The planet Jupiter showing two of its moons, Io (visible in front of the planet) and Europa at far right. The moons, held by the force of gravity, revolve around Jupiter. Galileo was the first to observe four of Jupiter's moons, a momentous discovery that he used to argue in favor of the Copernican system.

CHAPTER

5 CIRCULAR MOTION; GRAVITATION

An object moves in a straight line if the net force on it acts in the direction of motion, or is zero. If the net force acts at an angle to the direction of motion at any moment, then the object moves in a curved path. An example of the latter is projectile motion, which we have already discussed in Chapter 3. Another important case is that of an object moving in a circle, such as a ball at the end of a string revolving around one's head, or the nearly circular motion of the Moon about the Earth.

In this chapter, we study the circular motion of an object, and how Newton's laws of motion apply to it. We will also discuss how Newton conceived of another great law by applying the concepts of circular motion to the motion of the Moon and the planets. This is the law of universal gravitation, which was the capstone of Newton's analysis of the physical world.

5–1 Kinematics of Uniform Circular Motion

An object that moves in a circle at constant speed v is said to experience **uniform circular motion**. The *magnitude* of the velocity remains constant in this case, but the *direction* of the velocity is continuously changing as the object moves around the circle (Fig. 5–1). Since acceleration is defined as the rate of change of velocity, a change in direction of velocity constitutes an acceleration just as does a change in magnitude of velocity. Thus, an object revolving in a circle is continuously accelerating, even when the speed remains constant ($v_1 = v_2 = v$). We now investigate this acceleration quantitatively.

Acceleration is defined as

$$\mathbf{a} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t},$$

where $\Delta \mathbf{v}$ is the change in velocity during the short time interval Δt. We will eventually consider the situation when Δt approaches zero and thus obtain the instantaneous acceleration. But for purposes of making a clear drawing, Fig. 5–2, we consider a nonzero time interval. During the time Δt, the particle in Fig. 5–2a moves from point A to point B, covering a distance Δl *along the arc* which subtends an angle $\Delta\theta$. The change in the velocity vector is $\mathbf{v}_2 - \mathbf{v}_1 = \Delta \mathbf{v}$, and is shown in Fig. 5–2b. If we let Δt be very small (approaching zero), then Δl and $\Delta\theta$ are also very small and $\mathbf{v}_2$ will be almost parallel to $\mathbf{v}_1$ and $\Delta \mathbf{v}$ will be essentially perpendicular to them. Thus $\Delta \mathbf{v}$ points toward the center of the circle. Since $\mathbf{a}$, by its definition above, is in the same direction as $\Delta \mathbf{v}$, it too must point toward the center of the circle. Therefore, this acceleration is called **centripetal acceleration** ("center-seeking" acceleration) or **radial acceleration** (since it is directed along the radius, toward the center of the circle), and we denote it by $\mathbf{a}_R$.

We next determine the magnitude of the centripetal (radial) acceleration, a_R. Because CA is perpendicular to $\mathbf{v}_1$, and CB is perpendicular to $\mathbf{v}_2$, it follows that the angle $\Delta\theta$, defined as the angle between CA and CB, is also the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$. Hence the vectors $\mathbf{v}_2$, $\mathbf{v}_1$, and $\Delta \mathbf{v}$ in Fig. 5–2b form a triangle that is geometrically similar† to triangle ABC in Fig. 5–2a. Taking $\Delta\theta$ small (letting Δt be very small), we can write

$$\frac{\Delta v}{v} \approx \frac{\Delta l}{r},$$

where we have set $v = v_1 = v_2$ because the magnitude of the velocity is assumed not to change. This is an exact equality when Δt approaches zero, for then the arc length Δl equals the cord length AB. Since we want to find the instantaneous acceleration, for which Δt approaches zero, we write the above expression as an equality and solve for Δv:

$$\Delta v = \frac{v}{r}\,\Delta l.$$

To get the centripetal acceleration, a_R, we divide Δv by Δt:

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta l}{\Delta t};$$

and since $\Delta l/\Delta t$ is the linear speed, v, of the object,

$$a_R = \frac{v^2}{r}. \qquad (5\text{–}1)$$

CENTRIPETAL ACCELERATION

†Appendix A contains a review of geometry.

