + m_2\mathbf{v}_2$. After the collision, the balls each have a different velocity and momentum, which we will designate by a "prime" on the velocity: $m_1\mathbf{v}'_1$

FIGURE 7–3 Momentum is conserved in a collision of two balls.



and $m_2\mathbf{v}_2'$. The total momentum after the collision is $m_1\mathbf{v}_1' + m_2\mathbf{v}_2'$. No matter what the velocities and masses involved are, it is found that the total momentum before the collision is the same as afterwards, whether the collision is head-on or not, as long as no net external force acts:

$$\text{momentum before} = \text{momentum after}$$

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'. \quad \textbf{(7–3)}$$

CONSERVATION OF MOMENTUM (for two bodies colliding)

That is, the total vector momentum of the system of two balls is conserved: it stays constant.

Although the law of conservation of momentum was discovered experimentally, it is closely connected to Newton's laws of motion and they can be shown to be equivalent. We will do a simple derivation for the one-dimensional case illustrated in Fig. 7–3. We assume the force F that one ball exerts on the other during the collision is constant over the time of collision Δt. We use Newton's second law as expressed in Eq. 7–2, and rewrite it by multiplying both sides by Δt:

$$\Delta\mathbf{p} = \mathbf{F}\Delta t. \quad \textbf{(7–4)}$$

We apply this to ball 2, noting that the force $\mathbf{F}_{21}$ on ball 2 due to ball 1 during the collision is to the right ($+x$ direction—see Fig. 7–4):

$$\Delta\mathbf{p}_2 = m_2\mathbf{v}_2' - m_2\mathbf{v}_2 = \mathbf{F}_{21}\Delta t.$$

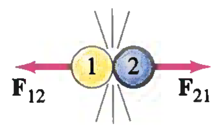


FIGURE 7–4 Forces on the balls during the collision of Fig. 7–3.

By Newton's third law, the force $\mathbf{F}_{12}$ on ball 1 due to ball 2 is $\mathbf{F}_{12} = -\mathbf{F}_{21}$ and acts to the left, so

$$\Delta\mathbf{p}_1 = m_1\mathbf{v}_1' - m_1\mathbf{v}_1 = \mathbf{F}_{12}\Delta t = -\mathbf{F}_{21}\Delta t.$$

We can combine these last two equations (their right sides differ only by a minus sign):

$$m_1\mathbf{v}_1' - m_1\mathbf{v}_1 = -(m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

or

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'$$

which is Eq. 7–3, the conservation of momentum.

The above derivation can be extended to include any number of interacting bodies. To show this in a simple way, we let $\mathbf{p}$ in Eq. 7–2 represent the total momentum of a system—that is, the vector sum of the momenta of all objects in the system. (For our two-body system above, $\mathbf{p} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$.) If the net force $\Sigma\mathbf{F}$ on the system is zero [as it was above for our two-body system, $\mathbf{F} + (-\mathbf{F}) = 0$], then from Eq. 7–2, $\Delta\mathbf{p} = \mathbf{F}\Delta t = 0$, so the total momentum doesn't change. Thus the general statement of the **law of conservation of momentum** is

LAW OF CONSERVATION OF LINEAR MOMENTUM

The total momentum of an isolated system of bodies remains constant.

By a **system**, we simply mean a set of objects that interact with each other. An **isolated system** is one in which the only forces present are those between the objects of the system. The sum of all these forces will be zero because of Newton's third law. If there are *external forces*—by which we mean forces exerted by objects outside the system—and they don't add up to zero (vectorially), then the total momentum won't be conserved. However, if the "system" can be redefined so as to include the other objects exerting these forces, then the conservation of momentum principle can apply. For example, if we take as our system a falling rock, it does n conserve momentum since an external force, the force of gravity exert by the Earth, is acting on it and its momentum changes. However, if we i clude the Earth in the system, the total momentum of rock plus Earth conserved. (This of course means that the Earth comes up to meet t ball. Since the Earth's mass is so great, its upward velocity is very tiny.)

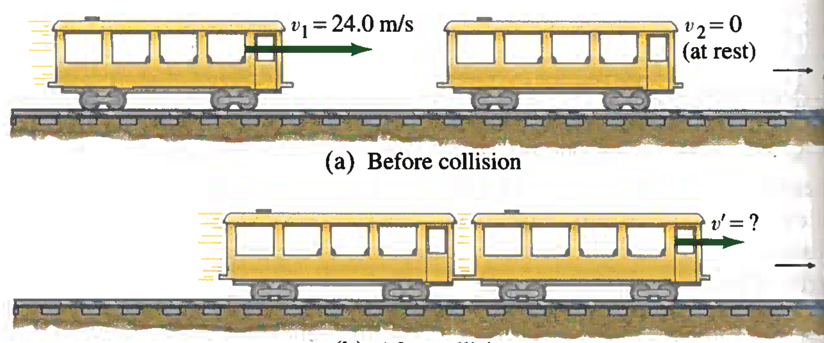


FIGURE 7–5 Example 7–3.

EXAMPLE 7–3 **Railroad cars collide: momentum conserved.** A 10,000-k railroad car traveling at a speed of 24.0 m/s strikes an identical car at res If the cars lock together as a result of the collision, what is their commo speed afterward? See Fig. 7–5.

SOLUTION The initial total momentum is

$$m_1 v_1 + m_2 v_2 = (10{,}000\text{ kg})(24.0\text{ m/s}) + (10{,}000\text{ kg})(0\text{ m/s})$$
$$= 2.40 \times 10^5\ \text{kg}\cdot\text{m/s}.$$

and is to the right in the $+x$ direction. After the collision, the total mo mentum will be the same, and it will be shared by both cars. Since th two cars become attached, they will have the same speed, call it v'. Then

$$(m_1 + m_2)v' = 2.40 \times 10^5\ \text{kg}\cdot\text{m/s}$$
$$v' = \frac{2.40 \times 10^5\ \text{kg}\cdot\text{m/s}}{2.00 \times 10^4\ \text{kg}} = 12.0\ \text{m/s},$$

to the right. Their mutual speed after collision is half the initial speed of car 1

The law of conservation of momentum is particularly useful when we are dealing with fairly simple systems such as collisions and certain types of explosions. For example, *rocket propulsion*, which we saw in Chapter 4 can be understood on the basis of action and reaction, can also be explained on the basis of the conservation of momentum. Before a rocket is fired, the total momentum of rocket plus fuel is zero. As the fuel burns, the total mo mentum remains unchanged: the backward momentum of the expelled gases is just balanced by the forward momentum gained by the rocket itself (see Fig. 7–6). Thus, a rocket can accelerate in empty space. There is no need for the expelled gases to push against the Earth or the air (as is some times erroneously thought), as we already discussed in Chapter 4. Similar examples are the recoil of a gun and the throwing of a package from a boat

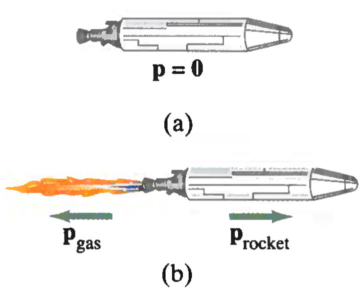


FIGURE 7–6 (a) A rocket, containing fuel, at rest in some reference frame. (b) In the same reference frame, the rocket fires and gases are expelled at high speed out the rear. The total vector momentum, $\mathbf{p}_{gas} + \mathbf{p}_{rocket}$, remains zero.

EXAMPLE 7–4 **Rifle recoil.** Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.050-kg bullet at a speed of 120 m/s, Fig. 7–7.

SOLUTION The total momentum of the system is conserved. We let the subscripts B represent the bullet and R the rifle; the final velocities are indicated by primes. Then conservation of momentum in the x direction gives

$$m_B v_B + m_R v_R = m_B v'_B + m_R v'_R$$

$$0 + 0 = (0.050\ \text{kg})(120\ \text{m/s}) + (5.0\ \text{kg})(v'_R)$$

$$v'_R = -\frac{(0.050\ \text{kg})(120\ \text{m/s})}{(5.0\ \text{kg})} = -1.2\text{m/s}.$$

Since the rifle has a much larger mass, its (recoil) velocity is much less than that of the bullet. The minus sign indicates that the velocity (and momentum) of the rifle is in the negative x direction, opposite to that of the bullet. Notice that it is the *vector sum* of the momenta that is conserved.

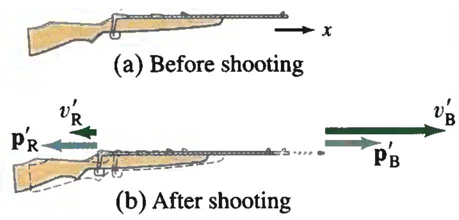


FIGURE 7–7 Example 7–4.

7–3 Collisions and Impulse

Conservation of momentum is a very useful tool for dealing with collision processes, as we already saw in the examples of the previous section. Collisions are a common occurrence in everyday life: a tennis racket or a baseball bat striking a ball, two billiard balls colliding, one railroad car striking another, a hammer hitting a nail. At the subatomic level, scientists learn about the structure of nuclei and their constituents, and about the nature of the forces involved, by careful study of collisions between nuclei and/or elementary particles.

In a collision of two ordinary objects, both objects are deformed, often considerably, because of the large forces involved (Fig. 7–8). When the collision occurs, the force usually jumps from zero at the moment of contact to a very large value within a very short time, and then abruptly returns to zero again. A graph of the magnitude of the force one object exerts on the other during a collision, as a function of time, is something like that shown by the red curve in Fig. 7–9. The time interval Δt is usually very distinct and usually very small.

From Newton's second law, Eq. 7–2, the *net* force on an object is equal to the rate of change of its momentum:

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

(We have written $\mathbf{F}$ instead of $\Sigma\mathbf{F}$ for the net force, which we assume is entirely due to the brief but large force that acts during the collision.) This equation applies, of course, to *each* of the objects in a collision. If we multiply both sides of this equation by the time interval Δt, we obtain

$$\text{Impulse} = \mathbf{F}\,\Delta t = \Delta \mathbf{p}. \qquad (7\text{–}5)$$

The quantity on the left, the product of the force $\mathbf{F}$ times the time Δt over which the force acts, is called the **impulse**. We see that the total change in momentum is equal to the impulse. The concept of impulse is of help

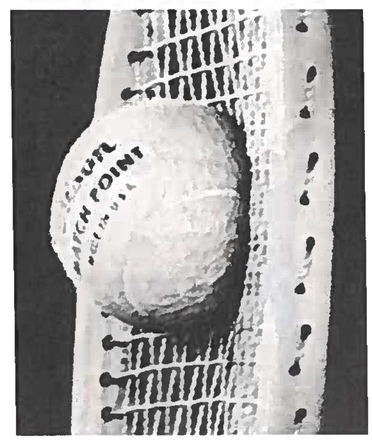
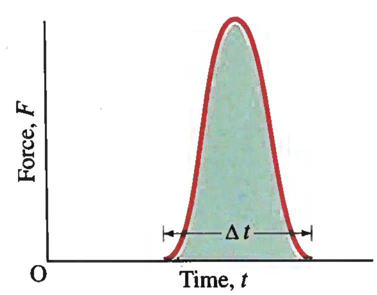


FIGURE 7–8 Tennis racket striking a ball. Note the deformation of both ball and racket due to the large force each exerts on the other.

FIGURE 7–9 Force as a function of time during a typical collision.



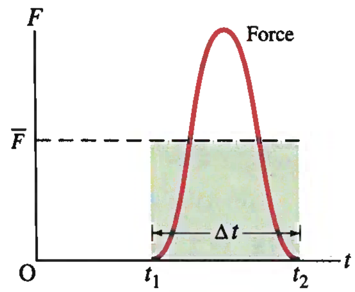
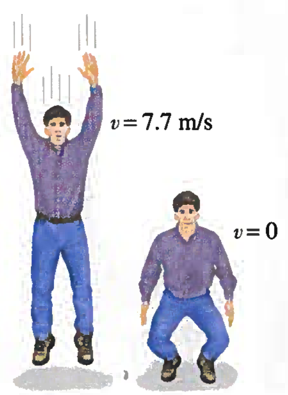


FIGURE 7–10 The average force $\overline{F}$ acting over an interval of time Δt gives the same impulse ($\overline{F}\,\Delta t$) as the actual force.

