A roller coaster at the highest point of its journey has its maximum potential energy (PE). As it rolls downhill, it loses PE and gains in kinetic energy (KE). Total energy is conserved. So if there is no friction, the loss in PE equals the gain in KE. If there is friction, the loss in PE equals the gain in KE plus the thermal energy produced by the work done by friction.

CHAPTER 6

WORK AND ENERGY

Until now we have been studying the motion of an object in terms of Newton's three laws of motion. In that analysis, *force* played a central role as the quantity determining the motion. In this chapter and the next, we discuss an alternative analysis of the motion of an object in terms of the quantities *energy* and *momentum*. The importance of these quantities is that they are *conserved*. That is, in quite general circumstances they remain constant. That conserved quantities exist not only gives us a deeper insight into the nature of the world, but also gives us another way to approach practical problems. We still consider only translational motion, without rotation, in this chapter.

The conservation laws of energy and momentum are especially valuable in dealing with systems of many objects in which a detailed consideration of the forces involved would be difficult.

This chapter is devoted to the very important concept of *energy* and the closely related concept of *work*, which are scalar quantities and thus have no direction associated with them. Since these two quantities are scalars, they are often easier to deal with than are vector quantities such as force and acceleration. Energy derives its importance from two sources. First, it is a conserved quantity. Second, energy is a concept that is useful not only in the study of motion, but in all areas of physics and other sciences as well. But before discussing energy itself, we first examine the concept of work.

6–1 Work Done by a Constant Force

The word *work* has a variety of meanings in everyday language. But in physics, work is given a very specific meaning to describe what is accomplished by the action of a force when it acts on an object as the object moves through a distance. Specifically, the **work** done on an object by a constant force (constant in both magnitude and direction) is defined to be *the product of the magnitude of the displacement times the component of the force parallel to the displacement.* In equation form, we can write

$$W = F_{\parallel} d$$

where $F_{\parallel}$ is the component of the constant force **F** parallel to the displacement **d**. We can also write

Work defined

$$W = Fd\cos\theta, \qquad (6\text{–}1)$$

where F is the magnitude of the constant force, d is the magnitude of the displacement of the object, and θ is the angle between the directions of the force and the displacement. The $\cos\theta$ factor appears in Eq. 6–1 because $F\cos\theta\ (= F_{\parallel})$ is the component of **F** parallel to **d** (Fig. 6–1). Work is a scalar quantity—it has only magnitude.

Let's first consider the case in which the motion and the force are in the same direction, so $\theta = 0$ and $\cos\theta = 1$, and then $W = Fd$. For example, if you push a loaded grocery cart a distance of 50 m by exerting a horizontal force of 30 N on the cart, you do 30 N × 50 m = 1500 N·m of work on the cart.

Units for work: the joule

As this example shows, in SI units, work is measured in newton-meters. A special name is given to this unit, the **joule** (J): 1 J = 1 N·m. In the cgs system, the unit of work is called the *erg* and is defined as 1 erg = 1 dyne·cm. In British units, work is measured in foot-pounds. It is easy to show that $1\ \text{J} = 10^7\ \text{erg} = 0.7376\ \text{ft·lb}$.

Force without work

A force can be exerted on an object and yet do no work. For example, if you hold a heavy bag of groceries in your hands at rest, you do no work on it. A force is exerted, but the displacement is zero, so the work $W = 0$. You also do no work on the bag of groceries if you carry it as you walk

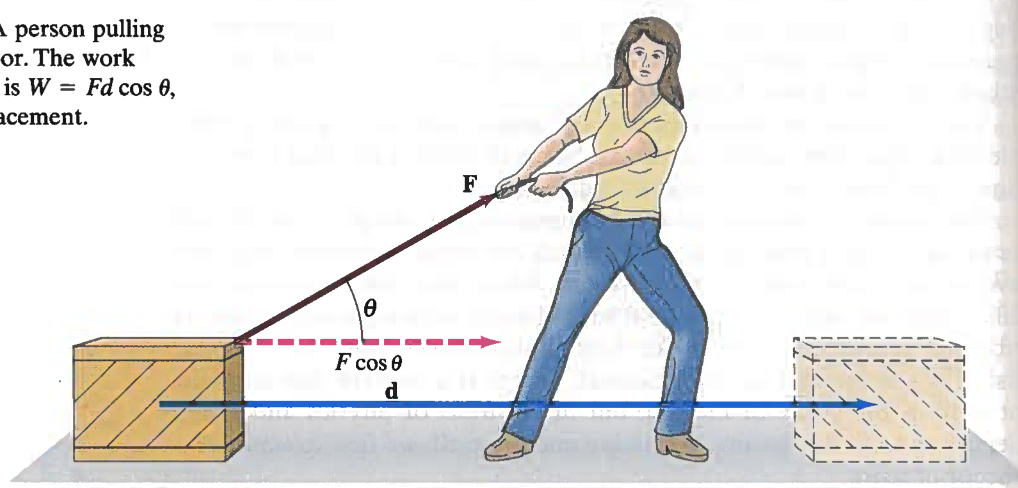


FIGURE 6–1 A person pulling a crate along the floor. The work done by the force **F** is $W = Fd\cos\theta$, where **d** is the displacement.

horizontally across the floor at constant velocity, as shown in Fig. 6–2. No horizontal force is required to move the package at a constant velocity. However, you do exert an upward force **F** on the package equal to its weight. But this upward force is perpendicular to the horizontal motion of the package and thus has nothing to do with that motion. Hence, the upward force is doing no work. This conclusion comes from our definition of work, Eq. 6–1: $W = 0$, because $\theta = 90°$ and $\cos 90° = 0$. Thus, when a particular force is perpendicular to the motion, no work is done by that force. (When you start or stop walking, there is a horizontal acceleration and you do briefly exert a horizontal force, and thus do work.)

When dealing with work, as with force, it is necessary to specify whether you are talking about work done *by* a specific object or done *on* a specific object. It is also important to specify whether the work done is due to one particular force (and which one), or work done by the total *net force* on the object.

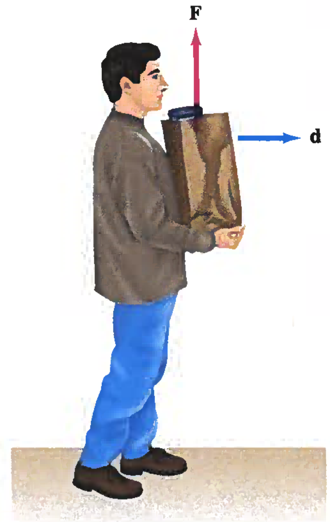


FIGURE 6–2 Work done on the bag of groceries in this case is zero since **F** is perpendicular to the displacement **d**.

EXAMPLE 6–1 Work done on a crate. A 50-kg crate is pulled 40 m along a horizontal floor by a constant force exerted by a person, $F_P = 100\,\text{N}$, which acts at a 37° angle as shown in Fig. 6–3. The floor is rough and exerts a friction force $F_{fr} = 50\,\text{N}$. Determine the work done by each force acting on the crate, and the net work done on the crate.

SOLUTION We choose our coordinate system so that **x** can be the vector that represents the 40-m displacement (that is, along the x axis). There are four forces acting on the crate, as shown in Fig. 6–3: the force exerted by the person $\mathbf{F}_P$; the friction force $\mathbf{F}_{fr}$; the crate's weight mg; and the normal force $\mathbf{F}_N$ exerted upward by the floor. The work done by the gravitational and normal forces is zero, since they are perpendicular to the displacement **x** ($\theta = 90°$ in Eq. 6–1):

$$W_G = mgx \cos 90° = 0$$

$$W_N = F_N x \cos 90° = 0.$$

The work done by $\mathbf{F}_P$ is

$$W_P = F_P x \cos\theta = (100\,\text{N})(40\,\text{m}) \cos 37° = 3200\,\text{J}.$$

The work done by the friction force is

$$W_{fr} = F_{fr} x \cos 180° = (50\,\text{N})(40\,\text{m})(-1) = -2000\,\text{J}.$$

The angle between the displacement **x** and $\mathbf{F}_{fr}$ is 180° because they point in opposite directions. Since the force of friction is opposing the motion, it does *negative* work on the crate.

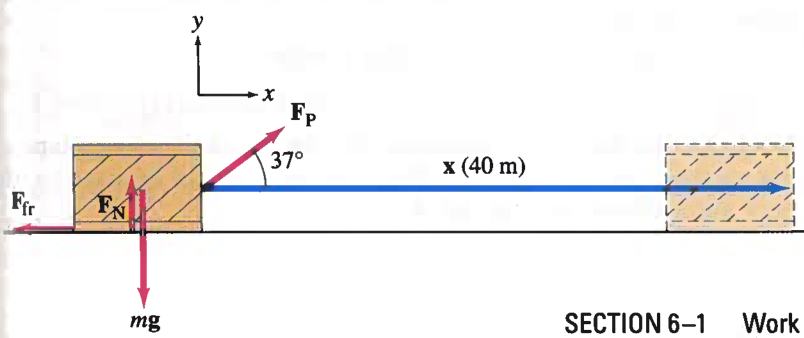


FIGURE 6–3 Example 6–1: 50-kg crate being pulled along a floor.

Finally, the net work can be calculated in two equivalent ways. (1) The net work done on an object is the algebraic sum of the work done by each force, since work is a scalar:

W_{net} is the work done by all the forces acting on the body

$$W_{net} = W_G + W_N + W_P + W_{fr}$$
$$= 0 + 0 + 3200\text{ J} - 2000\text{ J} = 1200\text{ J}.$$

(2) The net work can also be calculated by first determining the net force on the object and then taking its component along the displacement: $(F_{net})_x = F_P \cos\theta - F_{fr}$. Then the net work is

$$W_{net} = (F_{net})_x x = (F_P \cos\theta - F_{fr})x$$
$$= (100\text{ N} \cos 37° - 50\text{ N})(40\text{ m}) = 1200\text{ J}.$$

In the vertical (y) direction, there is no displacement and no work done.

Negative work

In Example 6–1 we saw that friction did negative work. In general, the work done by a force is negative whenever the force (or the component of the force, $F_{\parallel}$) acts in the direction opposite to the direction of motion.

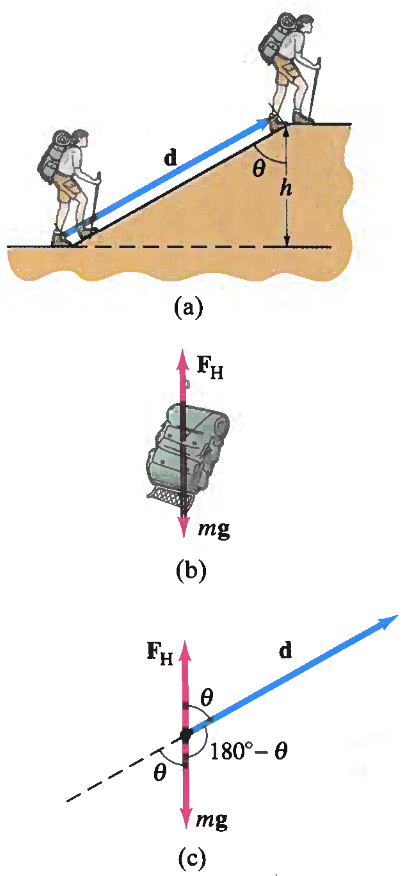


FIGURE 6–4 Example 6–2.

EXAMPLE 6–2 **Work on a backpack.** (a) Determine the work a hiker must do on a 15.0-kg backpack to carry it up a hill of height $h = 10.0$ m, as shown in Fig. 6–4a. Determine also (b) the work done by gravity on the backpack, and (c) the net work done on the backpack. For simplicity, assume the motion is smooth and at constant velocity (i.e., there is negligible acceleration).

SOLUTION (a) The forces on the backpack are shown in Fig. 6–4b: the force of gravity, $m\mathbf{g}$, acting downward; and $\mathbf{F}_H$, the force the hiker must exert upward to support the pack. Since we assume there is negligible acceleration, horizontal forces are negligible. In the vertical (y) direction, we choose up as positive. Newton's second law applied to the backpack gives

$$\Sigma F_y = ma_y$$
$$F_H - mg = 0.$$

Hence,

$$F_H = mg = (15.0\text{ kg})(9.80\text{ m/s}^2) = 147\text{ N}.$$

To calculate the work done by the hiker on the backpack, Eq. 6–1 can be written

$$W_H = F_H(d \cos\theta),$$

and we note from Fig. 6–4a that $d\cos\theta = h$. So the work done by the hiker can be written:

$$W_H = F_H(d\ \cos\theta) = F_H h = mgh$$
$$= (147\text{ N})(10.0\text{ m}) = 1470\text{ J}.$$

Note that the work done depends only on the change in elevation and not on the angle of the hill, θ. The same work would be done to lift the pack vertically the same height h.

(*b*) The work done by gravity is (from Eq. 6–1 and Fig. 6–4c):

$$W_G = (F_G)(d)\cos(180^\circ - \theta).$$

Since $\cos(180^\circ - \theta) = -\cos\theta$, we have

$$\begin{aligned} W_G = (F_G)(d)(-\cos\theta) &= mg(-d\cos\theta) \\ &= -mgh \\ &= -(15.0\,\text{kg})(9.80\,\text{m/s}^2)(10.0\,\text{m}) = -1470\,\text{J}. \end{aligned}$$

Note that the work done by gravity doesn't depend on the angle of the incline but only on the vertical height h of the hill. This is because gravity does work only in the vertical direction. We will make use of this important result later.

Work done by gravity depends on the height of the hill and not on the angle of incline

(*c*) The *net* work done on the backpack is $W_{net} = 0$, since the net force on the backpack is zero (it is assumed not to accelerate significantly). We can also determine the net work done by writing

$$W_{net} = W_G + W_H = -1470\,\text{J} + 1470\,\text{J} = 0$$

which is, as it should be, the same result.

Note in this example that even though the *net* work on the backpack is zero, the hiker nonetheless does do work on the backpack equal to 1470 J.

Hiker does work on pack, but the net work = 0

CONCEPTUAL EXAMPLE 6–3 **Does Earth do work on the Moon?** The Moon revolves around the Earth in a circular orbit, kept there by the gravitational force exerted by the Earth. Does gravity do (*a*) positive work, (*b*) negative work, or (*c*) no work at all on the Moon?

RESPONSE The gravitational force on the Moon (Fig. 6–5) acts toward the Earth (as a centripetal force), inward along the radius of the Moon's orbit. The Moon's displacement at any moment is along the circle, in the direction of its velocity, perpendicular to the radius and perpendicular to the force of gravity. Hence the angle θ between the force and the instantaneous displacement of the Moon is 90°, and the work done by gravity is therefore zero ($\cos 90^\circ = 0$).

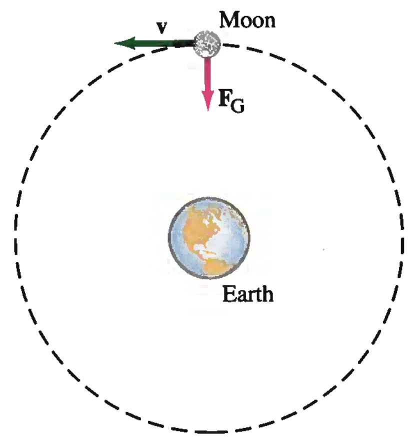


FIGURE 6–5 Conceptual Example 6–3.