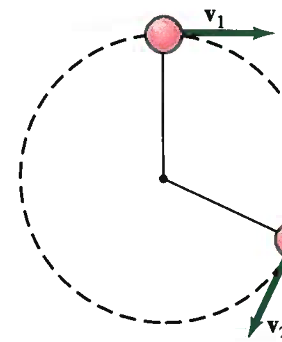
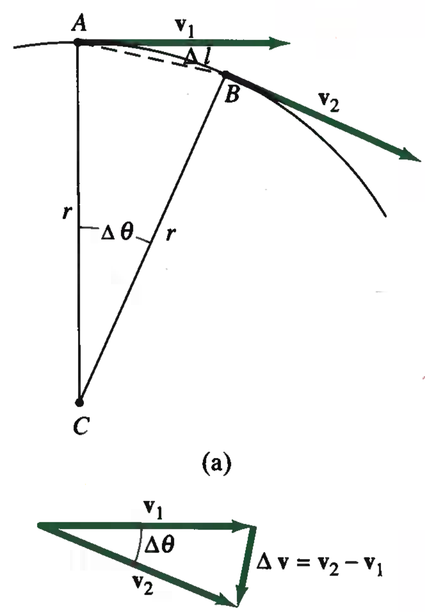


FIGURE 5–1 A small object moving in a circle, showing how the velocity changes. Note that at each point, the instantaneous velocity is in a direction tangent to the circular path.

FIGURE 5–2 Determining the change in velocity, $\Delta\mathbf{v}$, for a particle moving in a circle. The length Δl is the distance along the arc, from A to B.



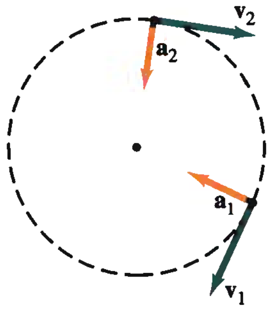


FIGURE 5–3 For uniform circular motion, **a** is always perpendicular to **v**.

To summarize, *an object moving in a circle of radius r with constant speed v has an acceleration whose direction is toward the center of the circle and whose magnitude is $a_R = v^2/r$.* It is not surprising that this acceleration depends on v and r. For the greater the speed v, the faster the velocity changes direction; and the larger the radius, the less rapidly the velocity changes direction.

The acceleration vector points toward the center of the circle. But the velocity vector always points in the direction of motion, which is tangential to the circle. Thus the velocity and acceleration vectors are perpendicular to each other at every point in the path for uniform circular motion (see Fig. 5–3). This is another example that illustrates the error in thinking that acceleration and velocity are always in the same direction. For an object falling vertically, **a** and **v** are indeed parallel. But in circular motion, **a** and **v** are not parallel—nor are they in projectile motion (Section 3–5), where the acceleration **a** = **g** is always downward but the velocity vector can have various directions (Figs. 3–18 and 3–20).

Period and frequency

Circular motion is often described in terms of the **frequency** f as so many revolutions per second. The **period** T of an object revolving in a circle is the time required for one complete revolution. Period and frequency are related by

$$T = \frac{1}{f}. \tag{5–2}$$

For example, if an object revolves at a frequency of 3 rev/s, then each revolution takes $\frac{1}{3}$ s. For an object revolving in a circle at constant speed v, we can write

$$v = \frac{2\pi r}{T}$$

since in one revolution the object travels one circumference ($=2\pi r$).

EXAMPLE 5–1 **Acceleration of a revolving ball.** A 150-g ball at the end of a string is revolving uniformly in a horizontal circle of radius 0.600 m, as in Fig. 5–1. The ball makes 2.00 revolutions in a second. What is its centripetal acceleration?

SOLUTION The centripetal acceleration is $a_R = v^2/r$. First, we determine the speed of the ball v. If the ball makes two complete revolutions per second, then the ball travels in a complete circle in 0.500 s, which is its period T. The distance traveled in this time is the circumference of the circle, $2\pi r$, where r is the radius of the circle. Therefore, the ball has speed

$$v = \frac{2\pi r}{T} = \frac{2(3.14)(0.600\text{ m})}{(0.500\text{ s})} = 7.54\text{ m/s}.$$

The centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(7.54\text{ m/s})^2}{(0.600\text{ m})} = 94.8\text{ m/s}^2.$$

EXAMPLE 5–2 **Moon's centripetal acceleration.** The Moon's nearly circular orbit about the Earth has a radius of about 384,000 km and a period T of 27.3 days. Determine the acceleration of the Moon toward the Earth.