FIGURE 7–11 Period during which impulse acts (Example 7–5).



mainly when dealing with forces that act over a short time, as when a bat hits a baseball. The force is generally not constant and often its variation in time is like that graphed in Fig. 7–10. It is often sufficient to approximate such a varying force by an average force $\overline{F}$ acting over a time Δt, as indicated by the dashed line in Fig. 7–10. $\overline{F}$ is chosen so that the area shown shaded in Fig. 7–10 (equal to $\overline{F} \times \Delta t$) is equal to the area under the actual curve of F vs. t (which area represents the impulse). Note from Eq. 7–5 that the same impulse, and same change in momentum, can be given to an object by a smaller force F if the time Δt over which it acts is greater, as long as the product, $F \times \Delta t$, remains the same.

EXAMPLE 7–5 **Bend your knees when landing.** (*a*) Calculate the impulse experienced when a 70-kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (*b*) stiff-legged, and (*c*) with bent legs. In the former case, assume the body moves 1.0 cm during impact, and in the second case, when the legs are bent, about 50 cm.

SOLUTION (*a*) Although we don't know F and thus can't calculate the impulse $F\Delta t$ directly, we can use the fact that the impulse equals the change in momentum of the object. We need to determine the velocity of the person just before striking the ground, which we can do using conservation of energy (Eq. 6–11a):

$$\Delta \text{KE} = -\Delta \text{PE}$$

$$\tfrac{1}{2}mv^2 - 0 = -mg(y - y_0),$$

where we assume he started from rest ($v_0 = 0$), and $y_0 = 3.0$ m and $y = 0$. Thus, after falling 3.0 m, the person's velocity just before hitting the ground will be

$$v = \sqrt{2g(y_0 - y)} = \sqrt{2(9.8\ \text{m/s}^2)(3.0\ \text{m})} = 7.7\ \text{m/s}.$$

As the person strikes the ground, the momentum is quickly brought to zero, Fig. 7–11. The impulse on the person is

$$\overline{F}\Delta t = \Delta p = p - p_0$$
$$= 0 - (70\ \text{kg})(7.7\ \text{m/s}) = -540\ \text{N·s}.$$

The negative sign tells us that the force is opposed to the original momentum—that is, the force acts upward.

(*b*) In coming to rest, the body decelerates from 7.7 m/s to zero in a distance $d = 1.0\ \text{cm} = 1.0 \times 10^{-2}$ m. The average speed during this brief period is

$$\overline{v} = \frac{(7.7\ \text{m/s} + 0\ \text{m/s})}{2} = 3.8\ \text{m/s},$$

so the collision lasts a time

$$\Delta t = \frac{d}{\overline{v}} = \frac{(1.0 \times 10^{-2}\ \text{m})}{(3.8\ \text{m/s})} = 2.6 \times 10^{-3}\ \text{s}.$$

Since the magnitude of the impulse is $\overline{F}\Delta t = 540$ N·s, and $\Delta t = 2.6 \times 10^{-3}$ s, the average net force $\overline{F}$ has magnitude

$$\overline{F} = \frac{540\ \text{N·s}}{2.6 \times 10^{-3}\ \text{s}} = 2.1 \times 10^5\ \text{N}.$$

The force $\overline{F}$ is the *net* force upward on the person (we calculated it from

Newton's second law). $\bar{F}$ equals the vector sum of the average force upward on the legs exerted by the ground, F_{grd}, which we take as positive, plus the downward force of gravity, $-mg$ (see Fig. 7–12):

$$\bar{F} = F_{grd} - mg.$$

Since $mg = (70\,\mathrm{kg})(9.8\,\mathrm{m/s^2}) = 690\,\mathrm{N}$, then

$$F_{grd} = \bar{F} + mg = 2.1 \times 10^5\,\mathrm{N} + 0.690 \times 10^3\,\mathrm{N} \approx 2.1 \times 10^5\,\mathrm{N}.$$

(*c*) This is just like part (*b*), except $d = 0.50\,\mathrm{m}$, so $\Delta t = (0.50\,\mathrm{m})/(3.8\,\mathrm{m/s}) = 0.13\,\mathrm{s}$, and

$$\bar{F} = \frac{540\,\mathrm{N \cdot s}}{0.13\,\mathrm{s}} = 4.2 \times 10^3\,\mathrm{N}.$$

The upward force exerted on the person's feet by the ground is, as in part (*b*):

$$F_{grd} = \bar{F} + mg = 4.2 \times 10^3\,\mathrm{N} + 0.69 \times 10^3\,\mathrm{N} = 4.9 \times 10^3\,\mathrm{N}.$$

Clearly, the force on the feet and legs is much less when the knees are bent. In fact, the ultimate strength of the leg bone (see Chapter 9, Table 9–2) is not great enough to support the force calculated in part (*b*), so the leg would likely break in such a stiff landing, whereas it probably wouldn't in part (*c*).

PROBLEM SOLVING

Free-body diagrams are always useful!

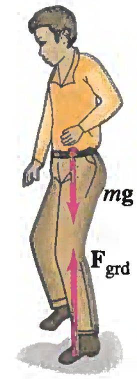


FIGURE 7–12 When the person lands on the ground, the average net force during impact is $\bar{F} = F_{grd} - mg$, where F_{grd} is the force the ground exerts upward on the person.

7–4 Conservation of Energy and Momentum in Collisions

During most collisions, we usually don't know how the collision force varies over time, and so analysis using Newton's second law becomes difficult or impossible. But we can still determine a lot about the motion after a collision, given the initial motion, by making use of the conservation laws for momentum and energy. We saw in Section 7–2 that in the collision of two objects such as billiard balls, the total momentum is conserved. If the two objects are very hard and no heat is produced in the collision, then kinetic energy is conserved as well. By this we mean that the sum of the kinetic energies of the two objects is the same after the collision as before. Of course, for the brief moment during which the two objects are in contact, some (or all) of the energy is stored momentarily in the form of elastic potential energy. But if we compare the total kinetic energy before the collision with the total after the collision, they are found to be the same. Such a collision, in which the total kinetic energy is conserved, is called an **elastic collision**. If we use the subscripts 1 and 2 to represent the two objects, we can write the equation for conservation of total kinetic energy as

Elastic collision

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2. \qquad \text{[elastic collision]} \qquad \textbf{(7–6)}$$

KE conserved in elastic collisions

Here, primed quantities (′) mean after the collision and unprimed mean before the collision, just as in Eq. 7–3 for conservation of momentum.

Although at the atomic level the collisions of atoms and molecules are often elastic, in the "macroscopic" world of ordinary objects, an elastic collision is an ideal that is never quite reached, since at least a little thermal energy (and perhaps sound and other forms of energy) is always produced during a collision. The collision of two hard elastic balls, such as billiard

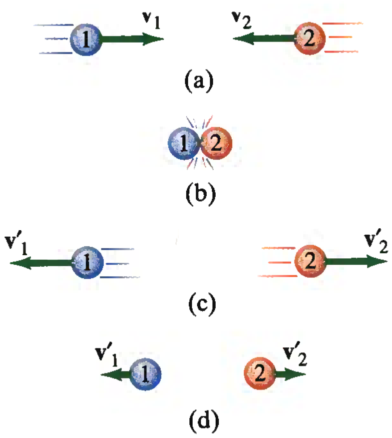


FIGURE 7–13 Two equal mass objects (a) approach each other with equal speeds, (b) collide, and then (c) bounce off with equal speeds in the opposite directions if the collision is elastic, or (d) bounce back much less or not at all, if the collision is inelastic.

balls, however, is very close to perfectly elastic, and we often treat it as such. Even when the kinetic energy is not conserved, the *total* energy is, of course, always conserved.

Collisions in which kinetic energy is not conserved are said to be **inelastic collisions**. The kinetic energy that is lost is changed into other forms of energy, often thermal energy, so that the total energy (as always) is conserved. In this case, we can write that

$$\text{KE}_1 + \text{KE}_2 = \text{KE}'_1 + \text{KE}'_2 + \text{thermal and other forms of energy.}$$

See Fig. 7–13.

7–5 Elastic Collisions in One Dimension—Solving Problems Using Energy and Momentum Conservation

We now apply the conservation laws for momentum and kinetic energy to an elastic collision between two small objects (particles) that collide head-on, so all the motion is along a line. Let us assume that both particles are initially moving with velocities v_1 and v_2 along the x axis, Fig. 7–14a. After the collision, their velocities are v'_1 and v'_2, Fig. 7–14b. For any $v > 0$, the particle is moving to the right (increasing x), whereas for $v < 0$, the particle is moving to the left (toward decreasing values of x).

From conservation of momentum, we have

Momentum conservation

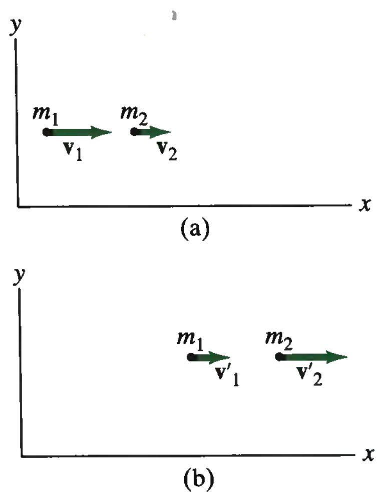
$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2.$$

Because the collision is assumed to be elastic, kinetic energy is also conserved:

KE conservation

$$\tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_1 v_1'^2 + \tfrac{1}{2} m_2 v_2'^2.$$

FIGURE 7–14 Two particles, of masses m_1 and m_2, (a) before the collision, and (b) after the collision.



We have two equations, so we can solve for two unknowns. If we know the masses and initial velocities, then we can solve these two equations for the velocities after the collision, v'_1 and v'_2. We will do this in a moment in some Examples, but first we derive a useful result. To do so we rewrite the momentum equation as

$$m_1(v_1 - v'_1) = m_2(v'_2 - v_2), \qquad \text{(i)}$$

and we rewrite the KE equation as

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

or [noting that $(a - b)(a + b) = a^2 - b^2$] we write this as

$$m_1(v_1 - v'_1)(v_1 + v'_1) = m_2(v'_2 - v_2)(v'_2 + v_2). \qquad \text{(ii)}$$

We divide Eq. (ii) by Eq. (i), and (assuming $v_1 \neq v'_1$ and $v_2 \neq v'_2$) and obtain

$$v_1 + v'_1 = v'_2 + v_2.$$

We can rewrite this equation as

$$v_1 - v_2 = v'_2 - v'_1$$
$$= -(v'_1 - v'_2). \qquad \text{[head-on elastic collision]} \quad \text{(7–7)}$$

This is an interesting result: it tells us that for any elastic head-on collision the relative speed of the two particles after the collision has the same magnitude as before (but opposite direction), no matter what the masses are.

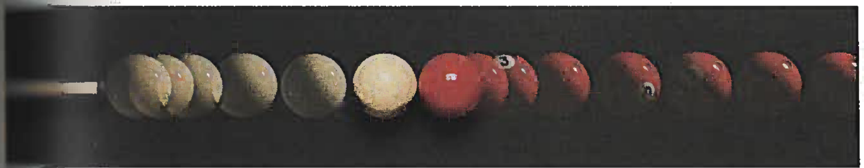


FIGURE 7–15 In this multi-flash photo of a head-on collision between two balls of equal mass, the white cue ball is accelerated from rest by the cue stick and then strikes the red ball, initially at rest. The white ball stops in its tracks and the (equal mass) red ball moves off with the same speed as the white ball had before the collision. See Example 7–6.

EXAMPLE 7–6 **Pool or billiards.** A billiard ball of mass m moving with speed v collides head-on with a second ball of equal mass at rest ($v_2 = 0$). What are the speeds of the two balls after the collision, assuming it is elastic?