PROBLEM SOLVING Work

1. Choose an xy coordinate system. If the body is in motion, it may be convenient to choose the direction of motion as one of the coordinate directions. [Thus, for an object on an incline, you might choose one coordinate axis to be parallel to the incline.]
2. Draw a free-body diagram showing all the forces acting on the body.
3. Determine any unknown forces using Newton's laws.
4. Find the work done *by* a specific force *on* the body by using $W = Fd\cos\theta$. Note that the work done is negative when a force tends to oppose the displacement.
5. To find the *net* work done on the body, either (a) find the work done by each force and add the results algebraically; or (b) find the net force on the object, F_{net}, and then use it to find the net work done:

$$W_{net} = F_{net}\,d\cos\theta.$$

* 6–2 Work Done by a Varying Force

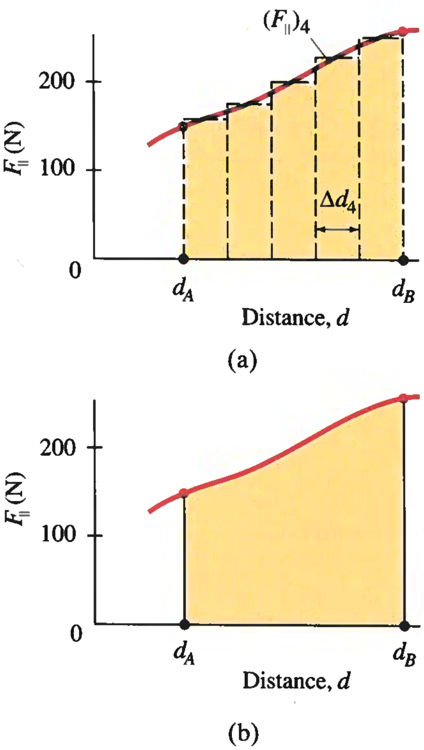


FIGURE 6–6 The work done by a force F can be calculated by taking: (a) the sum of the areas of the rectangles; (b) the area under the curve of $F_{\parallel}$ vs. d.

If the force acting on an object is constant, the work done by that force can be calculated using Eq. 6–1. But in many cases, the force varies in magnitude or direction during a process. For example, as a rocket moves away from Earth, work is done to overcome the force of gravity, which varies as the inverse square of the distance from the Earth's center. Other examples are the force exerted by a spring, which increases with the amount of stretch, or the work done by a varying force in pulling a box or cart up an uneven hill.

The work done by a varying force can be determined graphically. The procedure is like that for determining displacement when the velocity is known as a function of time (Section 2–8). To determine the work done by a variable force, we plot $F_{\parallel}(= F\cos\theta$, the component of **F** parallel to the direction of motion at any point) as a function of distance d, as in Fig. 6–6a. We divide the distance into small segments Δd. For each segment, we indicate the average of $F_{\parallel}$ by a horizontal dashed line. Then the work done for each segment is $\Delta W = F_{\parallel}\Delta d$, which is the area of a rectangle (Δd) wide and ($F_{\parallel}$) high. The total work done to move the object a total distance $d = d_B - d_A$ is the sum of the areas of the rectangles (five in the case shown in Fig. 6–6a). Usually, the average value of ($F_{\parallel}$) for each segment must be estimated, and a reasonable approximation of the work done can then be made. If we subdivide the distance into many more segments, Δd can be made smaller and our estimate of the work done more accurate. In the limit as Δd approaches zero, the total area of the many narrow rectangles approaches the area under the curve, Fig. 6–6b. That is, *the work done by a variable force in moving an object between two points is equal to the area under the $F_{\parallel}$ vs. d curve between those two points.*

6–3 Kinetic Energy, and the Work-Energy Principle

Energy is one of the most important concepts in science. Yet we cannot give a simple general definition of energy in only a few words. Nonetheless, each specific type of energy can be defined fairly simply. In this chapter, we define translational kinetic energy and potential energy. In later chapters, we will examine other types of energy, such as that related to heat (Chapters 14 and 15). The crucial aspect of all the types of energy is that the sum of all types, the *total energy*, remains the same after any process occurs as it was before: that is, the quantity "energy" can be defined so that it is a conserved quantity. More on this shortly.

For the purposes of this chapter, we can define energy in the traditional way as "the ability to do work." This simple definition is not very precise, nor is it really valid for all types of energy.[†] However, for mechanical energy which we discuss in this chapter, it serves to underscore the fundamental connection between work and energy. We now define and discuss one of the basic types of energy, kinetic energy.

A moving object can do work on another object it strikes. A flying cannonball does work on a brick wall it knocks down; a moving hammer does work on a nail it strikes. In either case, a moving object exerts a force on a

[†]Energy associated with heat is often not available to do work, as we will discuss in detail in Chapter 15.

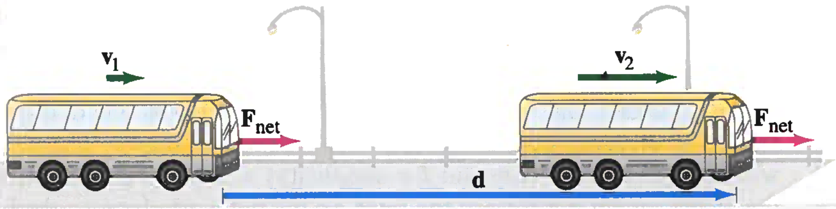


FIGURE 6–7 A constant net force F_{net} accelerates a bus from speed v_1 to speed v_2 over a distance d. The work done is $W = F_{net}d$.

second object and moves it through a distance. An object in motion has the ability to do work and thus can be said to have energy. The energy of motion is called **kinetic energy**, from the Greek word *kinetikos*, meaning "motion."

In order to obtain a quantitative definition for kinetic energy, let us consider an object of mass m that is moving in a straight line with an initial speed v_1. To accelerate it uniformly to a speed v_2, a constant net force F_{net} is exerted on it parallel to its motion over a distance d, Fig. 6–7. Then the net work done on the object is $W_{net} = F_{net}d$. We apply Newton's second law, $F_{net} = ma$, and use Eq. 2–10c, which we now write as $v_2^2 = v_1^2 + 2ad$, with v_1 as the initial speed and v_2 the final speed. We solve for a in Eq. 2–10c,

$$a = \frac{v_2^2 - v_1^2}{2d},$$

then substitute this into $F_{net} = ma$, and determine the work done:

$$W_{net} = F_{net}\,d = mad = m\left(\frac{v_2^2 - v_1^2}{2d}\right)d$$

or

$$W_{net} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad (6\text{–}2)$$

We *define* the quantity $\frac{1}{2}mv^2$ to be the **translational kinetic energy** (KE) of the object:

Kinetic energy defined

$$\text{KE} = \tfrac{1}{2}mv^2. \qquad (6\text{–}3)$$

(We call this "translational" kinetic energy to distinguish it from rotational kinetic energy, which we will discuss later, in Chapter 8.) Equation 6–2, derived here for one-dimensional motion, is valid in general for translational motion in three dimensions and even if the force varies. We can rewrite Eq. 6–2 as:

$$W_{net} = \text{KE}_2 - \text{KE}_1$$

or

WORK-ENERGY PRINCIPLE

$$W_{net} = \Delta\text{KE}. \qquad (6\text{–}4)$$

Equation 6–4 (or Eq. 6–2) is an important result. It can be stated in words:

WORK-ENERGY PRINCIPLE

The net work done on an object is equal to the change in its kinetic energy.

This is known as the **work-energy principle**. Notice, however, that we made use of Newton's second law, $F_{net} = ma$, where F_{net} is the *net* force—the sum of all forces acting on the object. Thus, the work-energy principle is valid only if W is the *net work* done on the object—that is, the work done by all forces acting on the object.

The work-energy principle tells us that if (positive) net work W is done on a body, its kinetic energy increases by an amount W. The principle

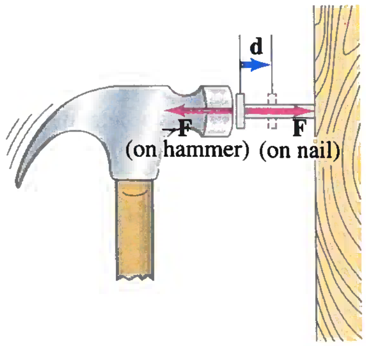


FIGURE 6–8 A moving hammer strikes a nail and comes to rest. The hammer exerts a force F on the nail; the nail exerts a force $-F$ on the hammer (Newton's third law). The work done on the nail is positive ($W_n = Fd > 0$). The work done on the hammer is negative ($W_h = -Fd$).

also holds true for the reverse situation: if negative net work W is done on the body, the body's kinetic energy decreases by an amount W. That is, a net force exerted on a body opposite to the body's direction of motion reduces its speed and its kinetic energy. An example is a moving hammer (Fig. 6–8) striking a nail. The net force on the hammer ($-\mathbf{F}$ in the figure, where $\mathbf{F}$ is assumed constant for simplicity) acts toward the left, whereas the displacement $\mathbf{d}$ is toward the right. So the net work done on the hammer, $W_h = (F)(d)(\cos 180°) = -Fd$, is negative and the hammer's kinetic energy decreases (usually to zero). Also note in this example that the hammer, as it slows down, does positive work on the nail: if the nail exerts a force $-\mathbf{F}$ on the hammer to slow it down, the hammer exerts a force $+\mathbf{F}$ on the nail (Newton's third law) through the distance d. Hence the net work done on the nail is $W_n = (+F)(+d) = Fd = -W_h$, and W_n is positive. Thus the decrease in kinetic energy of the hammer is also equal to the work the hammer can do on another object—which is consistent with energy being the ability to do work.

Note that whereas the translational kinetic energy ($=\frac{1}{2}mv^2$) is directly proportional to the mass of the object, it is proportional to the *square* of the speed. Thus, if the mass is doubled, the kinetic energy is doubled. But if the speed is doubled, the object has four times as much kinetic energy and is therefore capable of doing four times as much work.

Work-energy principle summarized

To summarize, the connection between work and kinetic energy (Eq. 6–4) operates both ways. If the net work W done on an object is positive, then the object's kinetic energy increases. If the net work W done on an object is negative, its kinetic energy decreases. If the net work done on the object is zero, its kinetic energy remains constant (which also means its speed is constant).

Energy units: the joule

Because of the direct connection between work and kinetic energy (Eq. 6–4), energy is measured in the same units as work: joules in SI units, ergs in the cgs, and foot-pounds in the British system. Like work, kinetic energy is a scalar quantity. The kinetic energy of a group of objects is the (algebraic) sum of the kinetic energies of the individual objects.

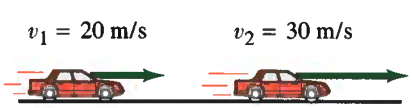
EXAMPLE 6–4 **KE and work done on a baseball.** A 145-g baseball is thrown with a speed of 25 m/s. (*a*) What is its kinetic energy? (*b*) How much work was done on the ball to make it reach this speed, if it started from rest?

SOLUTION (*a*) The kinetic energy is

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.145\ \text{kg})(25\ \text{m/s})^2 = 45\ \text{J}.$$

(*b*) Since the initial kinetic energy was zero, the net work done is just equal to the final kinetic energy, 45 J.

FIGURE 6–9 Example 6–5.



EXAMPLE 6–5 **Work on a car, to increase its KE.** How much work is required to accelerate a 1000-kg car from 20 m/s to 30 m/s (Fig. 6–9)?

SOLUTION The net work needed is equal to the increase in kinetic energy:

$$\begin{aligned} W &= \text{KE}_2 - \text{KE}_1 \\ &= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \\ &= \tfrac{1}{2}(1000\ \text{kg})(30\ \text{m/s})^2 - \tfrac{1}{2}(1000\ \text{kg})(20\ \text{m/s})^2 \\ &= 2.5 \times 10^5\ \text{J}. \end{aligned}$$

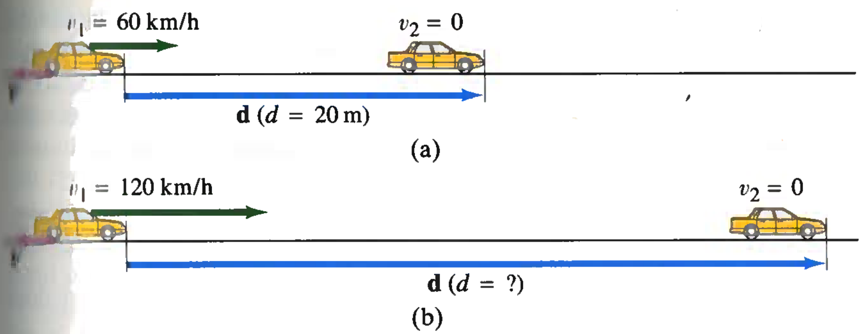


FIGURE 6–10 Conceptual Example 6–6.

CONCEPTUAL EXAMPLE 6–6 **Work to stop a car.** An automobile traveling 60 km/h can brake to a stop within a distance of 20 m (Fig. 6–10a). If the car is going twice as fast, 120 km/h, what is its stopping distance (Fig. 6–10b)? The maximum braking force is approximately independent of speed.

RESPONSE Since the stopping force F is approximately constant, the work needed to stop the car, Fd, is proportional to the distance traveled. We apply the work-energy principle, noting that **F** and **d** are in opposite directions and that the final speed of the car is zero:

$$W_{net} = Fd\cos 180° = -Fd$$
$$= \Delta \text{KE} = 0 - \tfrac{1}{2}mv^2.$$

Thus, since the force and mass are constant, we can see that the stopping distance, d, increases with the square of the speed:

Stopping distance ∝ initial speed squared

$$d \propto v^2.$$

If the car's initial speed is doubled, the stopping distance is $(2)^2 = 4$ times as great, or 80 m.

6–4 Potential Energy

Potential energy

We have just discussed how an object is said to have energy by virtue of its motion, which we call kinetic energy. But it is also possible to have **potential energy**, which is the energy associated with forces that depend on the position or configuration of a body (or bodies) and the surroundings. Various types of potential energy (PE) can be defined, and each type is associated with a particular force.

A wound-up clock spring is an example of potential energy. The clock spring acquired its potential energy because work was done *on* it by the person winding the clock. As the spring unwinds, it exerts a force and does work to move the clock hands around.

PE of gravity

Perhaps the most common example of potential energy is *gravitational potential energy*. A heavy brick held high in the air has potential energy because of its position relative to the Earth. It has the ability to do work, for if it is released, it will fall to the ground due to the gravitational force, and can do work on, say, a stake, driving it into the ground. Let us determine the gravitational potential energy of an object near the surface of the Earth. In order to

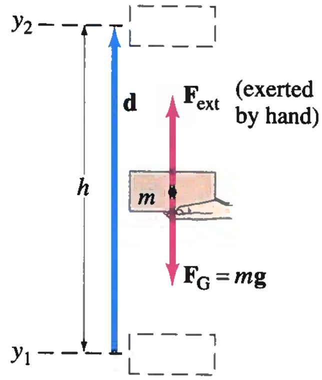


FIGURE 6–11 A person exerts an upward force $F_{ext} = mg$ to lift a brick from y_1 to y_2.

lift an object of mass m vertically, an upward force at least equal to its weight, mg, must be exerted on it, say, by a person's hand. In order to lift it without acceleration a height h, from position y_1 to y_2 in Fig. 6–11 (upward direction chosen positive), a person must do work equal to the product of the needed external force, $F_{ext} = mg$ upward, and the vertical distance h. That is,

$$W_{ext} = F_{ext}\, d \cos 0° = mgh$$
$$= mg\,(y_2 - y_1). \qquad \textbf{(6–5a)}$$

Gravity is also acting on the object as it moves from y_1 to y_2, and does work on it equal to

$$W_G = F_G d \cos\theta = mgh \cos 180°,$$

where $\theta = 180°$ because $\mathbf{F}_G$ and $\mathbf{d}$ point in opposite directions. So

$$W_G = -mgh$$
$$= -mg(y_2 - y_1). \qquad \textbf{(6–5b)}$$

If we now allow the object to start from rest and fall freely under the action of gravity, it acquires a velocity given by $v^2 = 2gh$ (Eq. 2–10c) after falling a height h. It then has kinetic energy $\frac{1}{2}mv^2 = \frac{1}{2}m(2gh) = mgh$, and if it strikes a stake it can do work on the stake equal to mgh (work-energy principle). Thus, to raise an object of mass m to a height h *requires* an amount of work equal to mgh (Eq. 6–5a). And once at height h, the object has the *ability* to do an amount of work equal to mgh.