SOLUTION In orbit around the Earth, the Moon travels a distance $2\pi r$, where $r = 3.84 \times 10^8$ m is the radius of its circular path. The speed of the Moon in its orbit about the Earth is $v = 2\pi r/T$. The period T in seconds is $T = (27.3\,\text{d})(24.0\,\text{h/d})(3600\,\text{s/h}) = 2.36 \times 10^6\,\text{s}$. Therefore,

$$a_R = \frac{v^2}{r} = \frac{(2\pi r)^2}{T^2 r} = \frac{[2(3.14)(3.84 \times 10^8\,\text{m})]^2}{(2.36 \times 10^6\,\text{s})^2(3.84 \times 10^8\,\text{m})}$$
$$= 0.00272\,\text{m/s}^2 = 2.72 \times 10^{-3}\,\text{m/s}^2.$$

We can write this in terms of $g = 9.80\,\text{m/s}^2$ (the acceleration of gravity at the Earth's surface) as†

$$a = 2.72 \times 10^{-3}\,\text{m/s}^2 \left(\frac{g}{9.80\,\text{m/s}^2}\right)$$
$$= 2.78 \times 10^{-4}\,g.$$

5–2 Dynamics of Uniform Circular Motion

According to Newton's second law ($\Sigma \mathbf{F} = m\mathbf{a}$), an object that is accelerating must have a net force acting on it. An object moving in a circle, such as a ball on the end of a string, must therefore have a force applied to it to keep it moving in that circle. That is, a net force is necessary to give it centripetal acceleration. The magnitude of the required force can be calculated using Newton's second law for the radial component, $\Sigma F_R = ma_R$, where a_R is the centripetal acceleration, $a_R = v^2/r$, and ΣF_R is the total (or net) force in the radial direction:

Force is needed to provide centripetal acceleration

$$\Sigma F_R = ma_R = m\frac{v^2}{r}. \qquad \text{[circular motion]} \qquad (5\text{–}3)$$

Since a_R is directed toward the center of the circle at any moment, the *net force too must be directed toward the center of the circle*. A net force is clearly necessary because otherwise, if no net force were exerted on the object, it would not move in a circle but in a straight line, as Newton's first law tells us. To pull an object out of its "natural" straight-line path, a net force to the side is necessary. For uniform circular motion, this sideways force must act toward the circle's center (see Fig. 5–4). The direction of the net force is thus continually changing so that it is always directed toward the center of the circle. This force is sometimes called a centripetal ("aiming toward the center") force. But be aware that "centripetal force" does not indicate some new kind of force. The term merely describes the direction of the net force: that the net force is directed toward the circle's center. The force *must be applied by other objects*. For example, when a

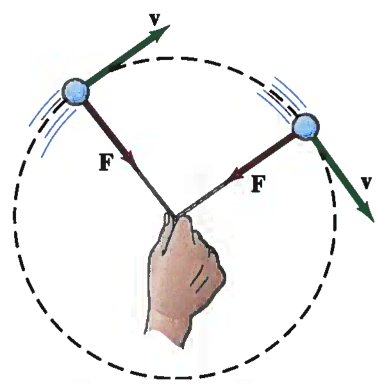


FIGURE 5–4 A force is required to keep an object moving in a circle. If the speed is constant, the force is directed toward the center of the circle.

Careful: Centripetal force is not a new kind of force

†Note: this acceleration $a = 2.78 \times 10^{-4}\,g$ is *not* the acceleration of gravity for objects at the Moon's surface due to the Moon's gravity. Rather it is the acceleration due to the *Earth's* gravity for any object (such as the Moon) that is 384,000 km from the Earth.

person swings a ball in a circle on the end of a string, the person pulls on the string and the string exerts the force on the ball.

There is no centrifugal force

There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: *there is no outward force*. Consider, for example, a person swinging a ball on the end of a string around her head (Fig. 5–5). If you have ever done this yourself, you know that you feel a force pulling outward on your hand. The misconception arises when this pull is interpreted as an outward "centrifugal" force pulling on the ball that is transmitted along the string to your hand. This is not what is happening at all. To keep the ball moving in a circle, you pull *inwardly* on the string, which in turn exerts the force on the ball. The ball exerts an equal and opposite force (Newton's third law), and *this* is the force your hand feels (see Fig. 5–5). The force *on the ball* is the one exerted *inwardly* on it by the string. To see even more convincing evidence that a "centrifugal force" does not act on the ball, consider what happens when you let go of the string. If a centrifugal force were acting, the ball would fly outward, as shown in Fig. 5–6a. But it doesn't; the ball flies off tangentially (Fig. 5–6b), in the direction of the velocity it had at the moment it was released, because the inward force no longer acts. Try it and see!