SOLUTION Since $v_1 = v$ and $v_2 = 0$, and $m_1 = m_2 = m$, then conservation of momentum gives

$$mv = mv_1' + mv_2'$$

or

$$v = v_1' + v_2'$$

since the m's cancel out. We have two unknowns (v_1' and v_2'), so we need a second equation, which could be the conservation of kinetic energy, or the simpler Eq. 7–7 we derived from it, which gives

$$v_1 - v_2 = v_2' - v_1'$$

or

$$v = v_2' - v_1'.$$

We subtract this equation from our momentum equation ($v = v_1' + v_2'$) and obtain

$$0 = 2v_1',$$

so $v_1' = 0$. This is one of our desired unknowns, and we can now solve for the other:

$$v_2' = v + v_1' = v + 0 = v.$$

To summarize, before the collision we have

$$v_1 = v, \qquad v_2 = 0$$

and after the collision

$$v_1' = 0, \qquad v_2' = v.$$

That is, ball 1 is brought to rest by the collision, whereas ball 2 acquires the original velocity of ball 1. This result is often observed by billiard and pool players, and is valid only if the two balls have equal masses (and no spin is given to the balls). See Fig. 7–15.

EXAMPLE 7–7 **A nuclear collision.** A proton of mass 1.01 u (unified atomic mass units) traveling with a speed of 3.60×10^4 m/s has an elastic head-on collision with a helium (He) nucleus ($m_{He} = 4.00$ u) initially at rest. What are the velocities of the proton and helium nucleus after the collision? (As mentioned in Chapter 1, $1\,u = 1.66 \times 10^{-27}$ kg, but we won't need this fact.)

SOLUTION Call the initial direction of motion the $+x$ direction. We have $v_2 = v_{He} = 0$ and $v_1 = v_p = 3.60 \times 10^4$ m/s. We want to find the velocities v'_p and v'_{He} after the collision. From conservation of momentum we have

$$m_p v_p + 0 = m_p v'_p + m_{He} v'_{He}.$$

Because the collision is elastic, kinetic energy is conserved and we can use Eq. 7–7, which becomes

$$v_p - 0 = v'_{He} - v'_p.$$

Thus

$$v'_p = v'_{He} - v_p,$$

and substituting this into the momentum equation we get

$$m_p v_p = m_p v'_{He} - m_p v_p + m_{He} v'_{He}.$$

Solving for v'_{He}, we obtain

$$v'_{He} = \frac{2 m_p v_p}{m_p + m_{He}} = \frac{2(1.01\,\text{u})(3.60 \times 10^4\,\text{m/s})}{5.01\,\text{u}} = 1.45 \times 10^4\,\text{m/s}.$$

The other unknown is v'_p, which we can now obtain from

$$\begin{aligned} v'_p &= v'_{He} - v_p \\ &= 1.45 \times 10^4\,\text{m/s} - 3.60 \times 10^4\,\text{m/s} = -2.15 \times 10^4\,\text{m/s}. \end{aligned}$$

The minus sign tells us that the proton reverses direction upon collision, and we see that its speed is less than its initial speed (see Fig. 7–16). This makes sense from ordinary experience: the lighter proton would be expected to "bounce back" somewhat from the more massive helium nucleus, but not with its full original velocity as it would from a rigid wall (which would correspond to extremely large, or infinite, mass).

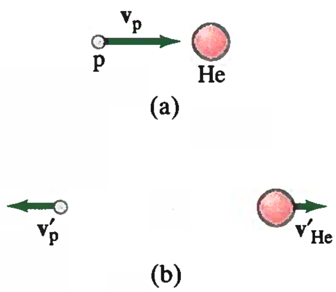


FIGURE 7–16 Example 7–7: (a) before collision, (b) after collision.

7–6 Inelastic Collisions

Completely inelastic collision

Collisions in which kinetic energy is not conserved are called inelastic collisions. Some of the initial kinetic energy in such collisions is transformed into other types of energy, such as thermal or potential energy, so the total final kinetic energy is less than the total initial kinetic energy. The inverse can also happen when potential energy (such as chemical or nuclear) is released, in which case the total final kinetic energy can be greater than the initial kinetic energy. Explosives are examples of this type. Typical macroscopic collisions are inelastic, at least to some extent, and often to a large extent. If two objects stick together as a result of a collision, the collision is said to be **completely inelastic.** Two colliding balls of putty that stick together or two railroad cars that couple together when they collide are examples of completely inelastic collisions. The kinetic energy in some cases is all transformed to other forms of energy in an inelastic collision, but in other cases only part of it is. In Example 7–3, for instance, we saw that when a traveling railroad car collided with a stationary one, the coupled cars traveled off with some kinetic energy.

In a completely inelastic collision, the maximum amount of kinetic energy is transformed to other forms consistent with conservation of momentum. Even though kinetic energy is not conserved in inelastic collisions, the total energy is conserved, and the total vector momentum is also always conserved.

EXAMPLE 7–8 **Railroad cars again.** For the completely inelastic collision of two railroad cars that we considered in Example 7–3, calculate how much of the initial kinetic energy is transformed to thermal or other forms of energy.

SOLUTION Initially, the total kinetic energy is

$$\tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}(10{,}000\text{ kg})(24.0\text{ m/s})^2 = 2.88 \times 10^6\text{ J}.$$

After the collision, the total kinetic energy is

$$\tfrac{1}{2}(20{,}000\text{ kg})(12.0\text{ m/s})^2 = 1.44 \times 10^6\text{ J}.$$

Hence the energy transformed to other forms is

$$2.88 \times 10^6\text{ J} - 1.44 \times 10^6\text{ J} = 1.44 \times 10^6\text{ J},$$

which is just half the original KE.

Ballistic pendulum

EXAMPLE 7–9 **Ballistic pendulum.** The *ballistic pendulum* is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m, is fired into a large block (of wood or other material) of mass M, which is suspended like a pendulum. (Usually, M is somewhat greater than m.) As a result of the collision, the pendulum-projectile system swings up to a maximum height h, Fig. 7–17. Determine the relationship between the initial speed of the projectile, v, and the height h.

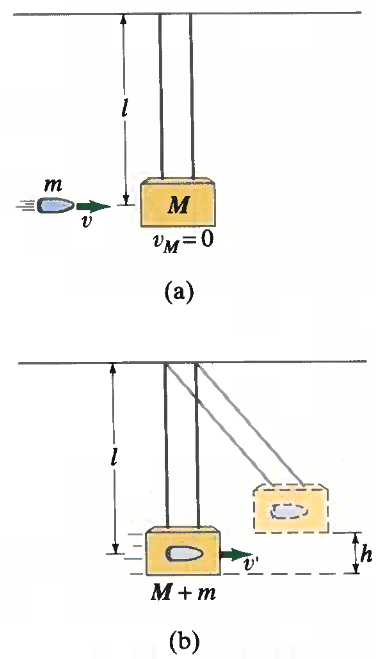


FIGURE 7–17 Ballistic pendulum (Example 7–9).

SOLUTION We analyze this process by dividing it into two parts: (1) the collision itself, and (2) the subsequent motion of the pendulum from the vertical hanging position to height h. In part (1), Fig. 7–17a, we assume the collision time is very short, and so the projectile comes to rest in the block before the block has moved significantly from its position directly below its support. Thus there is no net external force and momentum is conserved:

$$mv = (m + M)v', \qquad \textbf{(i)}$$

where v' is the speed of the block and embedded projectile just after the collision, before they have moved significantly. Once the pendulum begins to move (part 2, Fig. 7–17b), there will be a net external force (gravity, tending to pull it back to the vertical position). So, for part (2), we cannot use conservation of momentum. But we can use conservation of mechanical energy since the kinetic energy immediately after the collision is changed entirely to gravitational potential energy when the pendulum reaches its maximum height, h. Therefore (letting $y = 0$ for the pendulum in the vertical position):

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2$$

or

$$\tfrac{1}{2}(m + M)v'^2 + 0 = 0 + (m + M)gh, \qquad \textbf{(ii)}$$

so $v' = \sqrt{2gh}$. We combine equations (i) and (ii) to obtain

$$v = \frac{m + M}{m}v' = \frac{m + M}{m}\sqrt{2gh},$$

which is the final result. To obtain this result, we had to be opportunistic, in that we used whichever conservation laws we could: in (1) we could use only conservation of momentum, since the collision is inelastic and conservation of mechanical energy is not valid†; and in (2), conservation of mechanical energy is valid, but not conservation of momentum. In part (1), if there were significant motion of the pendulum during the deceleration of the projectile in the block, then there *would* be an external force during the collision—so conservation of momentum would not be valid, and this would have to be taken into account.

* 7–7 Collisions in Two or Three Dimensions

Conservation of momentum and energy can also be applied to collisions in two or three dimensions, and the vector nature of momentum is especially important. One common type of non-head-on collision is that in which a moving particle (called the "projectile") strikes a second particle initially at rest (the "target" particle). This is the common situation in games such as billiards, and for experiments in atomic and nuclear physics (the projectiles, from radioactive decay or a high-energy accelerator, strike a stationary target nucleus).

Figure 7–18 shows particle 1 (the projectile, m_1) heading along the x axis toward particle 2 (the target, m_2), which is initially at rest. If these are billiard balls, m_1 strikes m_2 and they go off at the angles θ_1' and θ_2', respectively, which are measured relative to m_1's initial direction (the x axis). The particles may begin to deflect even before they touch if electric, magnetic, or nuclear forces act between them.‡

Let us apply the law of conservation of momentum to a collision like that of Fig. 7–18. We choose the xy plane to be the plane in which the initial and final momenta lie. Because momentum is a vector, and is conserved, its components in the x and y directions remain constant. In the x direction,

$$p_{1x} + p_{2x} = p_{1x}' + p_{2x}'$$

or

p_x conserved

$$m_1 v_1 = m_1 v_1' \cos\theta_1' + m_2 v_2' \cos\theta_2'. \qquad \text{(7–8a)}$$

Because there is no motion in the y direction initially, the y component of

†Total energy of course is conserved.

‡You might think, for example, of two magnets oriented so that they repel each other: when one moves toward the other, the second moves away before the first one touches it.

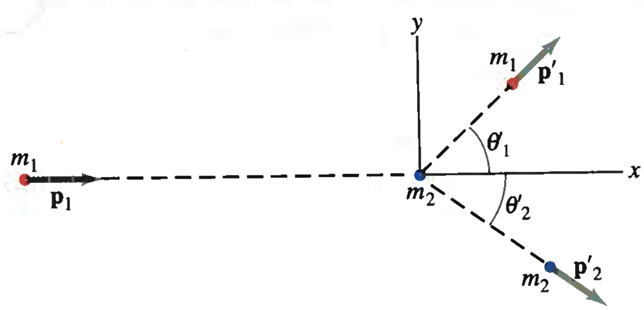


FIGURE 7–18 Particle 1, the projectile, collides with particle 2, the target. They move off, after the collision, with momenta $\mathbf{p}_1'$ and $\mathbf{p}_2'$ at angles θ_1' and θ_2'.

The total momentum is zero:

$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$$

or

$$0 = m_1 v'_1 \sin\theta'_1 + m_2 v'_2 \sin\theta'_2. \qquad \textbf{(7–8b)}$$

p_y conserved

EXAMPLE 7–10 **Billiard ball collision in 2-D.** A billiard ball moving with speed $v_1 = 3.0\,\text{m/s}$ in the $+x$ direction (Fig. 7–18) strikes an equal-mass ball initially at rest. The two balls are observed to move off at 45°, ball 1 above the x axis and ball 2 below. That is $\theta'_1 = 45°$ and $\theta'_2 = -45°$ in Fig. 7–18. What are the speeds of the two balls?

SOLUTION From symmetry we might guess that the two balls have the same speed. But let us not assume that now. Even though we aren't told whether the collision is elastic or inelastic, we can still always use conservation of momentum. So we can apply Eqs. 7–8a and b, and we want to solve for v'_1 and v'_2. We are given $m_1 = m_2 (= m)$ so

$$mv_1 = mv'_1\cos(45°) + mv'_2\cos(-45°)$$

and

$$0 = mv'_1\sin(45°) + mv'_2\sin(-45°).$$

The m's cancel out in both equations. The second equation yields [recall $\sin(-\theta) = -\sin\theta$]:

$$v'_2 = -v'_1\frac{\sin(45°)}{\sin(-45°)} = -v'_1\left(\frac{\sin 45°}{-\sin 45°}\right) = v'_1,$$

so they do have equal speeds as we guessed at first. The x component equation gives [recall $\cos(-\theta) = \cos\theta$]:

$$v_1 = v'_1\cos(45°) + v'_2\cos(45°) = 2v'_1\cos(45°)$$

so

$$v'_1 = v'_2 = \frac{v_1}{2\cos(45°)} = \frac{3.0\,\text{m/s}}{2(0.707)} = 2.1\,\text{m/s}.$$

When we have two independent equations, we can solve for, at most, two unknowns.