We therefore define the **gravitational potential energy** of a body as the product of its weight mg and its height y above some reference level (such as the ground):

Gravitational PE

$$\text{PE}_{grav} = mgy. \qquad \textbf{(6–6)}$$

The higher an object is above the ground, the more gravitational potential energy it has. We combine Eq. 6–5a with Eq. 6–6:

$$W_{ext} = mg\,(y_2 - y_1)$$
$$W_{ext} = \text{PE}_2 - \text{PE}_1 = \Delta\text{PE}. \qquad \textbf{(6–7a)}$$

That is, the work done by an external force to move the mass m from point 1 to point 2 (without acceleration) is equal to the change in potential energy of the object between points 1 and 2.

We can also write ΔPE in terms of the work done by gravity itself, starting from Eq. 6–5b, and obtain

$$W_G = -mg(y_2 - y_1)$$
$$W_G = -\Delta\text{PE}. \qquad \textbf{(6–7b)}$$

That is, the work done by gravity as the mass m moves from point 1 to point 2 is equal to the negative of the difference in potential energy between points 1 and 2.

Notice that the gravitational potential energy depends on the *vertical height* of the object *above some reference level* (Eq. 6–6), and in some situations, you may wonder from what point to measure the height y. The gravitational potential energy of a book held high above a table, for example, depends on whether we measure y from the top of the table, from the floor, or from some other reference point. What is physically important in any situation is the *change* in potential energy, ΔPE, because that is what is related to the work done, Eqs. 6–7, and it is ΔPE that can be measured. We can thus choose to measure y from any reference point that is convenient, but we must choose it at the start and be consistent throughout a calculation. The *change* in potential energy between any two points does not depend on this choice.

Change in PE *is what is physically meaningful*

An important result we discussed earlier (see Example 6–2 and Fig. 6–4) is that, since the gravity force does work only in the vertical direction, the work done by gravity depends only on the vertical height h, and not on the path taken, whether it be purely vertical motion or, say, motion along an incline. Thus, from Eqs. 6–7, we see that changes in gravitational potential energy depend only on the change in vertical height and not on the path taken.

EXAMPLE 6–7 **Potential energy changes for a roller coaster.** A 1000-kg roller-coaster car moves from point A, Fig. 6–12, to point B and then to point C. (*a*) What is its gravitational potential energy at B and C relative to point A? That is, take $y = 0$ at point A. (*b*) What is the change in potential energy when it goes from B to C? (*c*) Repeat parts (*a*) and (*b*), but take the reference point ($y = 0$) to be at point C.

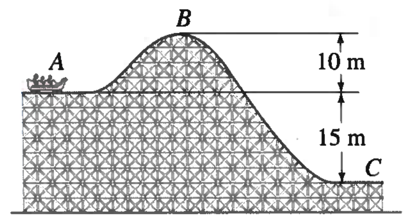


FIGURE 6–12 Example 6–7.

SOLUTION (*a*) We take upward as the positive direction, and measure the heights from point A, which means initially that the potential energy is zero. At point B, where $y_B = 10\,\text{m}$,

$$\text{PE}_B = mgy_B = (1000\ \text{kg})(9.8\ \text{m/s}^2)(10\ \text{m}) = 9.8 \times 10^4\ \text{J}.$$

At point C, $y_C = -15\,\text{m}$, since C is below A. Therefore,

$$\text{PE}_C = mgy_C = (1000\ \text{kg})(9.8\ \text{m/s}^2)(-15\ \text{m}) = -1.5 \times 10^5\ \text{J}.$$

(*b*) In going from B to C, the potential energy change ($\text{PE}_{\text{final}} - \text{PE}_{\text{initial}}$) is

$$\text{PE}_C - \text{PE}_B = (-1.5 \times 10^5\ \text{J}) - (9.8 \times 10^4\ \text{J})$$
$$= -2.5 \times 10^5\ \text{J}.$$

The gravitational potential energy decreases by 2.5×10^5 J.
(*c*) In this instance, $y_A = +15\,\text{m}$ at point A, so the potential energy initially (at A) is equal to

$$\text{PE}_A = (1000\ \text{kg})(9.8\ \text{m/s}^2)(15\ \text{m}) = 1.5 \times 10^5\ \text{J}.$$

At B, $y_B = 25\,\text{m}$, so the potential energy is

$$\text{PE}_B = 2.5 \times 10^5\ \text{J}.$$

At C, $y_C = 0$, so the potential energy is zero. The change in potential energy going from B to C is

$$\text{PE}_C - \text{PE}_B = 0 - 2.5 \times 10^5\ \text{J} = -2.5 \times 10^5\ \text{J},$$

which is the same as in part (*b*).

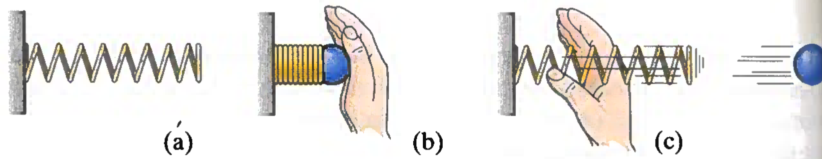


FIGURE 6–13 A spring (a) can store energy (elastic PE) when compressed as in (b), and can do work when released, as in (c).

PE defined in general

There are other kinds of potential energy besides gravitational. Each form of potential energy is associated with a particular force, and can be defined analogously to gravitational potential energy. In general, the *change in potential energy associated with a particular force, is equal to the negative of the work done by that force if the object is moved from one point to a second point* (as in Eq. 6–7b for gravity). Alternatively, we can define the *change in potential energy as the work required of an external force to move the object without acceleration between the two points*, as in Eq. 6–7a.

PE of elastic spring

We now consider another type of potential energy, that associated with elastic materials. This includes a great variety of practical applications. To take a simple example, consider the simple coil spring shown in Fig. 6–13. The spring has potential energy when compressed (or stretched), for when it is released, it can do work on a ball as shown. For a person to hold a spring either stretched or compressed an amount x from its normal (unstretched) length requires a force F_P that is directly proportional to x. That is,

$$F_P = kx,$$

where k is a constant, called the *spring constant*, and is a measure of the stiffness of the particular spring. The spring itself exerts a force in the opposite direction (Fig. 6–14),

$$F_S = -kx. \qquad \textbf{(6–8)}$$

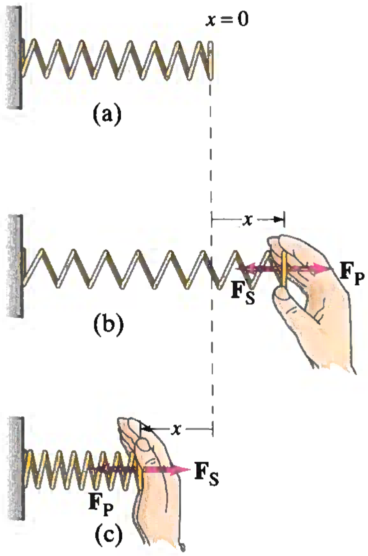


FIGURE 6–14 (a) Spring in normal (unstretched) position. (b) Spring is stretched by a person exerting a force $\mathbf{F}_P$ to the right (positive direction). The spring pulls back with a force $\mathbf{F}_S$ when $F_S = -kx$. (c) Person compresses the spring ($x < 0$), and the spring pushes back with a force $F_S = -kx$ where $F_S > 0$ because $x < 0$.

This force is sometimes called a "restoring force" because the spring exerts its force in the direction opposite the displacement (hence the minus sign), acting to return it to its normal length. Equation 6–8 is known as the **spring equation** and also as **Hooke's law** (see Chapter 9), and is accurate for springs as long as x is not too great.

In order to calculate the potential energy of a stretched spring, let us calculate the work required to stretch it (Fig. 6–14b). We might expect to use Eq. 6–1 for the work done on it, $W = Fx$, where x is the amount it is stretched from its normal length. But this would be incorrect since the force F_P $(= kx)$ is not constant but varies over this distance, becoming greater the more the spring is stretched, as shown graphically in Fig. 6–15. So let us use the average force, $\bar{F}$. Since F_P varies linearly—from zero at the unstretched position to kx when stretched to x—the average force is $\bar{F} = \frac{1}{2}[0 + kx] = \frac{1}{2}kx$, where x here is the final amount stretched (shown as x_f in Fig. 6–15 for clarity). The work done is then

$$W = \bar{F}_P x = (\tfrac{1}{2}kx)(x) = \tfrac{1}{2}kx^2.$$

Hence the *elastic potential energy* is proportional to the square of the amount stretched†:

Elastic PE

$$\text{elastic PE} = \tfrac{1}{2}kx^2. \qquad \textbf{(6–9)}$$

If a spring is *compressed* a distance x from its normal length, the force is still $F_P = kx$, and again the potential energy is given by this equation

†We can also obtain Eq. 6–9 using Section 6–2. The work done, and hence ΔPE, equals the area under the F vs. x graph of Fig. 6–15. This area is a triangle (colored in Fig. 6–15) of altitude kx and base x, and hence of area (for a triangle) equal to $\frac{1}{2}(kx)(x) = \frac{1}{2}kx^2$.

Thus x can be either the amount compressed or amount stretched from the normal position.

In each of the above examples of potential energy—from a brick held at a height y, to a stretched or compressed spring—an object has the capacity or *potential* to do work even though it is not yet actually doing it. From these examples, we can also see that energy can be *stored*, for later use, in the form of potential energy (Fig. 6–13, for example, for a spring). It is also worth noting that although there is a single universal formula for the translational kinetic energy of an object, $\frac{1}{2}mv^2$, there is no single formula for potential energy. Instead, the mathematical form of the potential energy depends on the force involved.

Potential energy belongs to a system, and not to a single object alone. Potential energy is associated with a force, and a force on one object is always exerted by some other object. Thus potential energy is a property of the system as a whole. For a particle raised to a height y above the Earth's surface, the potential energy change is mgy. The system here is the particle plus the Earth, and properties of both are involved: particle (m) and Earth (g).

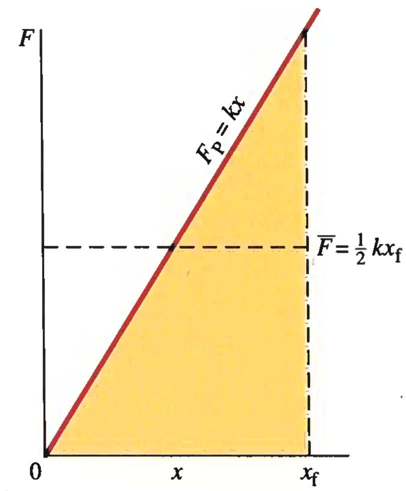


FIGURE 6–15 As a spring is stretched (or compressed), the force increases linearly as x increases: graph of $F = kx$ vs. x from $x = 0$ to $x = x_f$.

6–5 Conservative and Nonconservative Forces

The work done against gravity in moving an object from one point to another does not depend on the path taken. For example, it takes the same work ($=mgy$) to lift an object of mass m vertically a certain height as to carry it up an incline of the same vertical height, as in Fig. 6–4 (see Example 6–2). Forces such as gravity, for which the work done does not depend on the path taken but only on the initial and final positions, are called **conservative forces**. The elastic force of a spring (or other elastic material) in which $F = -kx$, is also a conservative force. Friction, on the other hand, is a **nonconservative** force since the work it does, for example, when a crate is moved across a floor from one point to another depends on whether the path taken is straight, or is curved or zigzag. As shown in Fig. 6–16, if a crate is pushed from point 1 to point 2 via the longer semicircular path rather than in the straight path, more work is done against friction, since the distance is greater and, unlike the gravitational force, the friction force is always directed opposite to the direction of motion. (The $\cos\theta$ term in Eq. 6–1 is always $\cos 180° = -1$ at all points on the path for the friction force.) Other forces that are nonconservative include the force exerted by a person and tension in a rope (see Table 6–1).

TABLE 6–1 Conservative and Nonconservative Forces

Conservative Forces	Nonconservative Forces
Gravitational	Friction
Elastic	Air resistance
Electric	Tension in cord
	Motor or rocket propulsion
	Push or pull by person

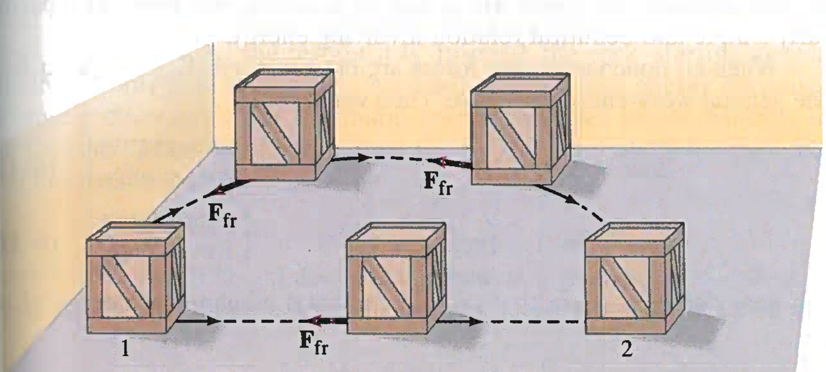


FIGURE 6–16 A crate is pulled across the floor from position 1 to position 2 via two paths, one straight and one curved. The friction force is always in the direction exactly opposed to the direction of motion. Hence, for a constant magnitude friction force, $W_{fr} = -F_{fr}d$, so if d is greater (as for the curved path), then W is greater.

Because potential energy is energy associated with the position or configuration of bodies, potential energy can only make sense if it can be stated uniquely for a given point. This cannot be done with nonconservative forces since the work done between two points depends not only on the two points but also on what path is taken (as in Fig. 6–16). Hence, *potential energy can be defined only for a conservative force*. Thus, although potential energy is always associated with a force, not all forces have a potential energy—for example, there is no potential energy for friction.

PE can be defined only for a conservative force

We can now extend the **work-energy principle** (discussed earlier in Section 6–3) to include potential energy. Suppose several forces act on an object which can undergo translational motion. And suppose only some of these forces are conservative, and we can write a potential-energy function for these conservative forces. We write the total (net) work W_{net} as a sum of the work done by conservative forces, W_C, and the work done by nonconservative forces, W_{NC}:

$$W_{net} = W_C + W_{NC}.$$

Then, from the work-energy principle, Eq. 6–4, we have

$$W_{net} = \Delta \text{KE}$$

$$W_C + W_{NC} = \Delta \text{KE}$$

where $\Delta \text{KE} = \text{KE}_2 - \text{KE}_1$, and so

$$W_{NC} = \Delta \text{KE} - W_C.$$

Work done by a conservative force can be written in terms of potential energy, as we saw in Eq. 6–7b for gravitational potential energy:

$$W_C = -\Delta \text{PE}.$$

We substitute this into the last equation above:

WORK-ENERGY PRINCIPLE (general form)

$$W_{NC} = \Delta \text{KE} + \Delta \text{PE}. \qquad \textbf{(6–10)}$$

Thus, *the work W_{NC} done by the nonconservative forces acting on an object is equal to the total change in kinetic and potential energy.*

It must be emphasized that *all* the forces acting on a body must be included in Eq. 6–10, either in the potential energy term on the right (if it is a conservative force), or in the work term, W_{NC}, on the left (but not in both!).

6–6 Mechanical Energy and Its Conservation

If only conservative forces are acting on a system, we arrive at a particularly simple and beautiful relation involving energy.