FIGURE 5–5 Swinging a ball on the end of a string.

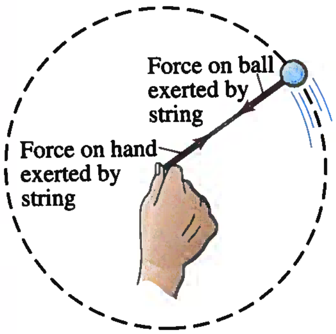
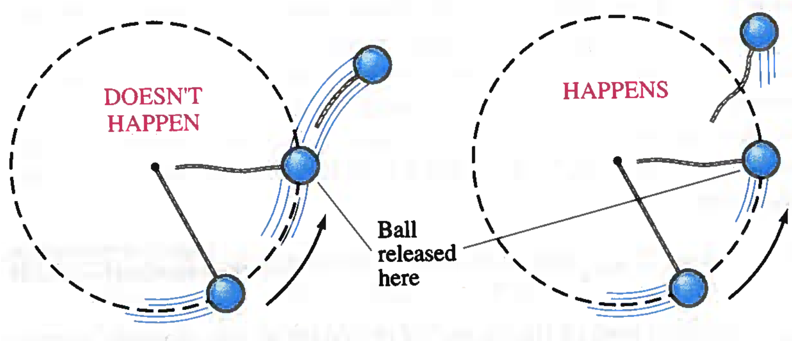


FIGURE 5–6 If centrifugal force existed, the ball would fly off as in (a) when released. In fact, it flies off tangentially as in (b). For example, in (c) sparks fly in straight lines tangentially from the edge of a rotating grinding wheel.



(c)

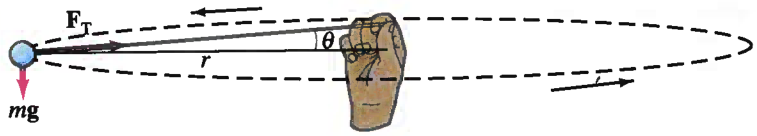


FIGURE 5–7 Example 5–3.

EXAMPLE 5–3 ESTIMATE **Force on revolving ball (horizontal).** Estimate the force a person must exert on a string attached to a 0.150-kg ball to make the ball revolve in a horizontal circle of radius 0.600 m, as in Example 5–1. The ball makes 2.00 revolutions per second.

SOLUTION First we draw the free-body diagram for the ball, Fig. 5–7, which shows the two forces acting on the ball: the force of gravity, $m\mathbf{g}$; and the tension force $\mathbf{F}_T$ that the string exerts (which occurs because the person exerts that same force on the string). The ball's weight complicates matters and makes it impossible to revolve a ball with the cord horizontal. But if the weight is small enough, we can ignore it. Then $\mathbf{F}_T$ will act nearly horizontally ($\theta \approx 0$ in Fig. 5–7) and provide the force necessary to give the ball its centripetal acceleration. We apply Newton's second law to the radial direction, which now is horizontal, so we call it x:

Tension in cord acts to provide the centripetal acceleration

$$\Sigma F_x = ma_x$$

or (recall from Example 5–1 that $v = 7.54\,\text{m/s}$),

$$F_{Tx} = m\frac{v^2}{r} = (0.150\ \text{kg})\frac{(7.54\ \text{m/s})^2}{(0.600\ \text{m})} \approx 14\ \text{N},$$

where we have rounded off because our estimate ignores the ball's mass.

CONCEPTUAL EXAMPLE 5–4 **Tetherball.** The game of tetherball is played with a ball tied to a pole with a string. When the ball is struck, it whirls around the pole as shown in Fig. 5–8. In what direction is the acceleration of the ball, and what causes the acceleration?

RESPONSE The acceleration points horizontally toward the center of the ball's circular path (not toward the top of the pole). The force responsible for the acceleration may not be obvious at first, since there seems to be no force pointing directly horizontally. But it is the *net* force (the sum of $m\mathbf{g}$ and $\mathbf{F}_T$ here) that must point in the direction of the acceleration. The vertical component of the string tension balances the ball's weight, $m\mathbf{g}$. The horizontal component of the string tension, F_{Tx}, is the force that produces the centripetal acceleration toward the center.