If we know that a collision is elastic, we can then apply conservation of kinetic energy and obtain a third equation:

$$\text{KE}_1 + \text{KE}_2 = \text{KE}'_1 + \text{KE}'_2$$

or, for the collision shown in Fig. 7–18,

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}m_1 v_1'^2 + \tfrac{1}{2}mv_2'^2. \qquad \text{[elastic collision]} \quad \textbf{(7–8c)}$$

KE conserved

If the collision is elastic, we have three independent equations and we can solve for three unknowns. If we are given m_1, m_2, v_1 (and v_2, if it is not zero), we cannot, for example, predict the final variables, v'_1, v'_2, θ'_1, and θ'_2, because there are four of them. However, if we measure one of these variables, say θ'_1, then the other three variables (v'_1, v'_2, and θ'_2) are uniquely determined, and we can determine them using Eqs. 7–8a, b, c.

PROBLEM SOLVING Momentum Conservation and Collisions

1. Be sure no significant external force acts on your chosen system. That is, the forces that act between the interacting bodies must be the only significant ones if momentum conservation is to be used. [Note: If this is valid for a portion of the problem, you can use momentum conservation for that portion only.]
2. Draw a diagram of the initial situation, just before the interaction (collision, explosion) takes place, and represent the momentum of each object with an arrow and label. Do the same for the final situation, just after the interaction.
3. Choose a coordinate system and "+" and "−" directions. (For a head-on collision, you will need only an x axis.) It is often convenient to choose the $+x$ axis in the direction of one object's initial velocity.
4. Write momentum conservation equation(s):

   total initial momentum = total final momentum

   You have one equation for each component (x, y, z); only one equation for a head-on collision. [Don't forget that it is the *total* momentum, not the individual momenta, that is conserved.]
5. If the collision is elastic, you can also write down a conservation of kinetic energy equation:

   total initial KE = total final KE.

   [Alternately, you could use Eq. 7–7: $v_1 - v_2 = v_2' - v_1'$, if the collision is one dimensional (head-on).]
6. Solve algebraically for the unknown(s).

7–8 Center of Mass (CM)

Until now, we have been mainly concerned with the motion of single particles. When we have dealt with an extended body (that is, a body that has size), we have assumed that it could be approximated as a point particle or that it underwent only translational motion. Real "extended" bodies, however, can undergo rotational and other types of motion as well. For example, the diver in Fig. 7–19a undergoes only translational motion (all parts of the body follow the same path), whereas the diver in Fig. 7–19b undergoes

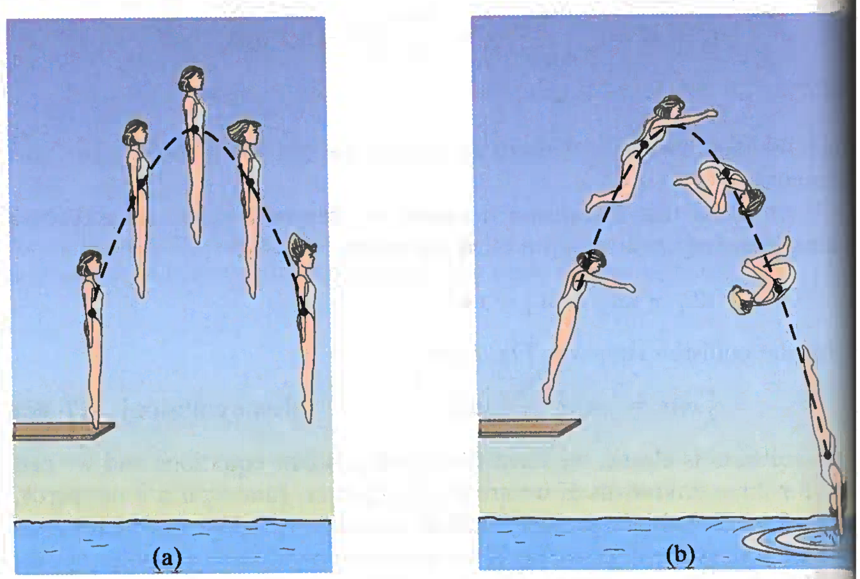


FIGURE 7–19 The motion of the diver is pure translation in (a), but is translation plus rotation in (b).

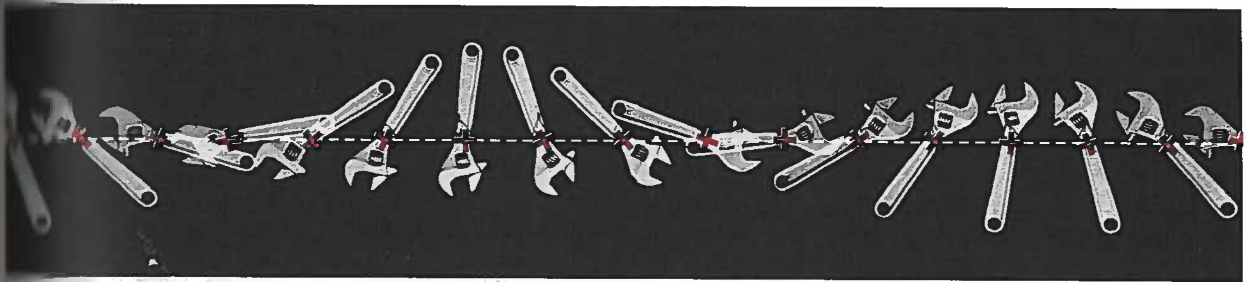


FIGURE 7–20 Translation plus rotation: a wrench moving over a horizontal surface. The CM, marked with a +, moves in a straight line.

both translational and rotational motion. We will refer to motion that is not pure translation as *general motion*.

Observations of the motion of bodies indicate that even if a body rotates, or there are several bodies that move relative to one another, there is one point that moves in the same path that a particle would if subjected to the same net force. This point is called the **center of mass** (abbreviated CM). The general motion of an extended body (or system of bodies) can be considered as *the sum of the translational motion of the* CM, *plus rotational, vibrational, or other types of motion about the* CM.

Center of mass and general motion

As an example, consider the motion of the center of mass of the diver in Fig. 7–19: the CM follows a parabolic path even when the diver rotates, as shown in Fig. 7–19b. This is the same parabolic path that a projected particle follows when acted on only by the force of gravity (that is, projectile motion). Other points in the rotating diver's body follow more complicated paths.

Figure 7–20 shows a wrench translating and rotating along a horizontal surface—note that its CM, marked by a red +, moves in a straight line, as shown by the dashed white line.

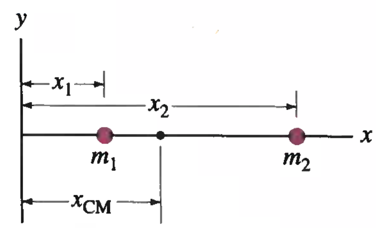


FIGURE 7–21 The center of mass of a two-particle system lies on the line joining the two masses.

The CM is defined in the following way. We can consider any extended body as being made up of many tiny particles. But first we consider a system made up of only two particles, of mass m_1 and m_2. We choose a coordinate system so that both particles lie on the x axis at positions x_1 and x_2, Fig. 7–21. The center of mass of this system is defined to be at the position, x_{CM}, given by

Center of mass (x coordinate)

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1x_1 + m_2x_2}{M}, \qquad \textbf{(7–9a)}$$

where $M = m_1 + m_2$ is the total mass of the system. The center of mass lies on the line joining m_1 and m_2. If the two masses are equal ($m_1 = m_2 = m$), x_{CM} is midway between them, since in this case

$$x_{CM} = \frac{m(x_1 + x_2)}{2m} = \frac{(x_1 + x_2)}{2}. \qquad \text{[equal masses]}$$

If one mass is greater than the other, say, $m_1 > m_2$, then the CM is closer to the larger mass. If there are more than two particles along a line, there will be additional terms in Eq. 7–9a, as the following Example shows.

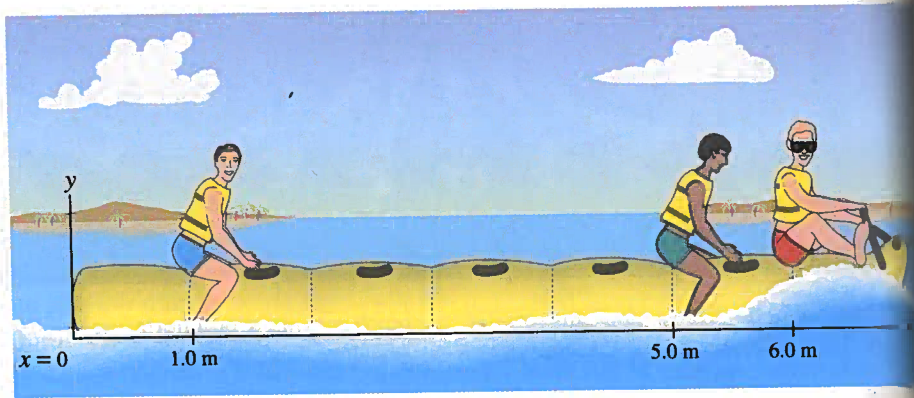


FIGURE 7–22
Example 7–11.

EXAMPLE 7–11 **CM of three guys on a raft.** Three people of rough equivalent mass m on a lightweight (air-filled) banana boat sit along th x axis at positions $x_1 = 1.0\,\text{m}$, $x_2 = 5.0\,\text{m}$, and $x_3 = 6.0\,\text{m}$ (Fig. 7–22). Find the position of the CM.

SOLUTION We use Eq. 7–9a with a third term:

$$x_{\text{CM}} = \frac{mx_1 + mx_2 + mx_3}{m + m + m} = \frac{m(x_1 + x_2 + x_3)}{3m}$$

$$= \frac{(1.0\,\text{m} + 5.0\,\text{m} + 6.0\,\text{m})}{3} = \frac{12.0\,\text{m}}{3} = 4.0\,\text{m}.$$

If the particles are spread out in two or three dimensions, then we need t specify not only the x coordinate of the CM (x_{CM}), but also the y and z coordi nates, which will be given by formulas just like Eq. 7–9a. For example, for tw particles of mass m_1 and m_2, whose y coordinates are y_1 and y_2, respectivel the y coordinate of their CM, y_{CM}, will be:

y coordinate of center of mass

$$y_{\text{CM}} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m_1 y_1 + m_2 y_2}{M}. \qquad \text{(7–9b)}$$

For more particles, there would be more terms in this formula.

Center of gravity

A concept similar to *center of mass* is **center of gravity** (CG). The CG of a body is that point at which the force of gravity can be considered to act. (O course, the force of gravity actually acts on all the different parts or particle of a body, but for purposes of determining the translational motion of body as a whole, we can assume that the entire weight of the body (which i the sum of the weights of all its parts) acts at the CG. Strictly speaking, ther is a conceptual difference between the center of gravity and the center o mass, but for practical purposes, they are generally at the same point.[†]

It is often easier to determine the CM or CG of an extended body experi mentally rather than analytically. If a body is suspended from any point, i will swing (Fig. 7–23) unless it is placed so its CG lies on a vertical lin directly below the point from which it is suspended. If the object is two dimensional, or has a plane of symmetry, it need only be hung from tw

[†]There would be a difference between the two points only if a body were large enough that the acceleration due to gravity was different at different parts of the body.

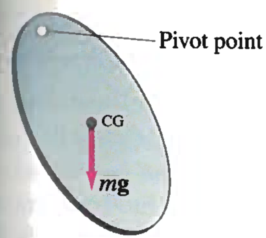


FIGURE 7–23 The force of gravity, considered to act at the CG, causes the body to rotate about the pivot point unless the CG is on a vertical line directly below the pivot, in which case the body remains at rest.