When no nonconservative forces are present, then $W_{NC} = 0$ in Eq. 6–10, the general work-energy principle. Then we have

$$\Delta \text{KE} + \Delta \text{PE} = 0 \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11a)}$$

or

$$(\text{KE}_2 - \text{KE}_1) + (\text{PE}_2 - \text{PE}_1) = 0. \qquad \begin{bmatrix}\text{conservative}\\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–11b)}$$

We now define a quantity E, called the **total mechanical energy** of our system, as the sum of the kinetic and potential energies at any moment

Total mechanical energy defined

$$E = \text{KE} + \text{PE}.$$

Now we can rewrite Eq. 6–11b as

CONSERVATION OF MECHANICAL ENERGY

$$\text{KE}_2 + \text{PE}_2 = \text{KE}_1 + \text{PE}_1 \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12a)}$$

or

$$E_2 = E_1 = \text{constant.} \qquad \begin{bmatrix}\text{conservative} \\ \text{forces only}\end{bmatrix} \qquad \textbf{(6–12b)}$$

Equations 6–12 express a useful and profound principle regarding the total mechanical energy—namely, that it is a **conserved quantity**. The total mechanical energy E remains constant as long as no nonconservative forces act: (KE + PE) at some initial point 1 is equal to the (KE + PE) at any later point 2. To say it another way, consider Eq. 6–11a which tells us $\Delta\text{PE} = -\Delta\text{KE}$; that is, if the kinetic energy KE increases, then the potential energy PE must decrease by an equivalent amount to compensate. Thus, the total, KE + PE, remains constant. This is called the **principle of conservation of mechanical energy** for conservative forces:

CONSERVATION OF MECHANICAL ENERGY

If only conservative forces are acting, the total mechanical energy of a system neither increases nor decreases in any process. It stays constant—it is conserved.

We now see the reason for the term "conservative force"—because for such forces, mechanical energy is conserved.

In the next Section we shall see the great usefulness of the conservation of mechanical energy principle in a variety of situations, and how it is often easier to use than the kinematic equations or Newton's laws. After that we will discuss how other forms of energy can be included in a grander conservation of energy law that includes energy associated with nonconservative forces.

6–7 Problem Solving Using Conservation of Mechanical Energy

A simple example of the conservation of mechanical energy is a rock allowed to fall from a height h under gravity (neglecting air resistance), as shown in Fig. 6–17. At the instant it is dropped, the rock, starting at rest, initially has only potential energy. As it falls, its potential energy decreases (because y decreases), but its kinetic energy increases to compensate, so that the sum of the two remains constant. At any point along the path, the total mechanical energy is given by

$$E = \text{KE} + \text{PE} = \tfrac{1}{2}mv^2 + mgy$$

where y is the rock's height above the ground at a given instant and v is its speed at that point. If we let the subscript 1 represent the rock at one

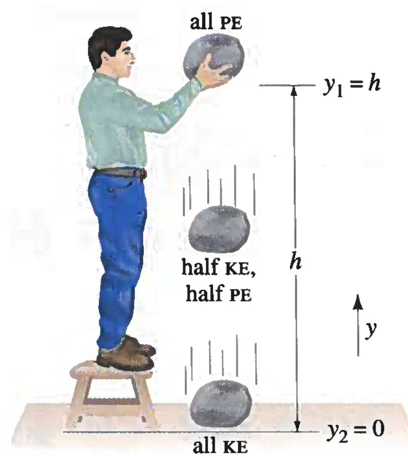


FIGURE 6–17 The rock's potential energy changes to kinetic energy as it falls.

point along its path (for example, the initial point), and 2 represent it at some other point, then we can write

$$\text{total mechanical energy at point 1} = \text{total mechanical energy at point 2}$$

or (see also Eq. 6–12a)

Conservation of energy when only gravity acts

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2. \qquad \text{[grav. PE only]} \quad \textbf{(6–13)}$$

Just before the rock hits the ground, all of its initial potential energy will have been transformed into kinetic energy. We can see this from Eq. 6–13: initially (point 1), we set $y_1 = h$ and $v_1 = 0$ (the rock starts from rest). Just before hitting the ground (point 2), we have $y_2 = 0$, so from Eq. 6–13 we have

$$0 + mgh = \tfrac{1}{2}mv_2^2 + 0,$$

or $\text{KE}_2 = \frac{1}{2}mv_2^2 = mgh = \text{PE}_1$; the original potential energy has become kinetic energy.

EXAMPLE 6–8 **Falling rock.** If the original height of the stone in Fig. 6–17 is $y_1 = h = 3.0\text{ m}$, calculate the stone's speed when it has fallen to 1.0 m above the ground.

SOLUTION Since $v_1 = 0$ (the moment of release), $y_2 = 1.0\text{ m}$, and $g = 9.8\text{ m/s}^2$, Eq. 6–13 gives

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$

$$0 + (m)(9.8\text{ m/s}^2)(3.0\text{ m}) = \tfrac{1}{2}mv_2^2 + (m)(9.8\text{ m/s}^2)(1.0\text{ m}).$$

The m's cancel out, and solving for v_2^2 (which we see doesn't depend on m), we find

$$v_2^2 = 2[(9.8\text{ m/s}^2)(3.0\text{ m}) - (9.8\text{ m/s}^2)(1.0\text{ m})] = 39.2\text{ m}^2/\text{s}^2,$$

and

$$v_2 = \sqrt{39.2}\text{ m/s} = 6.3\text{ m/s}.$$

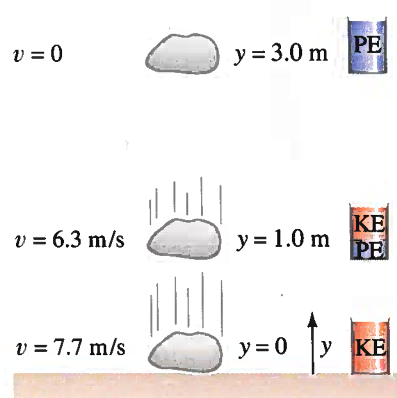


FIGURE 6–18 Energy buckets (for Example 6–8). Kinetic energy is red and potential energy is blue. The total (KE + PE) is the same for the three points shown. Note that the speed at $y = 0$, just before the rock hits, is $\sqrt{2(9.8\text{ m/s}^2)(3.0\text{ m})} = 7.7\text{ m/s}$.

A simple way to visualize energy conservation is with an "energy bucket" as shown in Fig. 6–18. At each point in the fall of the stone, the amount of kinetic energy and potential energy are shown as if they were two differently colored materials in the bucket. The total amount of material in the bucket (= total mechanical energy), remains constant.

Equation 6–13 can be applied to any object moving without friction under the action of gravity. For example, Fig. 6–19 shows a roller-coaster car starting from rest at the top of a hill, and coasting without friction to the bottom and up the hill on the other side.† Initially, the car has only potential energy. As it coasts down the hill, it loses potential energy and gains in kinetic energy, but the sum of the two remains constant. At the bottom of the hill it has its maximum kinetic energy, and as it climbs up the other side the kinetic energy changes back to potential energy. When the car comes to rest again, all of its energy will be potential energy. Given that the potential energy is proportional to the vertical height, energy conservation tells us that (in the

†The forces on the car are: gravity, the normal force exerted by the road, and friction (here assumed zero). The normal force acts perpendicular to the road, and so is always perpendicular to the motion and does no work. Thus $W_{NC} = 0$ in Eq. 6–10 (so mechanical energy is conserved) and we can use Eq. 6–13 with the potential energy being only gravitational potential energy. We will see how to deal with friction, for which $W_{NC} \neq 0$, in Section 6–9.

absence of friction) the car comes to rest at a height equal to its original height. If the two hills are the same height, the car will just barely reach the top of the second hill when it stops. If the second hill is lower than the first, not all of the car's kinetic energy will be transformed to potential energy and the car can continue over the top and down the other side. If the second hill is higher, the car will only reach a height on it equal to its original height on the first hill. This is true (in the absence of friction) no matter how steep the hill is, since potential energy depends only on the vertical height.

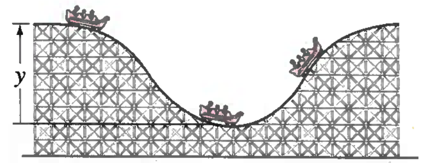


FIGURE 6–19 A roller-coaster car moving without friction illustrates the conservation of mechanical energy.

EXAMPLE 6–9 **Roller-coaster speed using energy conservation.** Assuming the height of the hill in Fig. 6–19 is 40 m, and the roller-coaster car starts from rest at the top, calculate (*a*) the speed of the roller-coaster car at the bottom of the hill, and (*b*) at what height it will have half this speed. Take $y = 0$ at the bottom of the hill.

SOLUTION (*a*) We use Eq. 6–13 with $v_1 = 0$, $y_1 = 40\,\text{m}$, and $y_2 = 0$. Then

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$

$$0 + (m)(9.8\,\text{m/s}^2)(40\,\text{m}) = \tfrac{1}{2}mv_2^2 + 0.$$

The m's cancel out and we find $v_2 = \sqrt{2(9.8\,\text{m/s}^2)(40\,\text{m})} = 28\,\text{m/s}$.

(*b*) We use the same equation, but now $v_2 = 14\,\text{m/s}$ (half of 28 m/s) and y_2 is unknown:

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$

$$0 + (m)(9.8\,\text{m/s}^2)(40\,\text{m}) = \tfrac{1}{2}(m)(14\,\text{m/s})^2 + (m)(9.8\,\text{m/s}^2)(y_2).$$

We cancel the m's and solve for y_2 and find $y_2 = 30\,\text{m}$. That is, the car has a speed of 14 m/s when it is 30 *vertical* meters above the lowest point, both when descending the left-hand hill and when ascending the right-hand hill.

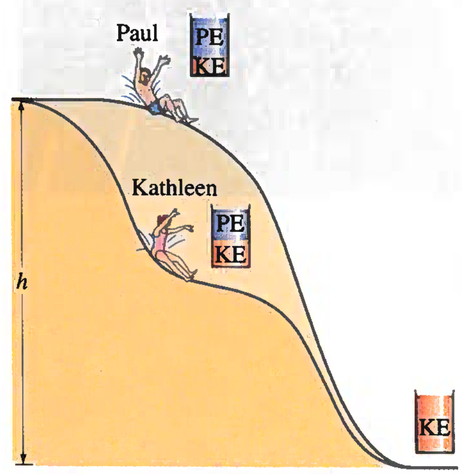
The mathematics of this Example is almost the same as that in Example 6–8. But there is an important difference between them. Example 6–8 could have been solved using force and acceleration. But here, where the motion is not vertical, using $F = ma$ would have been very difficult, whereas energy conservation readily gives us the answer.

CONCEPTUAL EXAMPLE 6–10 **Speeds on two water slides.** Two water slides at a pool are shaped differently but start at the same height h (Fig. 6–20). Two riders, Paul and Kathleen, start from rest at the same time on different slides. (*a*) Which rider, Paul or Kathleen, is traveling faster at the bottom? (*b*) Which rider makes it to the bottom first? Ignore friction.

RESPONSE (*a*) Each rider's initial potential energy mgh gets transformed to kinetic energy, so the speed v at the bottom is obtained from $\tfrac{1}{2}mv^2 = mgh$. The mass cancels in this equation and so the speed will be the same, regardless of the mass of the rider. Since they descend the same vertical height, they will finish with the same speed.

(*b*) Note that Kathleen is consistently at a lower elevation than Paul for the entire trip. This means she has converted her potential energy to kinetic energy earlier. Consequently, she is traveling faster than Paul for the whole trip, except toward the end where Paul finally gets up to the same speed. Since she was going faster for the whole trip, and the distance is roughly the same, Kathleen gets to the bottom first.

FIGURE 6–20 Conceptual Example 6–10.



PHYSICS APPLIED

Sport

There are many interesting examples of the conservation of energy in sports, one of which is the pole vault illustrated in Fig. 6–21. We often have to make approximations, but the sequence of events in broad outline for this case is as follows. The kinetic energy of the running athlete is transformed into elastic potential energy of the bending pole and, as the athlete leaves the ground, into gravitational potential energy. When the vaulter reaches the top and the pole has straightened out again, the energy has all been transformed into gravitational potential energy (if we ignore the vaulter's low horizontal speed over the bar). The pole does not supply any energy, but it acts as a device to *store* energy and thus aid in the transformation of kinetic energy into gravitational potential energy, which is the net result. The energy required to pass over the bar depends on how high the center of mass† (CM) of the vaulter must be raised. By bending their bodies, pole vaulters keep their CM so low that it can actually pass slightly beneath the bar (Fig. 6–22), thus enabling them to cross over a higher bar than would otherwise be possible.

FIGURE 6–21 Transformation of energy during a pole vault.

FIGURE 6–22 By bending their bodies, pole vaulters can keep their center of mass so low that it may even pass below the bar. By changing their kinetic energy (of running) into gravitational potential energy ($= mgy$) in this way, vaulters can cross over a higher bar than if the change in potential energy were accomplished without carefully bending the body.

EXAMPLE 6–11 ESTIMATE **Pole vault.** Estimate the kinetic energy and the speed required for a 70-kg pole vaulter to just pass over a bar 5.0 m high. Assume the vaulter's center of mass is initially 0.90 m off the ground and reaches its maximum height at the level of the bar itself.

SOLUTION We equate the total energy just before the vaulter places the end of the pole onto the ground (and the pole begins to bend and store potential energy) with the vaulter's total energy when passing over the bar (we ignore the small amount of kinetic energy at this point). We choose the initial position of the vaulter's center of mass to be $y_1 = 0$. The vaulter's body must then be raised to a height $y_2 = 5.0\,\text{m} - 0.9\,\text{m} = 4.1\,\text{m}$. Thus, using Eq. 6–13,

$$\tfrac{1}{2}mv_1^2 + 0 = 0 + mgy_2$$

and

$$\begin{aligned}\text{KE}_1 &= \tfrac{1}{2}mv_1^2 = mgy_2 \\ &= (70\,\text{kg})(9.8\,\text{m/s}^2)(4.1\,\text{m}) = 2.8 \times 10^3\,\text{J}.\end{aligned}$$

The speed is (solving for v_1 in $\text{KE}_1 = \tfrac{1}{2}mv_1^2$):

$$v_1 = \sqrt{\frac{2\text{KE}_1}{m}} = \sqrt{\frac{2(2800\,\text{J})}{70\,\text{kg}}} = 8.9\,\text{m/s}.$$

This is an approximation because we have ignored such things as the vaulter's speed while crossing over the bar, mechanical energy transformed when the pole is planted in the ground, and work done by the vaulter on the pole.

†The center of mass (CM) of a body is that point at which the entire mass of the body can be considered as concentrated, for purposes of describing its translational motion. (This is discussed in Chapter 7.) In Eq. 6–13, y is the position of the CM.

As another example of the conservation of mechanical energy, let us consider a mass m connected to a spring whose own mass can be neglected and whose stiffness constant is k. The mass m has speed v at any moment and the potential energy of the system is $\frac{1}{2}kx^2$, where x is the displacement of the spring from its unstretched length. If neither friction nor any other force is acting, the conservation-of-energy principle tells us that

Conservation of energy when PE is elastic

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \qquad \text{[elastic PE only]} \qquad \textbf{(6–14)}$$

where the subscripts 1 and 2 refer to the velocity and displacement at two different points.

EXAMPLE 6–12 **Toy dart gun.** A dart of mass 0.100 kg is pressed against the spring of a toy dart gun as shown in Fig. 6–23. The spring (with spring constant $k = 250\,\text{N/m}$) is compressed 6.0 cm and released. If the dart detaches from the spring when the latter reaches its normal length ($x = 0$), what speed does the dart acquire?