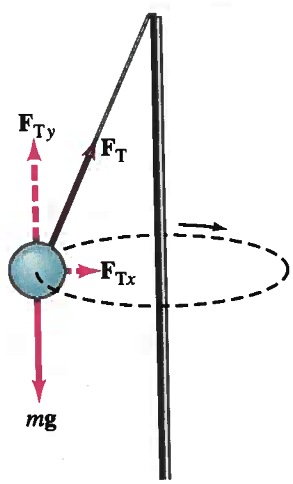


FIGURE 5–8 Conceptual Example 5–4.

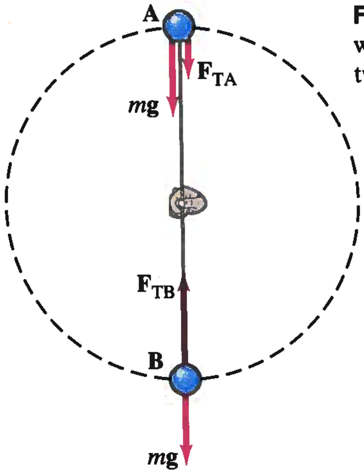


FIGURE 5–9 Example 5–5, with free-body diagrams at the two positions.

EXAMPLE 5–5 **Revolving ball (vertical circle).** A 0.150-kg ball on the end of a 1.10-m-long cord (negligible mass) is swung in a *vertical* circle. Determine the minimum speed the ball must have at the top of its arc so that it continues moving in a circle. (*b*) Calculate the tension in the cord at the bottom of the arc assuming the ball is moving at twice the speed of part (*a*).

Gravity and cord tension together provide centripetal acceleration

SOLUTION The free-body diagram is shown in Fig. 5–9 for both situations. (*a*) At the top (point A), two forces can act on the ball: **mg**, its weight; and $\mathbf{F}_{TA}$, the tension force the cord exerts at point A. Both act downward, and their vector sum acts to give the ball its centripetal acceleration a_R. We apply Newton's second law, for the vertical direction, choosing downward as positive (toward the center):

$$\Sigma F_R = ma_R$$

$$F_{TA} + mg = m\frac{v_A^2}{r},$$

From this equation we can see that the tension force F_{TA} at A will get larger if v_A (ball's speed at top of circle) is made larger, as expected. But we are asked for the *minimum* speed to keep the ball moving in a circle. The cord will remain taut as long as there is tension in it; but if the tension disappears (because v_A is too small) the cord can go limp, and the ball will fall out of its circular path. Thus, the minimum speed will occur if $F_{TA} = 0$, for which we have

Gravity provides centripetal acceleration

$$mg = m\frac{v_A^2}{r}.$$

We solve for v_A:

$$v_A = \sqrt{gr} = \sqrt{(9.80\ \text{m/s}^2)(1.10\ \text{m})} = 3.28\ \text{m/s}.$$

This is the minimum speed at the top of the circle if the ball is to continue moving in a circular path.

(*b*) At the bottom of the circle (see Fig. 5–9) the cord exerts its tension force F_{TB} upward whereas the force of gravity, **mg**, acts downward. So Newton's second law, this time choosing *upward* as positive (toward the center), gives

String tension and gravity acting in opposite directions provide centripetal acceleration

$$\Sigma F_R = ma_R$$

$$F_{TB} - mg = m\frac{v_B^2}{r}.$$

The speed v_B is given as twice what we found in (*a*), namely 6.56 m/s. [Note that the speed changes here because gravity acts on the ball at all points along the path, but Eq. 5–3 still remains valid, $\Sigma \mathbf{F}_R = mv^2/r$.] We solve for F_{TB} in the last equation:

$$F_{TB} = m\frac{v_B^2}{r} + mg$$

$$= (0.150\text{ kg})\frac{(6.56\text{ m/s})^2}{(1.10\text{ m})} + (0.150\text{ kg})(9.80\text{ m/s}^2) = 7.34\text{ N}.$$

Note that we could not simply set F_{TB} equal to mv_B^2/r; the latter is equal to the *net* force on the ball in the radial direction and so also includes gravity. Indeed, the cord's tension not only provides the centripetal acceleration, but must be even larger than ma_R to compensate for the downward force of gravity.