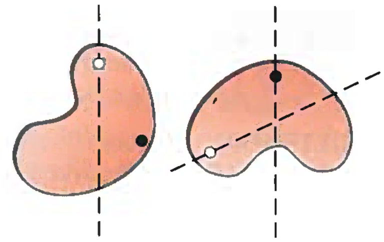


FIGURE 7–24 Finding the CG.

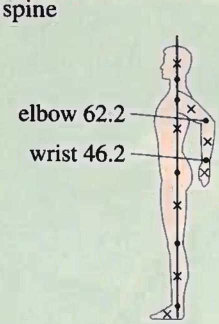
different pivot points and the respective vertical (plumb) lines drawn. Then the center of gravity will be at the intersection of the two lines, as in Fig. 7–24. If the object doesn't have a plane of symmetry, the CG with respect to the third dimension is found by suspending the object from at least three points whose plumb lines do not lie in the same plane. For symmetrically shaped bodies such as uniform cylinders (wheels), spheres, and rectangular solids, the CG is located at the geometric center of the body.

7–9 CM for the Human Body

If we have a group of extended bodies, each of whose CM is known, we can find the CM of the group using Eqs. 7–9a and b. As an example, we consider the human body. Table 7–1 indicates the CM and hinge points (joints) for the different components of a "representative" person. Of course, there are wide variations among people, so these data represent only a very rough average. Note that the numbers represent a percentage of the total height, which is regarded as 100 units; similarly the total mass is 100 units. Thus, for example, if a person is 1.70 m tall, his or her shoulder joint would be (1.70 m)(81.2/100) = 1.38 m above the floor.

TABLE 7–1
Center of Mass of Parts of Typical Human Body
(full height and mass = 100 units)

Distance of Hinge Points (%)	Hinge Points (•) (Joints)	Center of Mass (×) (% Height Above Floor)		Percent Mass
91.2	Base of skull on spine	Head	93.5	6.9
81.2	Shoulder joint	Trunk and neck	71.1	46.1
		Upper arms	71.7	6.6
		Lower arms	55.3	4.2
52.1	Hip	Hands	43.1	1.7
		Upper legs (thighs)	42.5	21.5
28.5	Knee			
		Lower legs	18.2	9.6
4.0	Ankle	Feet	1.8	3.4
			58.0	100.0



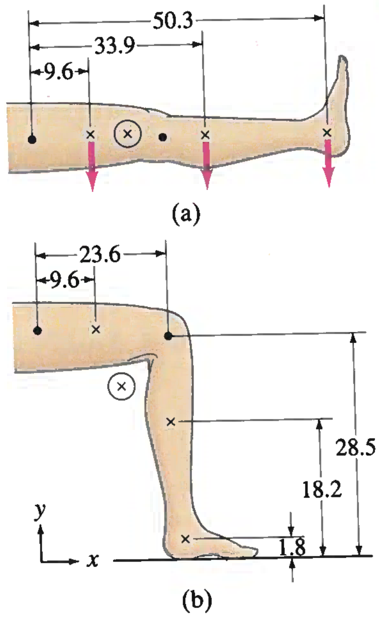


FIGURE 7–25 Example 7–12: finding the CM of a leg in two different positions (⊗ represents the calculated CM).

EXAMPLE 7–12 **A leg's CM.** Determine the position of the CM of the whole leg (*a*) when stretched out, and (*b*) when bent at 90°, as shown in Fig. 7–25. Assume the person is 1.70 m tall.

SOLUTION (*a*) Table 7–1 uses percentage units, meaning the person has a mass of 100 units and a height of 100 units. At the end we can multiply by (1.70 m/100). We measure the distance from the hip joint using Table 7–1 and obtain the numbers shown in Fig. 7–25a. Using Eq. 7–9a we obtain

$$x_{\mathrm{CM}} = \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4} = 20.4 \text{ units.}$$

Thus, the center of mass of the leg and foot is 20.4 units from the hip joint, or 52.1 − 20.4 = 31.7 units from the base of the foot. Since the person is 1.70 m tall, this is (1.70 m)(31.7/100) = 0.54 m.

(*b*) In this part, we have a two-dimensional problem. We use an *xy* coordinate system, as shown in Fig. 7–25b. First, we calculate how far to the right of the hip joint the CM lies:

$$x_{\mathrm{CM}} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{21.5 + 9.6 + 3.4} = 14.9 \text{ units.}$$

For our 1.70 m tall person, this is (1.70 m)(14.9/100) = 0.25 m. Next, we calculate the distance, y_{CM}, of the CM above the floor:

$$y_{\mathrm{CM}} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{21.5 + 9.6 + 3.4} = 23.1 \text{ units,}$$

or (1.70 m)(23.1/100) = 0.39 m. Thus, the CM is located 39 cm above the floor and 25 cm to the right of the hip joint.

CM can be outside a body

Note in this last Example that the CM can actually lie *outside* the body. Another example is a doughnut, whose CM is at the center of the hole.

Knowing the CM of the body when it is in various positions is of great use in studying body mechanics. One simple example from athletics is shown in Fig. 7–26. If high jumpers can get into the position shown, their CM can actually pass below the bar which their bodies go over, which means that for a particular takeoff speed, they can clear a higher bar. This is indeed what they try to do.

FIGURE 7–26 The CM of a high jumper may actually pass beneath the bar.

*7–10 Center of Mass and Translational Motion

As mentioned in Section 7–8, a major reason for the importance of the concept of center of mass is that the motion of the CM for a system of particles (or an extended body) is directly related to the net force acting on the system as a whole. We now show this, taking the simple case of one-dimensional motion (x direction) and only three particles, but the extension to more bodies and to three dimensions follows the same lines.

Suppose the three particles lie on the x axis and have masses m_1, m_2, m_3, and positions x_1, x_2, x_3. From Eq. 7–9a for the CM, we can write

$$Mx_{\text{CM}} = m_1x_1 + m_2x_2 + m_3x_3,$$

where $M = m_1 + m_2 + m_3$ is the total mass of the system. If these particles are in motion, say along the x axis with velocities v_1, v_2, and v_3, respectively, then in a short time Δt they each will have traveled a distance:

$$\Delta x_1 = x_1' - x_1 = v_1\,\Delta t$$
$$\Delta x_2 = x_2' - x_2 = v_2\,\Delta t$$
$$\Delta x_3 = x_3' - x_3 = v_3\,\Delta t,$$

where x_1', x_2', and x_3' represent their new positions after a time Δt. The position of the new CM is given by

$$Mx_{\text{CM}}' = m_1x_1' + m_2x_2' + m_3x_3'.$$

If we subtract the two CM equations, we get

$$M\,\Delta x_{\text{CM}} = m_1\,\Delta x_1 + m_2\,\Delta x_2 + m_3\,\Delta x_3.$$

During the time Δt, the CM will have moved a distance

$$\Delta x_{\text{CM}} = x_{\text{CM}}' - x_{\text{CM}} = v_{\text{CM}}\,\Delta t,$$

where v_{CM} is the velocity of the CM. Into the equation just before the last one, we substitute the relations for all the Δx's:

$$Mv_{\text{CM}}\,\Delta t = m_1v_1\,\Delta t + m_2v_2\,\Delta t + m_3v_3\,\Delta t.$$

We divide out Δt and get

$$Mv_{\text{CM}} = m_1v_1 + m_2v_2 + m_3v_3. \qquad \textbf{(7–10)}$$

Since $m_1v_1 + m_2v_2 + m_3v_3$ is the sum of the momenta of the particles of the system, it represents the *total momentum* of the system. Thus we see from Eq. 7–10 that *the total (linear) momentum of a system of particles is equal to the product of the total mass M and the velocity of the center of mass of the system*. Or, *the linear momentum of an extended body is the product of the body's mass and the velocity of its CM.*

Total momentum

If there are forces acting on the particles, then the particles may be accelerating. In a short time Δt, each particle's velocity will change by an amount

$$\Delta v_1 = a_1\,\Delta t, \qquad \Delta v_2 = a_2\,\Delta t, \qquad \Delta v_3 = a_3\,\Delta t.$$

If we now use the same reasoning as we did to derive Eq. 7–10, we obtain

$$Ma_{\text{CM}} = m_1a_1 + m_2a_2 + m_3a_3.$$

According to Newton's second law, $m_1a_1 = F_1$, $m_2a_2 = F_2$, and $m_3a_3 = F_3$, where F_1, F_2, and F_3 are the net forces on the three particles, respectively. Thus we get for the system as a whole:

Newton's second law for a system of particles or an extended body

$$Ma_{CM} = F_1 + F_2 + F_3 = F_{net}. \qquad (7\text{–}11)$$

That is, *the sum of all the forces acting on the system is equal to the total mass of the system times the acceleration of its center of mass*. This is **Newton's second law** for a system of particles, and it also applies to an extended body (which can be thought of as a collection of particles). Thus we conclude that the *center of mass of a system of particles (or of an extended body) with total mass M moves like a single particle of mass M acted upon by the same net external force*. That is, the system moves as if all its mass were concentrated at the CM and all the external forces acted at that point. We can thus treat the translational motion of any body or system of bodies as the motion of a particle (see Figs. 7–19 and 7–20). This theorem clearly simplifies our analysis of the motion of complex systems and extended bodies. Although the motion of various parts of the system may be complicated, we may often be satisfied with knowing the motion of the CM. This theorem also allows us to solve certain types of problems very easily, as illustrated by the following Example.

CONCEPTUAL EXAMPLE 7–13 **A two-stage rocket.** A rocket is shot into the air as shown in Fig. 7–27. At the moment it reaches its highest point, a horizontal distance d from its starting point, a prearranged explosion separates it into two parts of equal mass. Part I is stopped in midair and falls vertically to Earth. Where does part II land? Assume $\mathbf{g}$ = constant.

SOLUTION After the rocket is fired, the path of the CM of the system continues to follow the parabolic trajectory of a projectile acted on by only a constant gravitational force. The CM will thus arrive at a point $2d$ from the starting point. Since the masses of I and II are equal, the CM must be midway between them. Therefore, II lands a distance $3d$ from the starting point. (If part I had been given a kick up or down instead of merely falling, the solution would have been somewhat more complicated.)

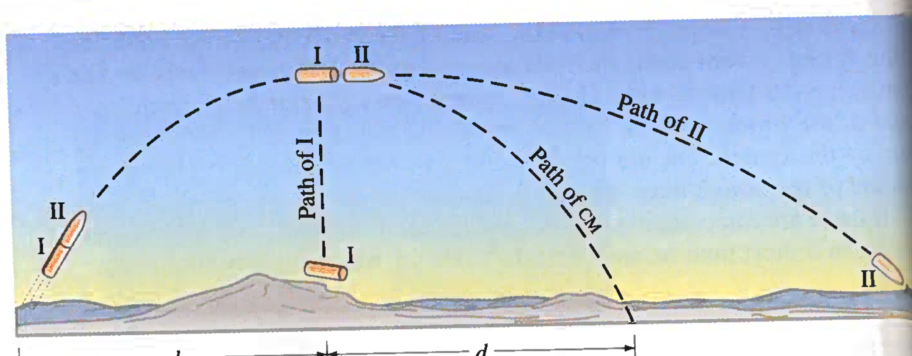


FIGURE 7–27 Example 7–13.

SUMMARY

The **momentum**, **p**, of a body is defined as the product of its mass times its velocity,

$$\mathbf{p} = m\mathbf{v}.$$

In terms of momentum, **Newton's second law** can be written as

$$\Sigma \mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

That is, the rate of change of momentum equals the net applied force.

The **law of conservation of momentum** states that the total momentum of an isolated system of objects remains constant. An isolated system is one on which the net external force is zero.

The law of conservation of momentum is very useful in dealing with **collisions**. In a collision, two (or more) bodies interact with each other for a very short time, and the force between them during this time is very large.