SOLUTION In the horizontal direction, the only force on the dart (neglecting friction) is the force exerted by the spring. Vertically, gravity is counterbalanced by the normal force exerted on the dart by the gun barrel. (After the dart leaves the barrel it will follow a projectile's path under gravity.) We use Eq. 6–14 with point 1 being at the maximum compression of the spring, so $v_1 = 0$ (dart not yet released) and $x_1 = -0.060\,\text{m}$. Point 2 we choose to be the instant the dart flies off the end of the spring (Fig. 6–23b), so $x_2 = 0$ and we want to find v_2. Thus Eq. 6–14 can be written

$$0 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + 0.$$

Then

$$v_2^2 = kx_1^2/m = (250\,\text{N/m})(0.060\,\text{m})^2/(0.100\,\text{kg}) = 9.0\,\text{m}^2/\text{s}^2$$

so $v_2 = \sqrt{v_2^2} = 3.0\,\text{m/s}$.

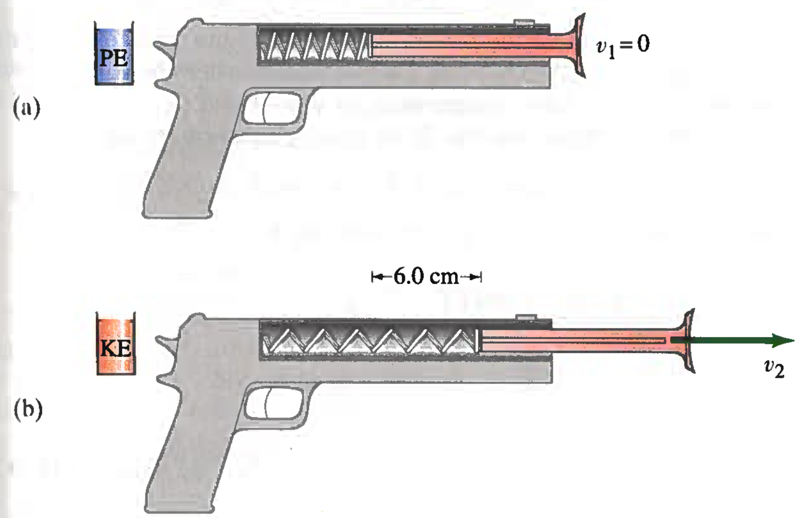


FIGURE 6–23 Example 6–12. (a) A dart is pushed against a spring, compressing it 6.0 cm. The dart is then released, and in (b) it leaves the spring at high velocity (v_2).

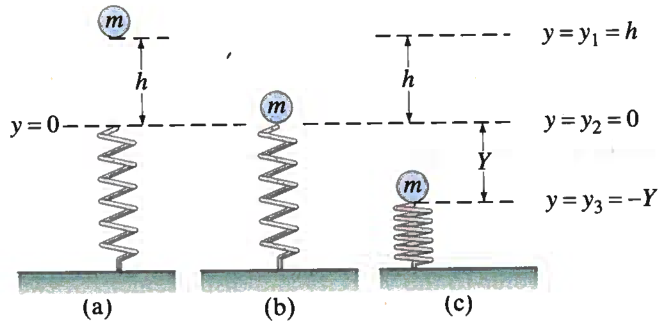


FIGURE 6–24
Example 6–13.

EXAMPLE 6–13 **Two kinds of PE.** A ball of mass $m = 2.60$ kg, starti from rest, falls a vertical distance $h = 55.0$ cm before striking a verti coiled spring, which it compresses (see Fig. 6–24) an amou $Y = 15.0$ cm. Determine the spring constant of the spring. Assume th spring has negligible mass. Measure all distances from the point whe the ball first touches the uncompressed spring ($y = 0$ at this point).

SOLUTION Since the motion is vertical, we use y instead of x (y po tive upward). We divide this solution into two parts. (See also altern solution below.)

Part 1: Let us first consider the energy changes of the ball as it falls fro a height $y_1 = h = 0.55$ m, Fig. 6–24a, to $y_2 = 0$, just as it touches th spring, Fig. 6–24b. Our system is the ball acted on by gravity (up to he the spring doesn't do anything) so

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2$$
$$0 + mgh = \tfrac{1}{2}mv_2^2 + 0$$

and $v_2 = \sqrt{2gh} = \sqrt{2(9.80\ \mathrm{m/s^2})(0.550\ \mathrm{m})} = 3.28$ m/s.

Part 2: Let's see what happens as the ball compresses the spring, Fi 6–24b to c. Now there are two conservative forces on the ball—gravi and the spring force. So our energy equation becomes

Conservation of energy: gravity and elastic PE

$$\mathrm{E}\,(\text{ball touches spring}) = \mathrm{E}\,(\text{spring compressed})$$
$$\tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ky_2^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2.$$

We take point 2 to be the instant when the ball just touches the spring, s $y_2 = 0$ and $v_2 = 3.28$ m/s. Point 3 we take to be when the ball comes to re and the spring is fully compressed, so $v_3 = 0$ and $y_3 = -Y = -0.150$ m (given). Putting these into the above energy equation we get

$$\tfrac{1}{2}mv_2^2 + 0 + 0 = 0 - mgY + \tfrac{1}{2}kY^2.$$

We know m, v_2, and Y, so we can solve for k:

$$k = \frac{2}{Y^2}\left[\tfrac{1}{2}mv_2^2 + mgY\right]$$
$$= \frac{m}{Y^2}\left[v_2^2 + 2gY\right]$$
$$= \frac{(2.60\ \mathrm{kg})}{(0.150\ \mathrm{m})^2}\left[(3.28\ \mathrm{m/s})^2 + 2(9.80\ \mathrm{m/s^2})(0.150\ \mathrm{m})\right] = 1580\ \mathrm{N/m}$$

PROBLEM SOLVING
Alternate Solution

Alternate solution: Instead of dividing the solution into two parts, w can do it all at once. After all, we get to choose what two points are use on the left and right of the energy equation. Let us write the energ equation for points 1 and 3 (Fig. 6–24). Point 1 is the initial point just before the ball starts to fall (Fig. 6–24a), so $v_1 = 0$, $y_1 = h = 0.550$ m; and point 3 is when the spring is fully compressed (Fig. 6–24c), so $v_3 = 0$, $y_3 = -Y = -0.150$ m. The forces on the ball in this process are gravity and (at least part of the time) the spring. So conservation of energy tells us

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}mv_3^2 + mgy_3 + \tfrac{1}{2}ky_3^2$$

$$0 \;+ mgh \;+ \quad 0 \quad = \quad 0 \;- mgY \;+ \tfrac{1}{2}kY^2$$

where we have set $y = 0$ for the spring at point 1 because it is not acting and is not compressed or stretched at this point. We solve for k:

$$k = \frac{2\,mg\,(h+Y)}{Y^2} = \frac{2(2.60\text{ kg})(9.80\text{ m/s}^2)(0.550\text{ m} + 0.150\text{ m})}{(0.150\text{ m})^2} = 1580\text{ N/m}$$

just as in our first method of solution.

EXAMPLE 6–14 **Bungee jump.** Dave jumps off a bridge with a bungee cord (a heavy stretchable cord) tied around his ankle (Fig. 6–25). He falls for 15 meters before the bungee cord begins to stretch. Dave's mass is 75 kg and we assume the cord obeys Hooke's law, $F = -kx$, with $k = 50$ N/m. If we neglect air resistance, estimate how far below the bridge Dave will fall before coming to a stop. Ignore the mass of the cord (not realistic, however).

PHYSICS APPLIED

Bungee jumping

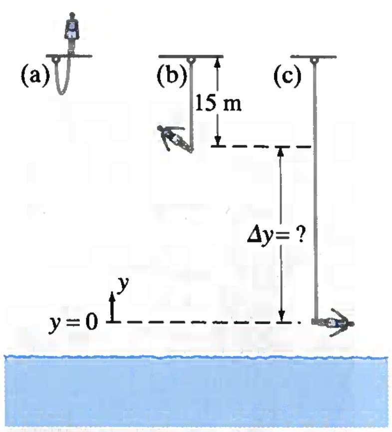


FIGURE 6–25 Example 6–14. (a) Bungee jumper about to jump. (b) Bungee cord at its unstretched length. (c) Maximum stretch of cord.

SOLUTION We first recognize that this problem is too hard to solve with the kinematic equations of Chapter 2. Those equations assumed a constant acceleration. But here the force the cord exerts on Dave gets stronger the farther he drops. But we can treat it readily using conservation of energy. Dave starts out with gravitational potential energy. The gravitational potential energy is tranformed into both kinetic energy and elastic potential energy. Assuming no frictional forces act on our system, the original total energy must be the same as the total energy at the end. If we define our coordinate system such that $y = 0$ at the lowest point in Dave's dive, and let the stretch of the cord at this point be represented by Δy, then the total fall is (see Fig. 6–25)

$$h = 15\text{ m} + \Delta y.$$

Conservation of energy then gives:

$$\text{KE}_1 + \text{PE}_1 = \text{KE}_2 + \text{PE}_2$$

$$0 + mg(15\text{ m} + \Delta y) = 0 + \tfrac{1}{2}k(\Delta y)^2.$$

To solve for Δy, we rewrite this in standard quadratic form ($ax^2 + bx + c = 0$):

$$(\Delta y)^2 - \frac{2\,mg}{k}\,\Delta y - \frac{2\,mg}{k}\,(15\text{ m}) = 0.$$

Using the quadratic formula, $\Delta y = (-b \pm \sqrt{b^2 - 4ac})/2a$, with $a = 1$, $b = (-2\,mg/k) = -29$ m, and $c = -2\,mg(15\text{ m})/k = -440\text{ m}^2$, we get two solutions:

$$\Delta y = 40\text{ m} \quad \text{and} \quad \Delta y = -11\text{ m}.$$

The negative solution is nonphysical, so the distance that Dave drops in this fall is:

$$h = 15\text{ m} + 40\text{ m} = 55\text{ m}.$$

6–8 Other Forms of Energy; Energy Transformations and the Law of Conservation of Energy

Besides the kinetic energy and potential energy of ordinary objects, other forms of energy can be defined as well. These include electric energy, nuclear energy, thermal energy, and the chemical energy stored in food and fuels. With the advent of the atomic theory, these other forms of energy have come to be considered as kinetic or potential energy at the atomic or molecular level. For example, according to the atomic theory, thermal energy is interpreted as the kinetic energy of rapidly moving molecules—when an object is heated, the molecules that make up the object move faster. On the other hand, the energy stored in food and fuel such as gasoline can be regarded as potential energy stored by virtue of the relative positions of the atoms within a molecule due to electric forces between the atoms (referred to as chemical bonds). For the energy in chemical bonds to be used to do work, it must be released, usually through chemical reactions. This is analogous to a compressed spring which, when released, can do work. Enzymes in our bodies allow the release of energy stored in food molecules. The violent spark of a spark plug in an automobile allows the mixture of gas and air to react chemically, releasing the stored energy which can then do work against the piston to propel the car forward. Electric, magnetic, and nuclear energy also can be considered examples of kinetic and potential (or stored) energy. We will deal with these other forms of energy in detail in later chapters.

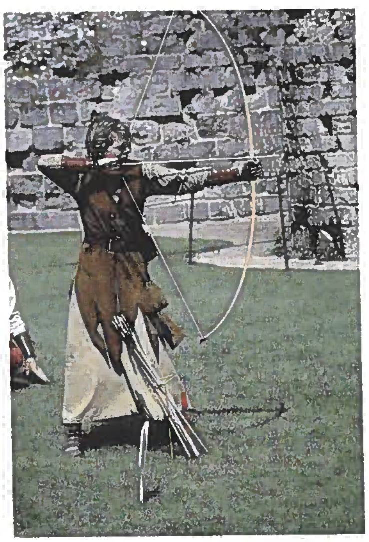


FIGURE 6–26 Potential energy of a bent bow about to be transformed into kinetic energy of an arrow.

Energy can be transformed from one form to another, and we have already encountered several examples of this. A stone held high in the air has potential energy; as it falls, it loses potential energy, since its height above the ground decreases. At the same time, it gains in kinetic energy, since its velocity is increasing. Potential energy is being transformed into kinetic energy.

Often the transformation of energy involves a transfer of energy from one body to another. The potential energy stored in the spring of Fig. 6–13b is transformed into kinetic energy of the ball, Fig. 6–13c. The kinetic energy of a running pole vaulter is transformed into elastic potential energy of the bending pole which in turn is transformed into the increasing potential energy of the rising athlete, Fig. 6–21. Water at the top of a dam has potential energy, which is transformed into kinetic energy as the water falls. At the base of the dam, the kinetic energy of the water can be transferred to turbine blades and further transformed into electric energy, as we shall see in a later chapter. The potential energy stored in a bent bow can be transformed into kinetic energy of the arrow (Fig. 6–26).

Work is done when energy is transferred

In each of these examples, the transfer of energy is accompanied by the performance of work. The spring of Fig. 6–13 does work on the ball. Water does work on turbine blades. A bow does work on an arrow. This observation gives us a further insight into the relation between work and energy: *work is done when energy is transferred from one object to another.* A person throwing a ball or pushing a grocery cart provides another example.

†If the objects are at different temperatures, heat can flow between them instead, or in addition. See Chapters 14 and 15.

The work done is a manifestation of energy being transferred from the person (ultimately derived from the chemical energy of food) to the ball or cart.

One of the great results of physics is that whenever energy is transferred or transformed, it is found that no energy is gained or lost in the process.

This is the **law of conservation of energy**, one of the most important principles in physics; it can be stated as:

The total energy is neither increased nor decreased in any process. Energy can be transformed from one form to another, and transferred from one body to another, but the total amount remains constant.

LAW OF CONSERVATION OF ENERGY

We have already discussed the conservation of energy for mechanical systems involving conservative forces, and we saw how it could be derived from Newton's laws and thus is equivalent to them. But in its full generality, the validity of the law of conservation of energy, encompassing all forms of energy including those associated with nonconservative forces like friction, rests on experimental observation. Even though Newton's laws are found to fail in the submicroscopic world of the atom, the law of conservation of energy has been found to hold in every experimental situation so far tested.

The law of conservation of energy is so wide-ranging that it plays an important role in other areas of physics (as we will see throughout this book), as well as in other sciences. We discuss another example of its usefulness in the next Section.

6–9 Energy Conservation with Dissipative Forces: Solving Problems

In our applications of energy conservation in Section 6–7, we neglected friction, a nonconservative force, but in many situations it cannot be ignored. In a real situation, the roller-coaster car in Fig. 6–19, for example, will not in fact reach the same height on the second hill as it had on the first hill because of friction. In this, and in other natural processes, the mechanical energy (sum of the kinetic and potential energies) does not remain constant but decreases. Because frictional forces reduce the total mechanical energy, they are called **dissipative forces**. Historically, the presence of dissipative forces hindered the formulation of a comprehensive conservation of energy law until well into the nineteenth century. It was not until then that heat, which is always produced when there is friction (try rubbing your hands together), was interpreted as a form of energy. Quantitative studies by nineteenth-century scientists (discussed in Chapters 14 and 15) demonstrated that if heat is considered as energy (properly called **thermal energy**), then the total energy is conserved in any process. For example, if the roller-coaster car in Fig. 6–19 is subject to frictional forces, then the initial total energy of the car will be equal to the kinetic plus potential energy of the car at any subsequent point along its path plus the amount of thermal energy produced in the process. The thermal energy produced by a constant friction force F_{fr} is equal to

Dissipative forces

the work done by this force. We now apply the general form of the work-energy principle, Eq. 6–10:

$$W_{NC} = \Delta \text{KE} + \Delta \text{PE}.$$

We can write $W_{NC} = -F_{fr}d$, where d is the distance over which the force acts. (**F** and **d** are in opposite directions, hence the minus sign.) Thus, with $\text{KE} = \frac{1}{2}mv^2$ and $\text{PE} = mgy$, we have

$$-F_{fr}d = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

or

Conservation of energy with gravity and friction

$$\tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 + F_{fr}d \qquad \begin{bmatrix}\text{gravity and}\\ \text{friction acting}\end{bmatrix} \qquad (6\text{–}15)$$

where d is the distance along the path traveled by the object in going from point 1 to point 2. Equation 6–15 can be seen to be Eq. 6–13 modified to include friction. It can be interpreted in a simple way: the initial mechanical energy of the car (point 1) equals the (reduced) final mechanical energy of the car plus the energy transformed by friction into thermal energy.