The **impulse** of a force on a body is defined as $\mathbf{F}\,\Delta t$, where **F** is the average force acting during the (usually short) time Δt. The impulse is equal to the change in momentum of the body:

$$\text{Impulse} = \mathbf{F}\,\Delta t = \Delta \mathbf{p}.$$

Total momentum is conserved in *any* collision. If $m_1\mathbf{v}_1$ and $m_2\mathbf{v}_2$ are the momenta of two objects before the collision and $m_1'\mathbf{v}_1'$ and $m_2'\mathbf{v}_2'$ are their momenta after, then

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'.$$

Total energy is also conserved, but this may not be helpful in problem solving unless the only type of energy transformation involves kinetic energy. In that case kinetic energy is conserved and the collision is called an **elastic collision**, and we can write

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2.$$

If kinetic energy is not conserved, the collision is called **inelastic**. A **completely inelastic** collision is one in which the colliding bodies stick together after the collision.

The **center of mass** (CM) of an object (or group of objects) is that point at which the net force can be considered to act for purposes of determining the translational motion of the body as a whole. The complete motion of a body can be described as the translational motion of its center of mass plus rotation (or other internal motion) about its center of mass.

QUESTIONS

1. We claim that momentum is conserved. Yet most moving objects eventually slow down and stop. Explain.
2. When a person jumps from a tree to the ground, what happens to the momentum of the person upon striking the ground?
3. Why, when you release an inflated but untied balloon, does it fly across the room?
4. It is said that in ancient times a rich man with a bag of gold coins froze to death stranded on the surface of a frozen lake. Because the ice was frictionless, he could not push himself to shore. What could he have done to save himself had he not been so miserly?
5. How can a rocket change direction when it is far out in space and is essentially in a vacuum?
6. According to Eq. 7–5, the longer the impact time of an impulse, the smaller the force can be for the same momentum change, and hence the smaller the deformation of the object on which the force acts. Explain on this basis the value of "air bags," which are intended to inflate during an automobile collision and reduce the possibility of fracture or death.
7. It used to be common wisdom to build cars to be as rigid as possible to withstand collisions. Today, though, cars are designed to have "crumple zones" that collapse upon impact. What advantage does this have?
8. Why is it easier to hit a home run from a pitched ball than from one tossed in the air by the batter?
9. The speed of a tennis ball on the return of a serve can be just as fast as the serve, even though the racket isn't swung very fast. How can this be?
10. Is it possible for a body to receive a larger impulse from a small force than from a large force?
11. A light body and a heavy body have the same kinetic energy. Which has the greater momentum?
12. Is it possible for an object to have momentum without having kinetic energy? Can it have kinetic energy but no momentum? Explain.
13. At a hydroelectric power plant, water is directed at high speed against turbine blades on an axle that turns an electric generator. Do you think the turbine blades should be designed so that the water is brought to a dead stop, or so that the water rebounds?

14. A superball is dropped from a height h onto a hard steel plate (fixed to the Earth), from which it rebounds at very nearly its original speed. (*a*) Is the momentum of the ball conserved during any part of this process? (*b*) If we consider the ball and Earth as our system, during what parts of the process is momentum conserved? (*c*) Answer part (*b*) for a piece of putty that falls and sticks to the steel plate.

15. Why do you tend to lean backward when carrying a heavy load in your arms?

16. Why is the CM of a 1-m length of pipe at its midpoint, whereas this is not true for your arm or leg?

17. Show on a diagram how your CM shifts when y change from a lying position to a sitting positio

18. Describe an analytic way of determining the CM any triangular-shaped, thin uniform plate.

*** 19.** A rocket following a parabolic path through the suddenly explodes into many pieces. What can y say about the motion of this system of pieces?

*** 20.** If only an external force can change the moment of the center of mass of an object, how can the int nal force of the engine accelerate a car?

PROBLEMS

SECTIONS 7–1 AND 7–2

1. (I) What is the magnitude of the momentum of a 22-g sparrow flying with a speed of 8.1 m/s?

2. (II) A child in a boat throws a 5.40-kg package out horizontally with a speed of 10.0 m/s, Fig. 7–28. Calculate the velocity of the boat immediately after, assuming it was initially at rest. The mass of the child is 26.0 kg and that of the boat is 55.0 kg.

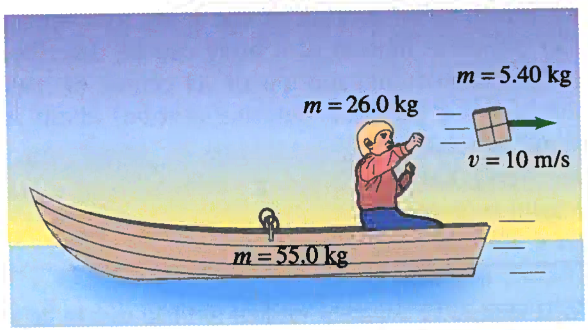


FIGURE 7–28 Problem 2.

3. (II) Calculate the force exerted on a rocket, given that the propelling gases are expelled at a rate of 1300 kg/s with a speed of 40,000 m/s (at the moment of takeoff).

4. (II) A halfback on an apparent breakaway for a touchdown is tackled from behind. If the halfback has a mass of 95 kg and was moving at 4.1 m/s when he was tackled by an 85-kg cornerback running at 5.5 m/s in the same direction, what was their mutual speed immediately after the touchdown-saving tackle?

5. (II) A 12,500-kg railroad car travels alone on a level frictionless track with a constant speed of 18.0 m/s. A 5750-kg additional load is dropped onto the car. What then will be the car's speed?

6. (II) A 9500-kg boxcar traveling at 16 m/s strikes a second car. The two stick together and move off with a speed of 6.0 m/s. What is the mass of the second car?

7. (II) A gun is fired vertically into a 1.40-kg block wood at rest directly above it. If the bullet has a mas 21.0 g and a speed of 210 m/s, how high will the bl rise into the air after the bullet becomes embedded in

8. (II) A 15-g bullet strikes and becomes embedded a 1.10-kg block of wood placed on a horizontal surf just in front of the gun. If the coefficient of kinetic f tion between the block and the surface is 0.25, and impact drives the block a distance of 9.5 m before comes to rest, what was the muzzle speed of the bull

9. (II) An atomic nucleus at rest decays radioactiv into an alpha particle and a smaller nucleus. Wh will be the speed of this recoiling nucleus if th speed of the alpha particle is 3.8×10^5 m/s? Assu the recoiling nucleus has a mass 57 times great than that of the alpha particle.

10. (II) An atomic nucleus initially moving at 420 m emits an alpha particle in the direction of its veloci and the new nucleus slows to 350 m/s. If the alph particle has a mass of 4.0 u and the original nucle has a mass of 222 u, what speed does the alpha part cle have when it is emitted?

11. (II) A 13-g bullet traveling 230 m/s penetrates a 2.0 block of wood and emerges going 170 m/s. If the blo is stationary on a frictionless surface when hit, ho fast does it move after the bullet emerges?

12. (II) A 975-kg two-stage rocket is traveling at a spe of 5.80×10^3 m/s with respect to Earth when a p designed explosion separates the rocket into tw sections of equal mass that then move with a relati speed (relative to each other) of 2.20×10^3 m along the original line of motion. (*a*) What is th speed and direction of each section (relative t Earth) after the explosion? (*b*) How much ener was supplied by the explosion? [*Hint*: What is th change in KE as a result of the explosion?]

13. (III) A rocket of total mass 3180 kg is traveling i outer space with a velocity of 115 m/s toward the Su It wishes to alter its course by 35.0°, and can do this firing its rockets briefly in a direction perpendicular its original motion. If the rocket gases are expelled at speed of 1750 m/s, how much mass must be expelled

SECTION 7–3

14. (I) A tennis ball may leave the racket of a top player on the serve with a speed of 65.0 m/s. If the ball's mass is 0.0600 kg and it is in contact with the racket for 0.0300 s, what is the average force on the ball? Would this force be large enough to lift a 60-kg person?

15. (I) A 0.145-kg baseball pitched at 39.0 m/s is hit on a horizontal line drive straight back toward the pitcher at 52.0 m/s. If the contact time between bat and ball is 1.00×10^{-3} s, calculate the average force between the ball and bat during contact.

16. (II) A golf ball of mass 0.045 kg is hit off the tee at a speed of 45 m/s. The golf club was in contact with the ball for 5.0×10^{-3} s. Find (*a*) the impulse imparted to the golf ball, and (*b*) the average force exerted on the ball by the golf club.

17. (II) A tennis ball of mass $m = 0.060$ kg and speed $v = 25$ m/s strikes a wall at a 45° angle and rebounds with the same speed at 45° (Fig. 7–29). What is the impulse given the wall?

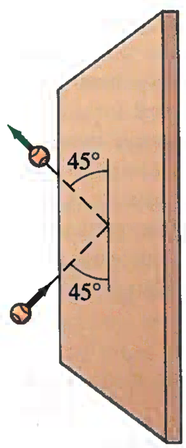


FIGURE 7–29 Problem 17.

18. (II) A 115-kg fullback is running at 4.0 m/s to the east and is stopped in 0.75 s by a head-on tackle by a tackler running due west. Calculate (*a*) the original momentum of the fullback, (*b*) the impulse exerted on the fullback, (*c*) the impulse exerted on the tackler, and (*d*) the average force exerted on the tackler.

19. (II) Suppose the force acting on a tennis ball (mass 0.060 kg) points in the $+x$ direction and is given by the graph of Fig. 7–30 as a function of time. Use graphical methods to estimate (*a*) the total impulse given the ball, and (*b*) the velocity of the ball after being struck, assuming the ball is being served so it is nearly at rest initially.

20. (III) From what maximum height can a 75-kg person jump without breaking the lower leg bone on either leg? Ignore air resistance and assume the CM of the person moves a distance of 0.60 m from the standing to the seated position (that is, in breaking the fall). Assume the breaking strength (force per unit area) of bone is $170 \times 10^6\ \mathrm{N/m^2}$, and its smallest cross-sectional area is $2.5 \times 10^{-4}\ \mathrm{m^2}$.

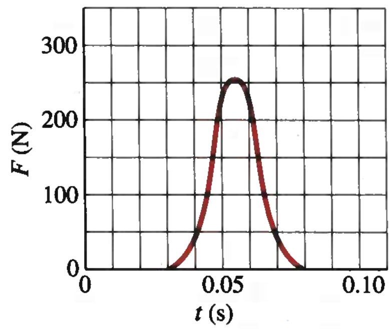


FIGURE 7–30 Problem 19.

SECTIONS 7–4 AND 7–5

21. (II) A ball of mass 0.440 kg moving east ($+x$ direction) with a speed of 3.70 m/s collides head-on with a 0.220-kg ball at rest. If the collision is perfectly elastic, what will be the speed and direction of each ball after the collision?

22. (II) A 0.450-kg ice puck, moving east with a speed of 3.00 m/s, has a head-on collision with a 0.900-kg puck initially at rest. Assuming a perfectly elastic collision, what will be the speed and direction of each object after the collision?

23. (II) Two billiard balls of equal mass undergo a perfectly elastic head-on collision. If the speed of one ball was initially 2.00 m/s, and of the other 3.00 m/s in the opposite direction, what will be their speeds after the collision?

24. (II) A 0.060-kg tennis ball, moving with a speed of 2.50 m/s, has a head-on collision with a 0.090-kg ball initially moving away from it at a speed of 1.00 m/s. Assuming a perfectly elastic collision, what is the speed and direction of each ball after the collision?

25. (II) A softball of mass 0.220 kg that is moving with a speed of 5.5 m/s collides head-on and elastically with another ball initially at rest. Afterward it is found that the incoming ball has bounced backward with a speed of 3.7 m/s. Calculate (*a*) the velocity of the target ball after the collision, and (*b*) the mass of the target ball.

26. (II) A pair of bumper cars in an amusement park ride collide elastically as one approaches the other directly from the rear (Fig. 7–31). One has a mass of 450 kg and the other 550 kg, owing to differences in passenger mass. If the lighter one approaches at 4.50 m/s and the other is moving at 3.70 m/s, calculate (*a*) their velocities after the collision, and (*b*) the change in momentum of each.

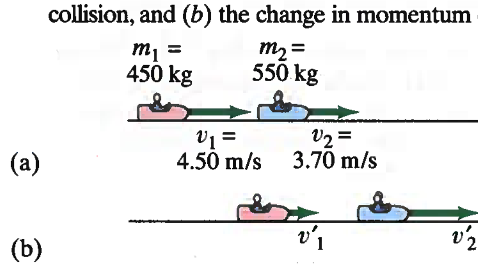


FIGURE 7–31 Problem 26: (a) before collision, (b) after collision.

27. (III) A 0.280-kg croquet ball makes an elastic head-on collision with a second ball initially at rest. The second ball moves off with half the original speed of the first ball. (*a*) What is the mass of the second ball? (*b*) What fraction of the original kinetic energy (ΔKE/KE) gets transferred to the second ball?

28. (III) In a physics lab, a small cube slides down a frictionless incline as shown in Fig. 7–32, and elastically strikes a cube at the bottom that is only one-half its mass. If the incline is 30 cm high and the table is 90 cm off the floor, where does each cube land?

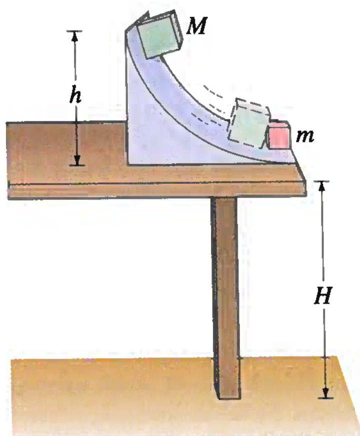


FIGURE 7–32 Problem 28.

29. (III) Take the general case of a body of mass m_1 and velocity v_1 elastically striking a stationary ($v_2 = 0$) body of mass m_2 head-on. (*a*) Show that the final velocities v_1' and v_2' are given by

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1,$$

$$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right)v_1.$$

(*b*) What happens in the extreme case when m_1 is much smaller than m_2? Cite a common example of this. (*c*) What happens in the extreme case when m_1 is much larger than m_2? Cite a common example of this. (*d*) What happens in the case when $m_1 = m_2$? Cite a common example of this.

SECTION 7–6

30. (II) An 18-g rifle bullet traveling 230 m/s buries itself in a 3.6-kg pendulum hanging on a 2.8-m-long string, which makes the pendulum swing upward in an arc. Determine the horizontal component of the pendulum's displacement.

31. (II) (*a*) Derive a formula for the fraction of kinetic energy lost, ΔKE/KE, for the ballistic pendulum collision of Example 7–9. (*b*) Evaluate for $m = 14.0$ g and $M = 380$ g.

32. (II) An explosion breaks an object into two pieces, one of which has 1.5 times the mass of the other. If 7500 J were released in the explosion, how much kinetic energy did each piece acquire?

33. (II) A 1.0×10^3-kg Toyota collides into the rear end of a 2.2×10^3-kg Cadillac stopped at a red light. The bumpers lock, the brakes are locked, and the two cars skid forward 2.8 m before stopping. The police officer, knowing that the coefficient of kinetic friction between tires and road is 0.40, calculates the speed of the Toyota at impact. What was that speed?

34. (II) A measure of inelasticity in a head-on collision of two bodies is the *coefficient of restitution*, *e*, defined as

$$e = \frac{v_1' - v_2'}{v_2 - v_1},$$

where $v_1' - v_2'$ is the relative velocity of the two bodies after and $v_2 - v_1$ is their relative velocity before the collision. (*a*) Show that for a perfectly elastic collision, $e = 1$, and for a completely inelastic collision, $e = 0$. (*b*) A simple method for measuring the coefficient of restitution for a body colliding with a very hard surface like steel is to drop the body onto a heavy steel plate, as shown in Fig. 7–33. Determine a formula for e in terms of the original height h and the maximum height reached after collision h'.

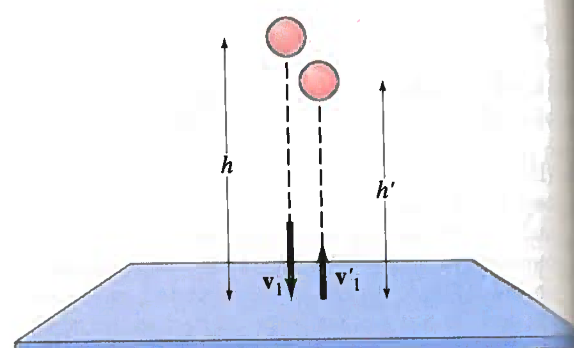


FIGURE 7–33 Problem 34. Measurement of coefficient of restitution.

35. (III) A wooden block is cut into two pieces, one with three times the mass of the other. A depression is made in both faces of the cut, so that a firecracker can be placed in it with the block reassembled. The reassembled block is set on a rough-surfaced table, and the fuse is lit. When the firecracker explodes, the two blocks separate. What is the ratio of distances each block travels?

36. (III) A 5.0-kg body moving in the $+x$ direction at 5.5 m/s collides head-on with a 3.0-kg body moving in the $-x$ direction at 4.0 m/s. Find the final velocity of each mass if: (*a*) the bodies stick together; (*b*) the collision is elastic; (*c*) the 5.0-kg body is at rest after the collision; (*d*) the 3.0-kg body is at rest after the collision; (*e*) the 5.0-kg body has a velocity of 4.0 m/s in the $-x$ direction after the collision. Are the results in (*c*), (*d*), and (*e*) "reasonable"? Explain.

SECTION 7–7

37. (II) A radioactive nucleus at rest decays into a second nucleus, an electron, and a neutrino. The electron and neutrino are emitted at right angles and have momenta of 9.30×10^{-23} kg·m/s, and 5.40×10^{-23} kg·m/s, respectively. What is the magnitude and direction of the momentum of the second (recoiling) nucleus?

38. (II) An eagle ($m_1 = 4.3$ kg) moving with speed $v_1 = 7.8$ m/s is on a collision course with a second eagle ($m_2 = 5.6$ kg) moving at $v_2 = 10.2$ m/s in a direction at right angles to the first. After they collide, they hold onto one another. In what direction, and with what speed, are they moving after the collision?

39. (II) A billiard ball of mass $m_A = 0.400$ kg moving with speed $v_A = 1.80$ m/s strikes a second ball, initially at rest, of mass $m_B = 0.500$ kg. As a result of the collision, the first ball is deflected off at an angle of 30.0° with a speed $v'_A = 1.10$ m/s. (*a*) Taking the x axis to be the original direction of motion of ball A, write down the equations expressing the conservation of momentum for the components in the x and y directions separately. (*b*) Solve these equations for the speed, v'_B, and angle, θ', of ball B. Do not assume the collision is elastic.

40. (III) An atomic nucleus of mass m traveling with speed v collides elastically with a target particle of mass $2m$ (initially at rest) and is scattered at 90°. (*a*) At what angle does the target particle move after the collision? (*b*) What are the final speeds of the two particles? (*c*) What fraction of the initial KE (ΔKE/KE) is transferred to the target particle?

41. (III) After a completely inelastic collision between two objects of equal mass, each having initial speed v, the two move off together with speed $v/3$. What was the angle between their initial directions?

42. (III) In order to convert a tough split in bowling, it is necessary to strike the pin a glancing blow as shown in Fig. 7–34. Assume that the bowling ball, initially traveling at 12.0 m/s, has five times the mass of a pin and that the pin goes off at 80° from the original direction of the ball. Calculate (*a*) the final speed of the pin, (*b*) the final speed of the ball, and (*c*) the angle through which the ball was deflected. Assume the collision is elastic.

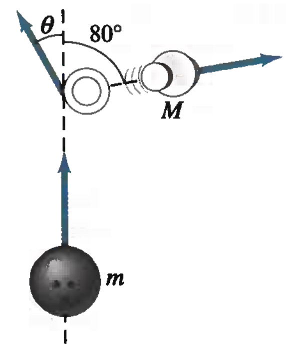


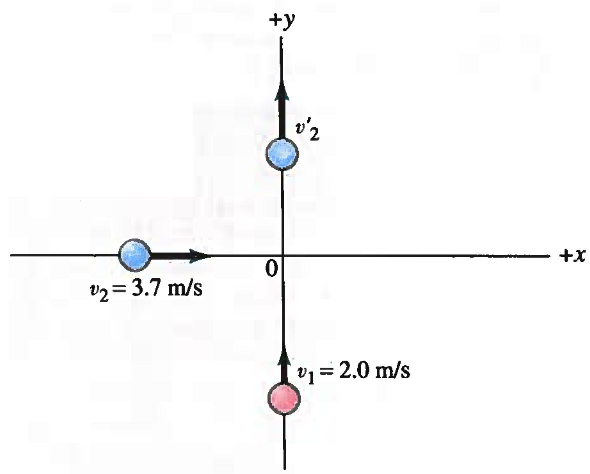
FIGURE 7–34 Problem 42.

* 43. (III) A neutron collides elastically with a helium nucleus (at rest initially) whose mass is four times that of the neutron. The helium nucleus is observed to rebound at an angle $\theta'_2 = 45°$. Determine the angle of the neutron, θ'_1, and the speeds of the two particles, v'_n and v'_{He}, after the collision. The neutron's initial speed is 6.2×10^5 m/s.

* 44. (III) Two billiard balls of equal mass move at right angles and meet at the origin of an xy coordinate system. One is moving upward along the y axis at 2.0 m/s, and the other is moving to the right along the x axis with speed 3.7 m/s. After the collision (assumed elastic), the second ball is moving along the positive y axis (Fig. 7–35). What is the final direction of the first ball, and what are their two speeds?

* 45. (III) Prove that in the elastic collision of two objects of identical mass, with one being a target initially at rest, the angle between their final velocity vectors is always 90°.

FIGURE 7–35 Problem 44. (Ball 1 after the collision is not shown.)



SECTION 7–8

46. (I) The distance between a carbon atom (m = 12 u) and an oxygen atom (m = 16 u) in the CO molecule, is 1.13×10^{-10} m. How far from the carbon atom is the center of mass of the molecule?

47. (I) An empty 1050-kg car has its CM 2.50 m behind the front of the car. How far from the front of the car will the CM be when two people sit in the front seat 2.80 m from the front of the car, and three people sit in the back seat 3.90 m from the front? Assume that each person has a mass of 70.0 kg.

48. (II) Three cubes, of side l_0, $2l_0$, and $3l_0$, are placed next to one another (in contact) with their centers along a straight line and the $l = 2l_0$ cube in the center (Fig. 7–36). What is the position, along this line, of the CM of this system? Assume the cubes are made of the same uniform material.

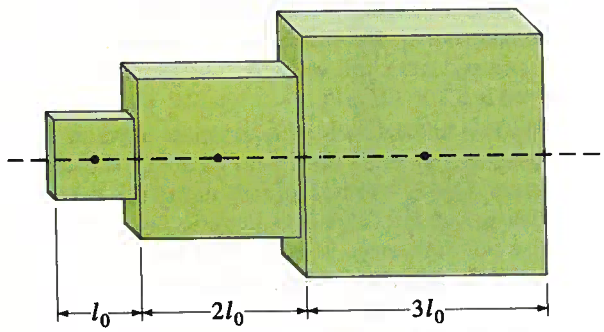


FIGURE 7–36 Problem 48.

49. (II) A square uniform raft, 18 m by 18 m, of mass 6200 kg, is used as a ferryboat. If three cars, each of mass 1200 kg, occupy the NE, SE, and SW corners, determine the CM of the loaded ferryboat.

50. (II) A (light) pallet has a load of cases of tomato paste (see Fig. 7–37), each of which is a cube of length l and has identical mass. Find the center of gravity in the horizontal plane, so that the crane operator can pick up the load without tipping it.

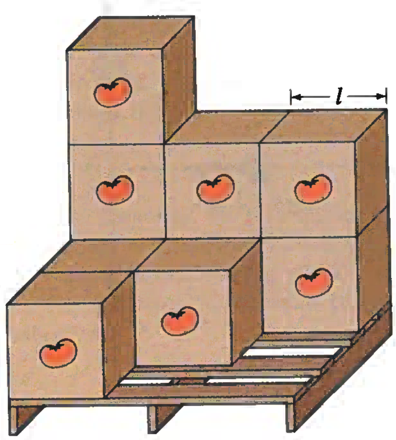


FIGURE 7–37 Problem 50.