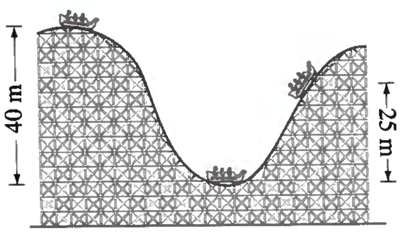


FIGURE 6–27 Example 6–15. Because of friction, a roller coaster car does not reach the original height on the second hill.

EXAMPLE 6–15 **Friction on the roller coaster.** The roller-coaster car in Example 6–9 is found to reach a vertical height of only 25 m on the second hill before coming to a stop (Fig. 6–27). It traveled a total distance of 400 m. Estimate the average friction force (assume constant) on the car, whose mass is 1000 kg.

SOLUTION We use conservation of energy, here in the form of Eq. 6–15, taking point 1 to be the instant when the car started coasting and point 2 to be the instant it stopped. Then $v_1 = 0$, $y_1 = 40$ m, $v_2 = 0$, $y_2 = 25$ m, and $d = 400$ m. Thus

$$0 + (1000\,\text{kg})(9.8\,\text{m/s}^2)(40\,\text{m}) = 0 + (1000\,\text{kg})(9.8\,\text{m/s}^2)(25\,\text{m}) + F_{fr}(400\,\text{m})$$

We solve this for F_{fr} to find $F_{fr} = 370$ N.

Another example of the transformation of kinetic energy into thermal energy occurs when an object, such as the rock of Fig. 6–17, strikes the ground. Mechanical energy is not conserved at this encounter, but the total energy is. Kinetic energy is transformed into thermal energy (and probably some sound energy) and both the rock and the ground will be slightly warmer as a result of their collision. A more apparent example of this transformation of kinetic energy into thermal energy can be observed by vigorously striking a nail several times with a hammer, and then gently touching the nail with your finger to see how hot it is.

When other forms of energy are involved, such as chemical or electrical energy, the total amount of energy is always found to be conserved. Hence the law of conservation of energy is believed to be universally valid.

PROBLEM SOLVING Conservation of Energy

1. Draw a picture.
2. Determine the system for which energy will be conserved: the object or objects and the forces acting.
3. Ask yourself what quantity you are looking for, and decide what are the initial (point 1) and final (point 2) locations.
4. If the body under investigation changes its height during the problem, then choose a $y = 0$ level for gravitational potential energy. This may be chosen for convenience; the lowest point in the problem is often a good choice.
5. If springs are involved, choose the unstretched spring position to be x (or y) $= 0$.
6. If no friction or other nonconservative forces act, then apply conservation of mechanical energy:

$$\mathrm{KE}_1 + \mathrm{PE}_1 = \mathrm{KE}_2 + \mathrm{PE}_2,$$

7. Solve for the unknown quantity.
8. If friction or other nonconservative forces are present, then an additional term (W_{NC}) will be needed:

$$W_{NC} = \Delta\mathrm{KE} + \Delta\mathrm{PE}.$$

To be sure which sign to give W_{NC}, or on which side of the equation to put it, use your intuition: is the total mechanical energy increased or decreased in the process?

Problem solving is not a process that can be done by following a set of rules. The box above is thus not a prescription, but is a *summary* of steps to help you get started in solving problems involving energy.

6–10 Power

Power defined

Average **power** is defined as the *rate at which work is done* (= work done divided by the time to do it), or as the *rate at which energy is transformed*. That is:

Average power

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}. \qquad (6\text{–}16)$$

The power of a horse refers to how much work it can do per unit time. The power rating of an engine refers to how much chemical or electrical energy can be transformed into mechanical energy per unit time. In SI units, power is measured in joules per second, and this unit is given a special name, the **watt** (W): 1 W = 1 J/s. We are most familiar with the watt for measuring the rate at which an electric lightbulb or heater changes electric energy into light or heat energy, but it is used for other types of energy transformations as well. In the British system, the unit of power is the foot-pound per second (ft·lb/s). For practical purposes, a larger unit is often used, the **horsepower**. One horsepower† (hp) is defined as 550 ft·lb/s, which equals 746 W.

Power units: the watt

It is very important to see the distinction between energy and power. To help make this distinction consider the following example. A person is limited in the work he or she can do, not only by the total energy required, but also by how fast this energy is transformed: that is, by power. For example, a person may be able to walk a long distance or climb many flights of stairs before having to stop because so much energy has been expended. On the other hand, a person who runs very quickly upstairs may fall exhausted after only a flight or two. He or she is limited in this case by power, the rate at which his or her body can transform chemical energy into mechanical energy.

Power and energy distinguished

† The unit was first chosen by James Watt (1736–1819), who needed a way to specify the power of his newly developed steam engines. He found by experiment that a good horse can work all day at an average rate of about 360 ft·lb/s. So as not to be accused of exaggeration in the sale of his steam engines, he multiplied this by $1\frac{1}{2}$ when he defined the hp.

EXAMPLE 6–16 **Stair-climbing power.** A 70-kg jogger runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. (*a*) Estimate the jogger's power output in watts and horsepower. (*b*) How much energy did this require?

SOLUTION (*a*) The work done is against gravity, and equals $W = mgy$. Then the average power output was

$$\overline{P} = \frac{W}{t} = \frac{mgy}{t} = \frac{(70\ \text{kg})(9.8\ \text{m/s}^2)(4.5\ \text{m})}{4.0\ \text{s}} = 770\ \text{W}.$$

Since there are 746 W in 1 hp, the jogger is doing work at a rate of just over 1 hp. It is worth noting that a human cannot do work at this rate for very long.

(*b*) The energy required is $E = \overline{P}t$ (Eq. 6–16). Since $\overline{P} = 770\ \text{W} = 770\ \text{J/s}$, then $E = (770\ \text{J/s})(4.0\ \text{s}) = 3100\ \text{J}$. [Note that the person had to transform more energy than this. The total energy transformed by a person or an engine always includes some thermal energy (recall how hot you get running up stairs).]

CONCEPTUAL EXAMPLE 6–17 **Energy of a powerful laser.** The Nova laser at Lawrence Livermore National Lab has ten beams, each of which has a power output greater than that of all the power plants in the United States. Where does this power come from?

RESPONSE Power companies do not really sell power. They sell energy. When you get your bill at the end of each month, you are charged for kilowatt-hours. A kilowatt is a unit of energy per time (power), so multiplying this by time will yield energy. Power stations are rated by power output, or how quickly they can deliver the energy. The Nova laser buys energy during the course of an afternoon and stores it. It then dumps the energy into the laser beams in a very short time interval, on the order of 1 ns (10^{-9} s). The *energy* delivered to the target is not so great, on the order of 10^5 J or what you get eating a doughnut. But it is delivered in so short a time that the power is extremely high, on the order of 10^{14} W.

Automobiles do work to overcome the force of friction (and air resistance), to climb hills, and to accelerate. A car is limited by the rate it can do work, which is why automobile engines are rated in horsepower. A car needs power most when it is climbing hills and when accelerating. In the next Example, we will calculate how much power is needed in these situations for a car of reasonable size. Even when a car travels on a level road at constant speed, it needs some power just to do work to overcome the retarding forces of internal friction and air resistance. These forces depend on the conditions and speed of the car, but are typically in the range 400 N to 1000 N.

It is often convenient to write the power in terms of the net force F applied to an object and its speed v. This is readily done since $\overline{P} = W/t$ and $W = Fd$ where d is the distance traveled. Then

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v} \qquad \text{(6–17)}$$

where $\overline{v} = d/t$ is the average speed of the object.

EXAMPLE 6–18 **Power needs of a car.** Calculate the power required of a 1400-kg car under the following circumstances: (*a*) the car climbs a 10° hill (a fairly steep hill) at a steady 80 km/h; and (*b*) the car accelerates along a level road from 90 to 110 km/h in 6.0 s to pass another car. Assume the retarding force on the car is $F_R = 700\,\text{N}$ throughout. See Fig. 6–28. (Be careful not to confuse $\mathbf{F}_R$, which is due to air resistance and friction that retards the motion, with the force $\mathbf{F}$ needed to accelerate the car, which is the frictional force exerted by the road on the tires—the reaction to the motor-driven tires pushing against the road.)

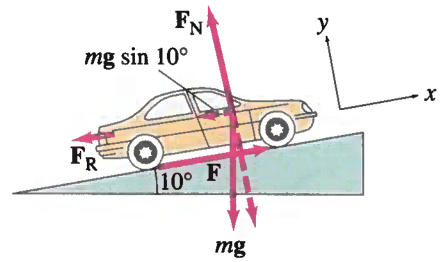


FIGURE 6–28 Example 6–18: calculation of power needed for a car (*a*) to climb a hill, (*b*) to pass another car.

SOLUTION (*a*) To move at a steady speed up the hill, the car must exert a force equal to the sum of the retarding force, 700 N, and the component of gravity parallel to the hill, $mg \sin 10° = (1400\,\text{kg})(9.80\,\text{m/s}^2)(0.174) = 2400\,\text{N}$. Since $\bar{v} = 80\,\text{km/h} = 22\,\text{m/s}$ and is parallel to $\mathbf{F}$, then (Eq. 6–17):

$$\overline{P} = F\bar{v}$$

$$= (2400\,\text{N} + 700\,\text{N})(22\,\text{m/s}) = 6.80 \times 10^4\,\text{W}$$

$$= 91\,\text{hp}.$$

(*b*) The car accelerates from 25.0 m/s to 30.6 m/s (90 to 110 km/h). Thus the car must exert a force that overcomes the 700 N retarding force plus that required to give it the acceleration $\bar{a}_x = (30.6\,\text{m/s} - 25.0\,\text{m/s})/6.0\,\text{s} = 0.93\,\text{m/s}^2$. We apply Newton's second law with x being the direction of motion:

$$ma_x = \Sigma F_x = F - F_R.$$

Then the force required, F, is

$$F = ma_x + F_R$$

$$= (1400\,\text{kg})(0.93\,\text{m/s}^2) + 700\,\text{N}$$

$$= 1300\,\text{N} + 700\,\text{N} = 2000\,\text{N}.$$

Since $\overline{P} = F\bar{v}$, the required power increases with speed and the motor must be able to provide a maximum power output of

$$\overline{P} = (2000\,\text{N})(30.6\,\text{m/s})$$

$$= 6.12 \times 10^4\,\text{W}$$

$$= 82\,\text{hp}.$$

Even taking into account the fact that only 60 to 80 percent of the engine's power output reaches the wheels, it is clear from these calculations that an engine of 100 to 150 hp is quite adequate from a practical point of view.

SUMMARY

Work is done on an object by a force when the object moves through a distance, d. If the direction of a constant force F makes an angle θ with the direction of motion, the work done by this force is

$$W = Fd\cos\theta.$$

Energy can be defined as the ability to do work. In SI units, work and energy are measured in **joules** (1 J = 1 N·m).

Kinetic energy (KE) is energy of motion. A body of mass m and speed v has translational kinetic energy

$$\text{KE} = \tfrac{1}{2}mv^2.$$

Potential energy (PE) is energy associated with forces that depend on the position or configuration of bodies. Gravitational potential energy is

$$\text{PE}_{\text{grav}} = mgy,$$

where y is the height of the object of mass m above an arbitrary reference point. Elastic potential energy is equal to $\frac{1}{2}kx^2$ for a stretched or compressed spring, where x is the displacement from the unstretched position. Other potential energies include chemical, electrical, and nuclear energy. The change in potential energy when an object changes position is equal to the external work needed to take the object from one position to the other.

The **work-energy principle** states that the net work done on a body (by the *net* force) equals the change in kinetic energy of that body:

$$W_{\text{net}} = \Delta\text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2.$$

The **law of conservation of energy** states that energy can be transformed from one type to another, but the total energy remains constant. It is valid even when friction is present, since the heat generated can be considered a form of energy. In the absence of friction and other nonconservative forces, the total mechanical energy is conserved:

$$\text{KE} + \text{PE} = \text{constant}.$$

When nonconservative forces act, then

$$W_{\text{NC}} = \Delta\text{KE} + \Delta\text{PE},$$

where W_{NC} is the work done by nonconservative forces.

Power is defined as the rate at which work is done, or the rate at which energy is transformed. The SI unit of power is the **watt** (1 W = 1 J/s).

QUESTIONS

1. In what ways is the word "work" as used in everyday language the same as defined in physics? In what ways is it different?
2. Can a centripetal force ever do work on an object? Explain.
3. Can the normal force on an object ever do work? Explain.
4. A woman swimming upstream is not moving with respect to the shore. Is she doing any work? If she stops swimming and merely floats, is work done on her?
5. Is the work done by kinetic friction forces always negative? [*Hint*: Consider what happens to the dishes when pulling a tablecloth from under your mom's best china.]
6. Why is it tiring to push hard against a solid wall even though no work is done?
7. You have two springs that are identical except that spring 1 is stiffer than spring 2 ($k_1 > k_2$). On which spring is more work done (*a*) if they are stretched using the same force, (*b*) if they are stretched the same distance?
8. Can kinetic energy ever be negative? Explain.
9. A hand exerts a constant horizontal force on a block that is free to slide on a frictionless surface, as shown below (Fig. 6–29). The block starts from rest at point A, and by the time it has traveled a distance d to point B it is traveling with speed v_B. When the block has traveled another distance d to point C, will its speed be greater than, less than, or equal to $2v_B$? Explain your reasoning.

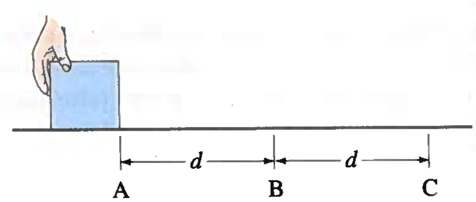


FIGURE 6–29 Question 9.

10. By approximately how much does your gravitational potential energy change when you jump as high as you can?

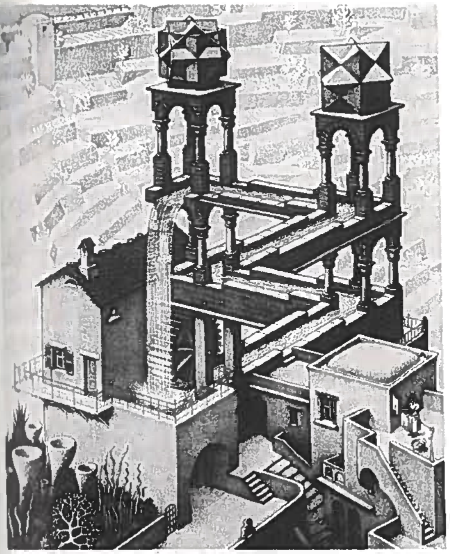


FIGURE 6–30 Question 11.

11. Describe precisely what is "wrong" physically in the famous Escher drawing shown in Fig. 6–30.

12. In Fig. 6–31, water balloons are tossed from the roof of a building, all with the same speed but with different launch angles. Which one has the highest speed on impact?