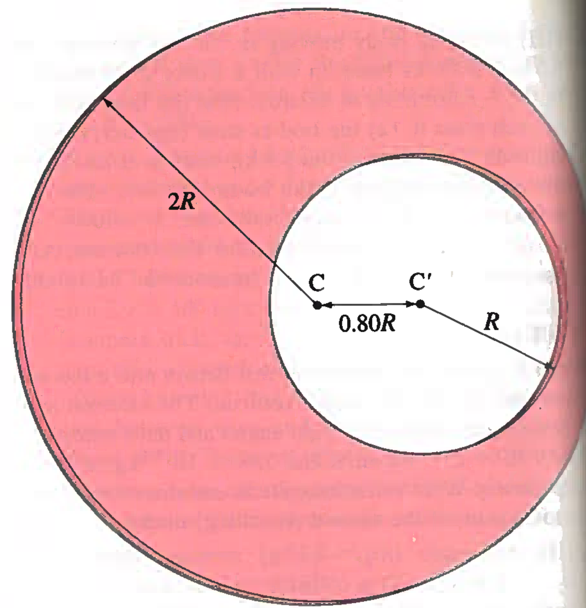


FIGURE 7–38 Problem 51.

51. (III) A uniform circular plate of radius $2R$ has a ci cular hole of radius R cut out of it. The center of th smaller circle is a distance $0.80R$ from the center o the larger circle, Fig. 7–38. What is the position of th center of mass of the plate? [*Hint:* Try subtraction.]

*SECTION 7–9

*52. (I) Assume that your proportions are the same as tho in Table 7–1, and calculate the mass of one of your le

*53. (I) Determine the CM of an outstretched arm usin Table 7–1.

*54. (II) Use Table 7–1 to calculate the position of the C of an arm bent at a right angle. Assume that the pe son is 155 cm tall.

*55. (II) Calculate how far below the torso's median li the CM will be when a jumper is in a position suc that his arms and legs are hanging vertically, and hi trunk and head are horizontal. Will this be outsid the body? Use Table 7–1.

*SECTION 7–10

*56. (II) The masses of the Earth and Moon ar 5.98×10^{24} kg and 7.35×10^{22} kg, respectively, an their centers are separated by 3.84×10^{8} m (*a*) Where is the CM of this system located? (*b*) Wh can you say about the motion of the Earth-Moo system about the Sun, and of the Earth and Moo separately about the Sun?

*57. (II) A 55-kg woman and a 90-kg man stand 10.0 m apart on frictionless ice. (*a*) How far from the woma is their CM? (*b*) If they hold on to the two ends of rope, and the man pulls on the rope so that he mov 2.5 m, how far from the woman will he be now (*c*) How far will the man have moved when he collide with the woman?

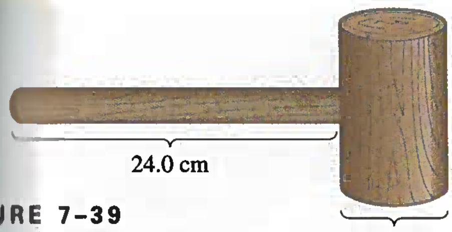


FIGURE 7–39 Problem 58.

58. (II) A mallet consists of a uniform cylindrical head of mass 2.00 kg and a diameter 0.0800 m mounted on a uniform cylindrical handle of mass 0.500 kg and length 0.240 m, as shown in Fig. 7–39. If this mallet is tossed, spinning, into the air, how far above the bottom of the handle is the point that will follow a parabolic trajectory?

*59. (II) (*a*) Suppose that in Example 7–13 (Fig. 7–27), $m_{II} = 3m_I$. Where then would m_{II} land? (*b*) What if $m_I = 3m_{II}$?

*60. (III) A helium balloon and its gondola, of mass M, are in the air and stationary with respect to the ground. A passenger, of mass m, then climbs out and slides down a rope with speed v, measured with respect to the balloon. With what speed and direction (relative to Earth) does the balloon then move? What happens if the passenger stops?

GENERAL PROBLEMS

61. During a Chicago storm, winds can whip horizontally at speeds of 100 km/h. If the air strikes a person at the rate of 40 kg/s per square meter and is brought to rest, calculate the force of the wind on a person. Assume the person's area to be 1.50 m high and 0.50 m wide. Compare to the typical maximum force of friction ($\mu \approx 1.0$) between the person and the ground, if the person has a mass of 70 kg.

62. A 5800-kg open railroad car coasts along with a constant speed of 8.60 m/s on a level track. Snow begins to fall vertically and fills the car at a rate of 3.50 kg/min. Ignoring friction with the tracks, what is the speed of the car after 90.0 min?

63. A 0.145-kg pitched baseball moving horizontally at 35.0 m/s strikes a bat and is popped straight up to a height of 55.6 m before turning around. If the contact time is 0.50 ms, calculate the average force on the ball during the contact.

64. A rocket of mass m traveling with speed v_0 along the x axis suddenly shoots out fuel, equal to one third of its mass, parallel to the y axis with speed $2v_0$. Give the components of the final velocity of the rocket.

65. A novice pool player is faced with the corner pocket shot shown in Fig. 7–40. The relative size of some of the dimensions (the units aren't important, only their ratios) are also shown. Should the player be worried about this being a "scratch shot," one where the cue ball will also fall into a pocket? Give details.

66. A 140-kg astronaut (including space suit) acquires a speed of 2.50 m/s by pushing off with his legs from an 1800-kg space capsule. (*a*) What is the change in speed of the space capsule? (*b*) If the push lasts 0.500 s, what is the average force exerted by each on the other? As the reference frame, use the position of the capsule before the push.

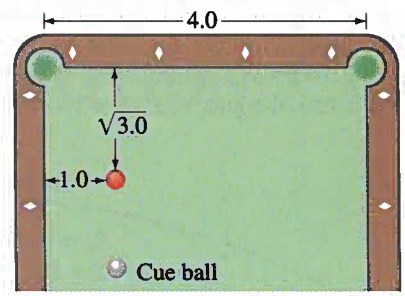


FIGURE 7–40 Problem 65.

67. A golf ball rolls off the top of a flight of concrete stairs of total vertical height 4.00 m. The ball hits four times on the way down, each time striking the horizontal part of a different step. If all collisions are perfectly elastic, what is the bounce height on the fifth bounce when the ball reaches the bottom of the stairs?

68. A ball of mass m makes a head-on elastic collision with a second ball (at rest) and rebounds with a speed equal to one-fourth its original speed. What is the mass of the second ball?

69. You have been hired as an expert witness in a court case involving an automobile accident. The accident involved a car of mass 2000 kg (car A) which approached a stationary car of mass 1000 kg (car B). The driver of car A applied his brakes 15 m before he crashed into car B. After the collision, car A slid 15 m while car B slid 30 m. The coefficient of kinetic friction between the locked wheels and the road was measured to be 0.60. Prove to the court that the driver of car A was exceeding the 55-mph speed limit before applying the brakes.

* **70.** Two people, one of mass 75 kg and the other of mass 60 kg, sit in a rowboat of mass 80 kg. With the boat initially at rest, the two people, who have been sitting at opposite ends of the boat 2.0 m apart from each other, now exchange seats. How far and in what direction will the boat move?

71. A meteor whose mass was about 10^8 kg struck the Earth ($m = 6.0 \times 10^{24}$ kg) with a speed of about 15 km/s and came to rest in the Earth. (*a*) What was the Earth's recoil speed? (*b*) What fraction of the meteor's kinetic energy was transformed to KE of the Earth? (*c*) By how much did the Earth's KE change as a result of this collision?

72. An explosion breaks an object, originally at rest, into two fragments. One fragment acquires twice the kinetic energy of the other. What is the ratio of their masses?

73. The force on a bullet is given by the formula $F = 580 - 1.8 \times 10^5 t$ over the time interval $t = 0$ to $t = 3.0 \times 10^{-3}$ s. In this formula, t is in seconds and F is in newtons. (*a*) Plot a graph of F vs. t for $t = 0$ to $t = 3.0$ ms. (*b*) Estimate, using graphical methods, the impulse given the bullet. (*c*) If the bullet achieves a speed of 220 m/s as a result of this impulse, given to it in the barrel of a gun, what must its mass be?

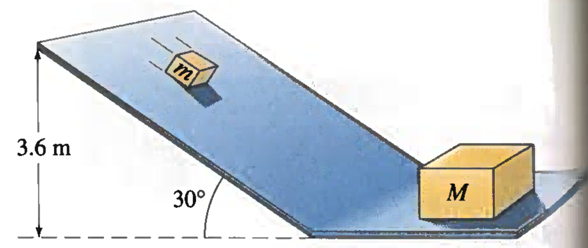


FIGURE 7–41 Problems 74 and 75.

75. In Problem 74 (Fig. 7–41), what is the upper limit on the mass m if it is to rebound from M, slide up the incline, stop, slide down the incline, and collide with M again?

76. A 0.25-kg skeet (clay target) is fired at an angle of 30° to the horizon with a speed of 30 m/s (Fig. 7–42). When it reaches the maximum height, it is hit from below by a 15-g pellet traveling vertically upward at a speed of 200 m/s. The pellet is embedded in the skeet. (*a*) How much higher did the skeet go up? (*b*) How much extra distance, Δx, does the skeet travel because of the collision?

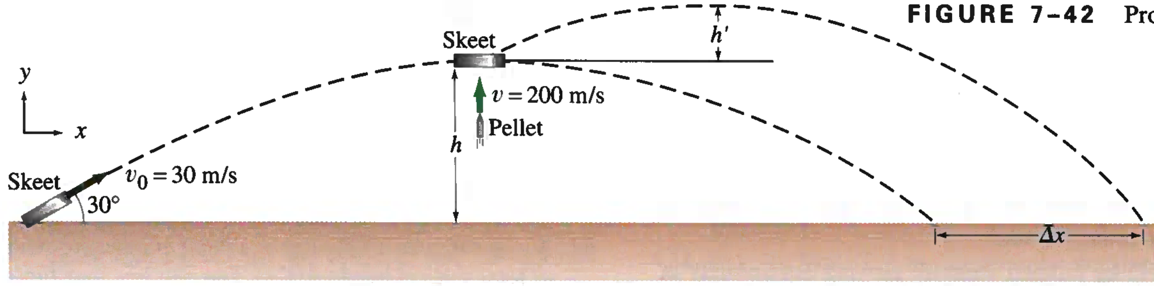


FIGURE 7–42 Problem 76.

74. A block of mass $m = 2.20$ kg slides down a 30.0° incline which is 3.60 m high. At the bottom, it strikes a block of mass $M = 7.00$ kg which is at rest on a horizontal surface, Fig. 7–41. (Assume a smooth transition at the bottom of the incline.) If the collision is elastic, and friction can be ignored, determine (*a*) the speeds of the two blocks after the collision, and (*b*) how far back up the incline the smaller mass will go.

77. The gravitational slingshot effect. Figure 7–43 shows the planet Saturn moving in the negative x direction at its orbital speed (with respect to the sun) of 9.6 km/s. The mass of Saturn is 5.69×10^{26} kg. A spacecraft with mass 825 kg approaches Saturn, moving initially in the $+x$ direction at 10.4 km/s. The gravitational attraction of Saturn (a conservative force) causes the spacecraft to swing around it (orbit shown as dashed line) and head off in the opposite direction. Estimate the final speed of the spacecraft after it is far enough away to be nearly free of Saturn's gravitational pull.

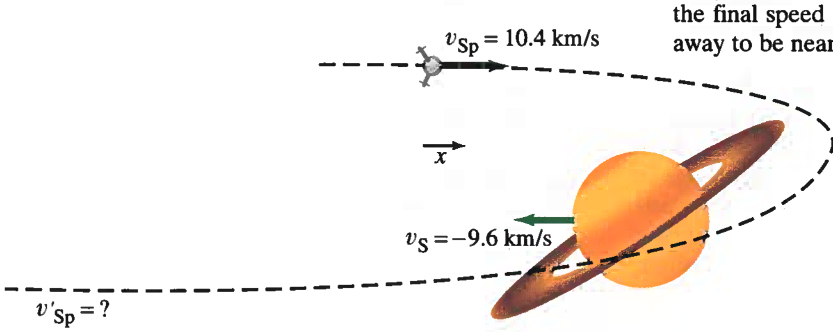


FIGURE 7–43 Problem 77.