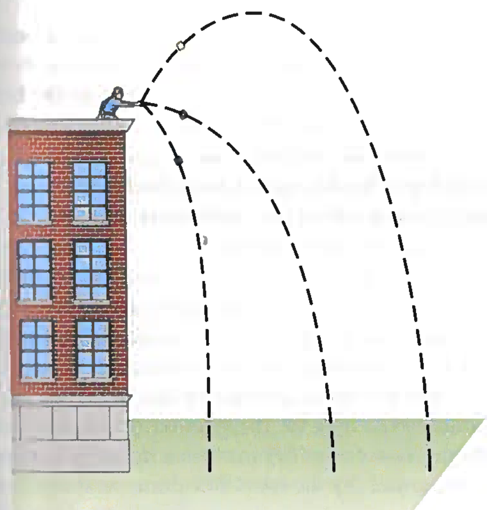


FIGURE 6–31 Question 12.

13. A pendulum is launched from a point that is a height h above its lowest point in 2 different ways (Fig. 6–32). During both launches, the bob is given an initial speed of 3.0 m/s. On the first launch, the initial velocity of the bob is directed upward along the trajectory, and on the second launch it is directed downward along the trajectory. Which launch will cause the pendulum to swing the largest angle from the equilibrium position?

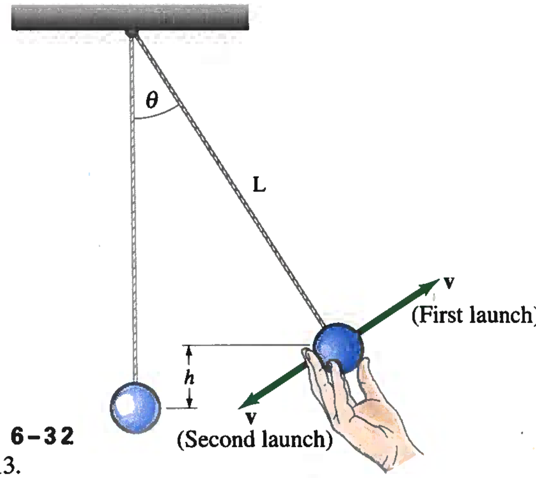


FIGURE 6–32 Question 13.

14. A coil spring of mass m rests upright on a table. If you compress the spring by pressing down with your hand and then release it, can the spring actually leave the table? Explain, using the law of conservation of energy.

15. In a well-known lecture demonstration, a bowling ball is hung from the ceiling by a steel wire (Fig. 6–33). The lecturer pulls the ball back and stands against the side wall of the lecture hall with the ball against his nose. To avoid injury the lecturer is supposed to release the ball, but not push it. Why?

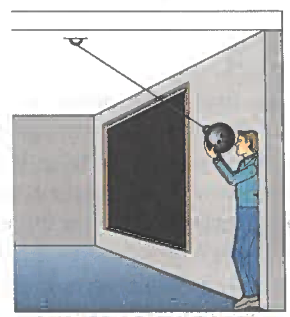


FIGURE 6–33 Question 15.

16. What happens to the gravitational potential energy when water at the top of a waterfall falls to the pool below?

17. Describe the energy transformations when a child hops around on a pogo stick.

18. Describe the energy transformations that take place when a skier starts skiing down a hill, but after a time is brought to rest by striking a snowdrift.

19. A hill has a height h. A child on a sled (total mass m) slides down starting from rest at the top. Does the velocity at the bottom depend on the angle of the hill if (*a*) it is icy and there is no friction, and (*b*) there is friction (deep snow)?

20. Seasoned hikers prefer to step over a fallen log in their path rather than stepping on top and jumping down on the other side. Explain.

21. (*a*) Where does the kinetic energy come from when a car accelerates uniformly starting from rest? (*b*) How is the increase in kinetic energy related to the friction force the road exerts on the tires?

22. Two identical arrows, one with twice the speed of the other, are fired into a bale of hay. Assuming the hay exerts a constant frictional force on the arrows, the faster arrow will penetrate how much farther than the slower arrow? Explain.

23. Analyze the motion of a simple swinging pendulum in terms of energy, (*a*) ignoring friction, and (*b*) taking it into account. Explain why a grandfather clock has to be wound up.

24. When a "superball" is dropped, can it rebound to a height greater than its original height?

25. Suppose you lift a suitcase from the floor to a table. Does the work you do on the suitcase depend on (*a*) whether you lift it straight up or along a more complicated path, (*b*) the time it takes, (*c*) the height of the table, and (*d*) the weight of the suitcase?

26. Repeat the previous question for the *power* needed rather than the work.

27. Why is it easier to climb a mountain via a zigzag trail rather than to climb straight up?

28. Recall from Chapter 4, Example 4–12, that you c[an] use a pulley and ropes to decrease the force needed [to] raise a heavy load (see Fig. 6–34). But for every met[er] the load is raised, how much rope must be pulled u[p]? Account for this, using energy concepts.

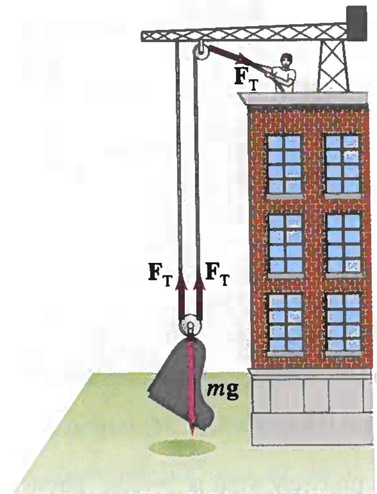


FIGURE 6–34 Question 28.

PROBLEMS

SECTION 6–1

1. (I) A 75.0-kg firefighter climbs a flight of stairs 10.0 m high. How much work is required?

2. (I) A 900-N crate rests on the floor. How much work is required to move it at constant speed (*a*) 6.0 m along the floor against a friction force of 180 N, and (*b*) 6.0 m vertically?

3. (I) How much work did the movers do (horizontally) pushing a 150-kg crate 12.3 m across a rough floor without acceleration, if the effective coefficient of friction was 0.70?

4. (I) A car does 7.0×10^4 J of work in traveling 2.8 km at constant speed. What was the average retarding force (from all sources) acting on the car?

5. (I) How high will a 0.325-kg rock go if thrown straight up by someone who does 115 J of work on it? Neglect air resistance.

6. (I) A hammerhead with a mass of 2.0 kg is allowed to fall onto a nail from a height of 0.40 m. What is the maximum amount of work it could do on the nail? Why do people not just "let it fall" but add their own force to the hammer as it falls?

7. (II) What is the minimum work needed to push a 1000-kg car 300 m up a 17.5° incline? (*a*) Ignore friction. (*b*) Assume the effective coefficient of friction is 0.25.

8. (II) A grocery cart with mass of 18 kg is pushed [at] constant speed along an aisle by a force $F = 12$ [N]. The applied force acts at a 20° angle to the horizo[n]tal. Find the work done by each of the forces on t[he] cart if the aisle is 15 m long.

9. (II) Eight books, each 4.6 cm thick with mass 1.8 k[g], lie flat on a table. How much work is required to st[ack] them one on top of another?

10. (II) A 280-kg piano slides 4.3 m down a 30° incli[ne] and is kept from accelerating by a man who is pu[sh]ing back on it *parallel to the incline* (Fig. 6–35). Th[e] effective coefficient of kinetic friction is 0.40. Calc[u]late: (*a*) the force exerted by the man, (*b*) the wo[rk] done by the man on the piano, (*c*) the work done b[y] the friction force, (*d*) the work done by the force o[f] gravity, and (*e*) the net work done on the piano.

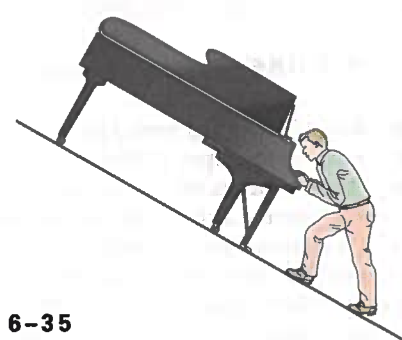
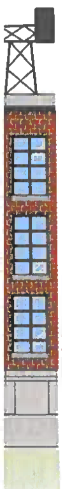


FIGURE 6–35
Problem 10.

-12, that you can
e force needed to
ıt for every meter
ust be pulled up?
pts.



ion 28.

18 kg is pushed at
a force $F = 12$ N
gle to the horizon
the forces on the

with mass 1.8 kg
is required to stack

lown a 30° incline
man who is push
e (Fig. 6–35). The
tion is 0.40. Calcu
man, (b) the work
the work done by
ne by the force of
on the piano.

12. (II) (*a*) Find the force required to give a helicopter of mass M an acceleration of $0.10\,g$ upward. (*b*) Find the work done by this force as the helicopter moves a distance h upward.

SECTION 6–2

12. (II) In Fig. 6–6a, assume the distance axis is linear and that $d_A = 10.0$ m and $d_B = 35.0$ m. Estimate the work done by this force in moving a 2.80-kg object from d_A to d_B.

13. (II) The x component of the force on an object varies as shown in Fig. 6–36. Determine the work done by this force to move the object (*a*) from $x = 0.0$ to $x = 10.0$ m, and (*b*) from $x = 0.0$ to $x = 15.0$ m.

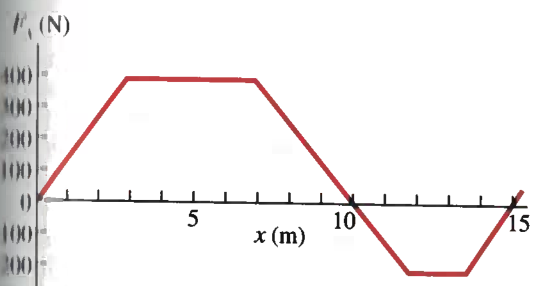


FIGURE 6–36 Problem 13.

14. (II) A spring has $k = 88$ N/m. Use a graph to determine the work needed to stretch it from $x = 3.8$ cm to $x = 5.8$ cm, where x is the displacement from its unstretched length.

15. (III) The x component of force exerted on a particle increases linearly from zero at $x = 0$, to 24.0 N at $x = 3.0$ m. It remains constant at 24.0 N from $x = 3.0$ m to $x = 8.0$ m, and then decreases linearly to zero at $x = 11.0$ m. Determine the work done to move the particle from $x = 0$ to $x = 11.0$ m graphically by determining the area under the F_x vs. x graph.

16. (III) A 1300-kg space vehicle falls from a vertical height of 2500 km above the Earth's surface. Use Eq. 5–4 to estimate how much work is done by the force of gravity in bringing the vehicle to the Earth's surface. (First construct an F vs. r graph, where r is the distance from the Earth's center; then determine the work graphically.)

SECTION 6–3

17. (I) At room temperature, an oxygen molecule, with mass of 5.31×10^{-26} kg, typically has a KE of about 6.21×10^{-21} J. How fast is it moving?

18. (I) (*a*) If the KE of an arrow is doubled, by what factor has its speed increased? (*b*) If its speed is doubled, by what factor does its KE increase?

19. (I) How much work is required to stop an electron ($m = 9.11 \times 10^{-31}$ kg) which is moving with a speed of 1.90×10^6 m/s?

20. (I) How much work must be done to stop a 1000-kg car traveling at 110 km/h?

21. (II) An automobile is traveling along a highway at 90 km/h. If it travels instead at 100 km/h, what is the percent increase in the automobile's kinetic energy?

22. (II) An 80-g arrow is fired from a bow whose string exerts an average force of 95 N on the arrow over a distance of 80 cm. What is the speed of the arrow as it leaves the bow?

23. (II) A baseball ($m = 140$ g) traveling 35 m/s moves a fielder's glove backward 25 cm when the ball is caught. What was the average force exerted by the ball on the glove?

24. (II) If the speed of a car is increased by 50%, by what factor will its minimum braking distance be increased, assuming all else is the same? Ignore the driver's reaction time.

25. (II) At an accident scene on a level road, investigators measure a car's skid mark to be 88 m long. It was a rainy day and the coefficient of friction was estimated to be 0.42. Use these data to determine the speed of the car when the driver slammed on (and locked) the brakes. (Why does the car's mass not matter?)

26. (II) A softball having a mass of 0.25 kg is pitched at 95 km/h. By the time it reaches the plate, it may have slowed by 10 percent. Neglecting gravity, estimate the average force of air resistance during a pitch, if the distance between the plate and the pitcher is about 15 m.

27. (III) One car has twice the mass of a second car, but only half as much kinetic energy. When both cars increase their speed by 5.0 m/s, they then have the same kinetic energy. What were the original speeds of the two cars?

28. (III) A 220-kg load is lifted 21.0 m vertically with an acceleration $a = 0.150\,g$ by a single cable. Determine (*a*) the tension in the cable, (*b*) the net work done on the load, (*c*) the work done by the cable on the load, (*d*) the work done by gravity on the load, and (*e*) the final speed of the load assuming it started from rest.

SECTIONS 6–4 AND 6–5

29. (I) A spring has a spring constant, k, of 440 N/m. How much must this spring be stretched to store 25 J of potential energy?

30. (I) A 6.0-kg monkey swings from one branch to another 1.2 m higher. What is the change in potential energy?

31. (I) By how much does the gravitational potential energy of a 64-kg pole vaulter change if his center of mass rises about 4.0 m during the jump?

32. (II) In starting an exercise, a 1.60-m tall person lifts a 2.10-kg book on the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to (*a*) the ground, and (*b*) the top of the person's head? (*c*) How is the work done by the person related to the answers in parts (*a*) and (*b*)?

33. (II) A 55-kg hiker starts at an elevation of 1600 m and climbs to the top of a 3100-m peak. (*a*) What is the hiker's change in potential energy? (*b*) What is the minimum work required of the hiker? (*c*) Can the actual work done be more than this? Explain why.

34. (II) (*a*) A spring of spring constant k is initially compressed a distance x_0 from its unstretched length. What is the change in potential energy if it is then compressed to an amount x from its unstretched length? (*b*) Suppose the spring is then *stretched* a distance x_0 from the unstretched length. What is the change in potential energy as compared to when it is compressed by an amount x_0?

SECTIONS 6–6 AND 6–7

35. (I) Jane, looking for Tarzan, is running at top speed (5.6 m/s) and grabs a vine hanging vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine (or rope) affect your answer?

36. (I) A novice skier, starting from rest, slides down a frictionless 35.0° incline whose vertical height is 125 m. How fast is she going when she reaches the bottom?

37. (I) A sled is initially given a shove up a frictionless 25.0° incline. It reaches a maximum vertical height 1.35 m higher than where it started. What was its initial speed?

38. (II) In the high jump, the kinetic energy of an athlete is transformed into gravitational potential energy without the aid of a pole. With what minimum speed must the athlete leave the ground in order to lift his center of mass 2.10 m and cross the bar with a speed of 0.70 m/s?

39. (II) A 75-kg trampoline artist jumps vertically upward from the top of a platform with a speed of 5.0 m/s. (*a*) How fast is he going as he lands on the trampoline, 3.0 m below (Fig. 6–37)? (*b*) If the trampoline behaves like a spring of spring constant 5.2×10^4 N/m, how far does he depress it?

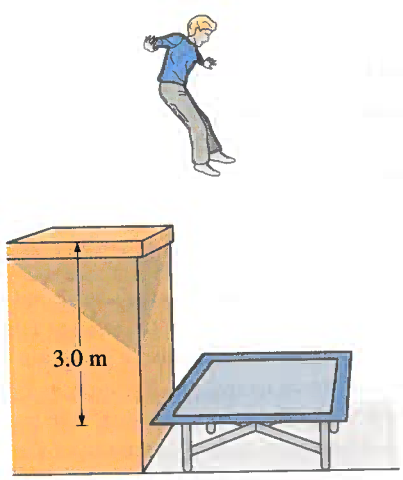


FIGURE 6–37 Problem 39.

40. (II) A roller coaster, shown in Fig. 6–38, is pulled u[p] to point A where it and its screaming occupants a[re] released from rest. Assuming no friction, calcul[ate] the speed at points B, C, D.

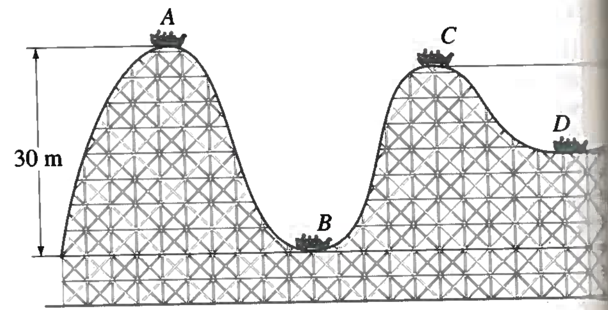


FIGURE 6–38 Problems 40 and 53.

41. (II) A projectile is fired at an upward angle of 45.0° from the top of a 265-m cliff with a speed of 185 m/s. What will be its speed when it strikes the ground below? (Use conservation of energy.)

42. (II) A 60-kg bungee jumper jumps from a bridg[e]. She is tied to a 12-m-long bungee cord and falls a total of 31 m. (*a*) Calculate the spring constant k of the bungee cord. (*b*) Calculate the maximum accel[eration] experienced by the jumper.

43. (II) A vertical spring (ignore its mass), whose sprin[g] constant is 900 N/m, is attached to a table and is compressed 0.150 m. (*a*) What speed can it give to a 0.300-kg ball when released? (*b*) How high above it[s] original position (spring compressed) will the ba[ll] fly?

44. (II) A small mass m slides without friction along th[e] looped apparatus shown in Fig. 6–39. If the object is to remain on the track, even at the top of the circle (whose radius is r), from what minimum height h must it be released?

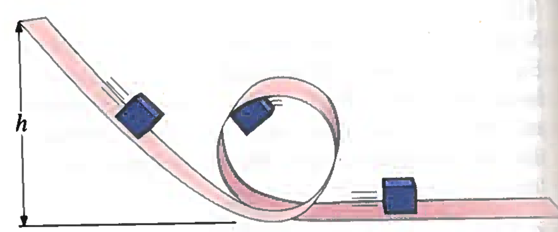


FIGURE 6–39 Problems 44 and 79.

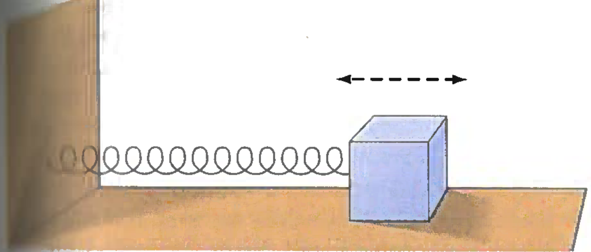


FIGURE 6–40 Problems 45, 55, 56, and 78.

45. (II) A mass m is attached to the end of a spring (constant k), Fig. 6–40. The mass is given an initial displacement x_0, after which it oscillates back and forth. Write a formula for the total mechanical energy (ignore friction and mass of the spring) in terms of position x and speed v.

46. (III) An engineer is designing a spring to be placed at the bottom of an elevator shaft. If the elevator cable should happen to break at a height h above the top of the spring, calculate the value that the spring constant k should have so that passengers undergo an acceleration of no more than $5.0\,g$ when brought to rest. Let M be the total mass of the elevator and passengers.

47. (III) What should be the spring constant k of a spring designed to bring a 1200-kg car to rest from a speed of 100 km/h so that the occupants undergo a maximum acceleration of $5.0\,g$?

48. (III) A cyclist intends to cycle up a 7.50° hill whose vertical height is 120 m. Assuming the mass of bicycle plus person is 75.0 kg, (*a*) calculate how much work must be done against gravity. (*b*) If each complete revolution of the pedals moves the bike 5.10 m along its path, calculate the average force that must be exerted on the pedals tangent to their circular path. Neglect work done by friction and other losses. The pedals turn in a circle of diameter 36.0 cm.

SECTIONS 6–8 AND 6–9

49. (I) Two railroad cars, each of mass 6500 kg and traveling 95 km/h, collide head-on and come to rest. How much thermal energy is produced in this collision?

50. (II) A 17-kg child descends a slide 3.5 m high and reaches the bottom with a speed of 2.5 m/s. How much thermal energy due to friction was generated in this process?

51. (II) A ski starts from rest and slides down a 20° incline 100 m long. (*a*) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (*b*) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.

52. (II) A 90-kg crate, starting from rest, is pulled across a floor with a constant horizontal force of 350 N. For the first 15 m the floor is frictionless, and for the next 15 m the coefficient of friction is 0.30. What is the final speed of the crate?

53. (II) Suppose the roller coaster in Fig. 6–38 passes point A with a speed of 1.70 m/s. If the average force of friction is equal to one fifth of its weight, with what speed will it reach point B? The distance traveled is 45.0 m.

54. (II) A skier traveling 12.0 m/s reaches the foot of a steady upward 18° incline and glides 12.2 m up along this slope before coming to rest. What was the average coefficient of friction?

55. (III) A 0.520-kg wood block is firmly attached to a very light horizontal spring ($k = 180\,\mathrm{N/m}$) as shown in Fig. 6–40. It is noted that the block-spring system, when compressed 5.0 cm and released, stretches out 2.3 cm beyond the equilibrium position before stopping and turning back. What is the coefficient of kinetic friction between the block and the table?

56. (III) A 180-g wood block is firmly attached to a very light horizontal spring, Fig. 6–40. The block can slide along a table where the coefficient of friction is 0.30. A force of 20 N compresses the spring 18 cm. If the spring is released from this position, how far beyond its equilibrium position will it stretch on its first swing?

57. (III) In early test flights for the space shuttle using a "glider" (mass of 1000 kg including pilot), it was noted that after a horizontal launch at 500 km/h at a height of 3500 m, the glider eventually landed at a speed of 200 km/h. (*a*) What would its landing speed have been in the absence of air resistance? (*b*) What was the average force of air resistance exerted on it if it came in at a constant glide of 10° to the Earth?

SECTION 6–10

58. (I) How long will it take a 1750-W motor to lift a 285-kg piano to a sixth-story window 16.0 m above?

59. (I) If a car generates 18 hp when traveling at a steady 90 km/h, what must be the average force exerted on the car due to friction and air resistance?

60. (I) (*a*) Show that a British horsepower (550 ft·lb/s) is equal to 746 W. (*b*) What is the horsepower rating of a 100-W lightbulb?

61. (II) Electric energy units are often expressed in the form of "kilowatt-hours." (*a*) Show that one kilowatt-hour (kWh) is equal to 3.6×10^6 J. (*b*) If the typical family of four in the United States uses electric energy at an average rate of 500 W, how many kWh would their electric bill be for one month, and (*c*) how many joules would this be? (*d*) At a cost of \$0.12 per kWh, what would their monthly bill be in dollars? Does the monthly bill depend on the *rate* at which they use the electric energy?

62. (II) A driver notices that her 1000-kg car slows down from 90 km/h to 70 km/h in about 6.0 s on the level when it is in neutral. Approximately what power (watts and hp) is needed to keep the car traveling at a constant 80 km/h?

63. (II) How much work can a 3.0-hp motor do in 1.0 h?

64. (II) A shot-putter accelerates a 7.3-kg shot from rest to 14 m/s. If this motion takes 2.0 s, what average power was developed?

65. (II) A pump is to lift 8.00 kg of water per minute through a height of 3.50 m. What output rating (watts) should the pump motor have?

66. (II) During a workout, the football players at State U ran up the stadium stairs in 61 s. The stairs are 140 m long and inclined at an angle of 30°. If a typical player has a mass of 105 kg, estimate the average power output on the way up. Ignore friction and air resistance.

67. (II) How fast must a cyclist climb a 6.0° hill to maintain a power output of 0.25 hp? Neglect work done by friction and assume the mass of cyclist plus bicycle is 70 kg.

68. (II) A 1000-kg car has a maximum power output of 120 hp. How steep a hill can it climb at a constant speed of 70 km/h if the frictional forces add up to 600 N?

69. (II) Squaw Valley ski area in California claims that its lifts can move 47,000 people per hour. If the average lift carries people about 200 m (vertically) higher, estimate the maximum total power needed.

70. (III) A bicyclist coasts down a 7.0° hill at a steady speed of 5.0 m/s. Assuming a total mass of 75 kg (bicycle plus rider), what must be the cyclist's power output to climb the same hill at the same speed?

GENERAL PROBLEMS

71. A paratrooper fell 370 m after jumping from an aircraft without his parachute opening. He landed in a snowbank, creating a crater 1.1 m deep, but survived with only minor injuries. Assuming the paratrooper's mass was 80 kg and his terminal velocity was 30 m/s, estimate: (*a*) the work done by the snow in bringing him to rest; (*b*) the average force exerted on him by the snow to stop him; and (*c*) the work done on him by air resistance as he fell.

72. Designers of today's cars have built "5 mi/h (8 km/h) bumpers" that are designed to elastically compress and rebound without any physical damage at speeds below 8 km/h. If the material of the bumpers permanently deforms after a compression of 1.5 cm, but remains like an elastic spring up to that point, what must the effective spring constant of the bumper material be, assuming the car has a mass of 1400 kg and is tested by ramming into a solid wall?

73. In a certain library the first shelf is 10.0 cm off the ground, and the remaining 4 shelves are each spaced 30.0 cm above the previous one. If the average book has a mass of 1.5 kg with a height of 20 cm, and an average shelf holds 25 books, how much work is required to fill this bookshelf from scratch, assuming the books are all laying flat on the floor to start?

74. In a film of Jesse Owens's famous long jump in the 1936 Olympics, it is observed that his center of mass rose 1.1 m from launch point to the top of the arc. What minimum speed did he need at launch if he was also noted to be traveling at 6.5 m/s at the top of the arc?

75. A 0.20-kg pinecone falls from a branch 18 m above the ground. (*a*) With what speed would it hit the ground if air resistance could be ignored? (*b*) If it actually hits the ground with a speed of 10.0 m/s, what was the average force of air resistance exerted on it?

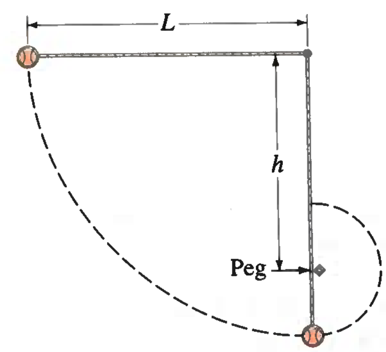


FIGURE 6–41 Problem 76.

76. A ball is attached to a horizontal cord of length L whose other end is fixed, Fig. 6–41. (*a*) If the ball is released, what will be its speed at the lowest point of its path? (*b*) A peg is located a distance h directly below the point of attachment of the cord. If $h = 0.80\,L$, what will be the speed of the ball when it reaches the top of its circular path about the peg?

77. A 65-kg hiker climbs to the top of a 3900-m-high mountain. The climb is made in 5.0 h starting at an elevation of 2200 m. Calculate (*a*) the work done against gravity, (*b*) the average power output in watts and in horsepower, and (*c*) assuming the body is 15% efficient, what rate of energy input was required.

78. A mass m is attached to the end of a spring (constant k) as shown in Fig. 6–40. The mass is given an initial displacement x_0 from equilibrium, and an initial speed v_0. Ignoring friction and the mass of the spring, use energy methods to find (*a*) its maximum speed, and (*b*) its maximum stretch from equilibrium in terms of the given quantities.

79. The small mass m sliding without friction along the looped apparatus shown in Fig. 6–39 is to remain on the track at all times, even at the very top of the loop of radius r. (*a*) Calculate, in terms of the given quantities, the minimum release height h (as in Problem 44). Next, if the actual release height is $2h$, calculate (*b*) the normal force exerted by the track at the bottom of the loop, (*c*) the normal force exerted by the track at the top of the loop, and (*d*) the normal force exerted by the track after the block exits the loop onto the flat section.

80. An elevator cable breaks when a 900-kg elevator is 30 m above a huge spring ($k = 4.0 \times 10^5\,\mathrm{N/m}$) at the bottom of the shaft. Calculate (*a*) the work done by gravity on the elevator before it hits the spring, (*b*) the speed of the elevator just before striking the spring, and (*c*) the amount the spring compresses (note that work is done by both the spring and gravity in this part).

81. Water flows over a dam at the rate of 550 kg/s and falls vertically 80 m before striking the turbine blades. Calculate: (*a*) the speed of the water just before striking the turbine blades (neglect air resistance), and (*b*) the rate at which mechanical energy is transferred to the turbine blades, assuming 60% efficiency.

82. A bicyclist of mass 75 kg (including the bicycle) can coast down a 4.0° hill at a steady speed of 10 km/h. Pumping hard, the cyclist can descend the hill at a speed of 30 km/h. Using the same power, at what speed can the cyclist climb the same hill? Assume the force of friction is proportional to the square of the speed v; that is, $F_{fr} = bv^2$, where b is a constant.

83. Show that on a roller coaster with a circular vertical loop (Fig. 6–42), the difference in your apparent weight at the top of the loop and the bottom of the loop is 6 g's—that is, six times your weight. Ignore friction. Show also that as long as your speed is above the minimum needed, this answer doesn't depend on the size of the loop or how fast you go through it.

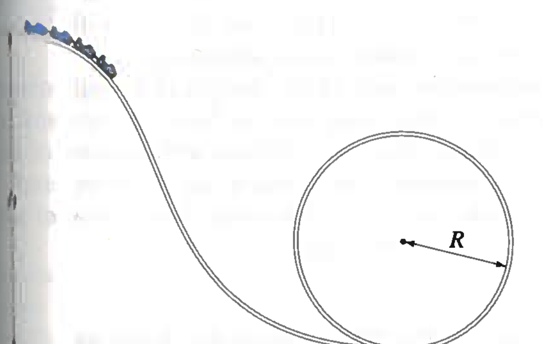


FIGURE 6–42 Problem 83.

84. If you stand on a bathroom scale, the spring inside the scale compresses 0.50 mm, and it tells you your weight is 700 N. Now if you jump on the scale from a height of 1.0 m, what does the scale read at its peak?

85. A 75-kg student runs at 5.0 m/s, grabs a rope, and swings out over a lake (Fig. 6–43). He releases the rope when his velocity is zero. (*a*) What is the angle θ when he releases the rope? (*b*) What is the tension in the rope just before he releases it? (*c*) What is the maximum tension in the rope?

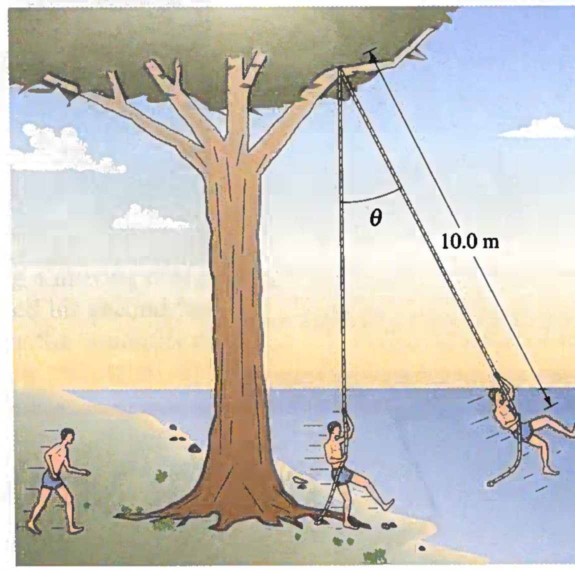


FIGURE 6–43 Problem 85.

86. In the rope climb, a 70-kg athlete climbs a vertical distance of 5.0 m in 9.0 s. What minimum power output was used to accomplish this feat?