CONCEPTUAL EXAMPLE 5–6 **Ferris wheel.** A rider on a Ferris wheel moves in a vertical circle of radius r at constant speed v (Fig. 5–10). Is the normal force that the seat exerts on the rider at the top of the circle (*a*) less than, (*b*) more than, or (*c*) the same as, the force the seat exerts at the bottom of the circle?

RESPONSE The free-body diagram is shown in Fig. 5–10 and is similar to that for Example 5–5, with $\mathbf{F}_N$ replacing $\mathbf{F}_T$. Because the acceleration points radially *toward* the center, Newton's second law tells us that $F_N < mg$ at the top but $F_N > mg$ at the bottom. So the correct answer is (*a*).

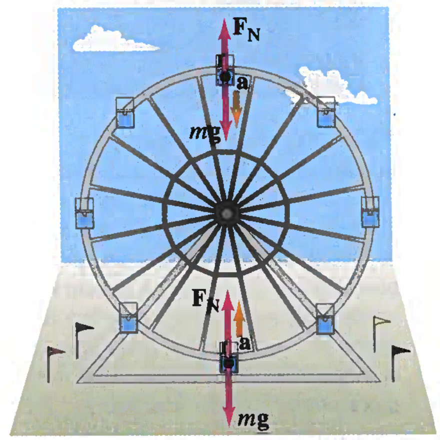


FIGURE 5–10 Conceptual Example 5–6.

5–3 A Car Rounding a Curve

An example of centripetal acceleration occurs when an automobile rounds a curve. In such a situation, you may feel that you are thrust outward. But there is not some mysterious centrifugal force pulling on you. What is happening is that you tend to move in a straight line, whereas the car has begun to follow a curved path. To make you go in the curved path, the seat (friction) or the door of the car (direct contact) exerts a force on you (Fig. 5–11). The car itself must have an inward force exerted on it if it is to move in a curve. On a flat road, this force is supplied by friction between the tires and the pavement. (It is static friction as long

PHYSICS APPLIED
Driving on a curve

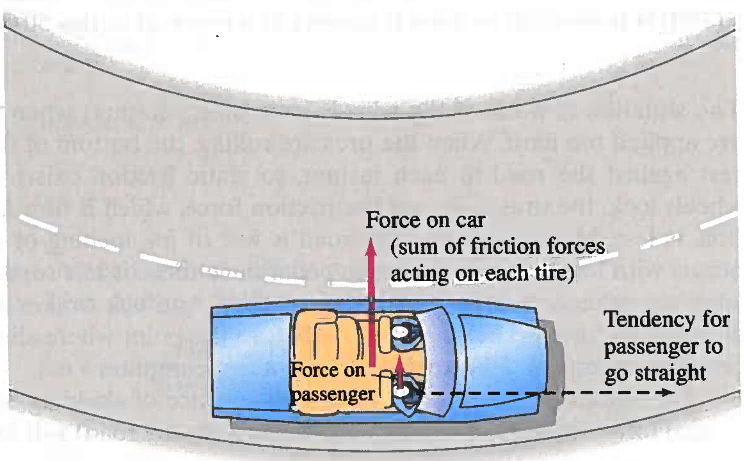
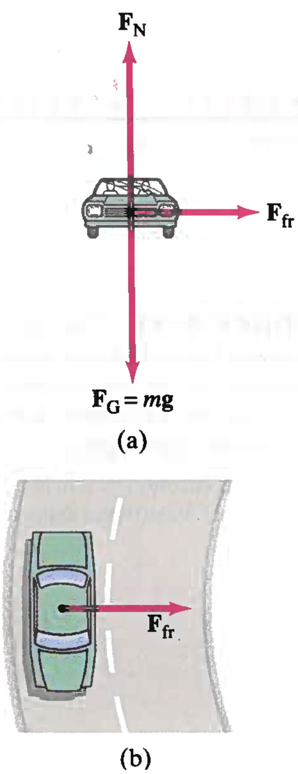


FIGURE 5–11 The road exerts an inward force (friction against the tires) on a car to make it move in a circle; and the car exerts an inward force on the passenger.

FIGURE 5–12 Race car heading down into a curve. From the tire marks we can see that most cars experienced a sufficient friction force to give them the needed centripetal acceleration for rounding the curve safely. But, we can also see a few tire tracks of cars on which there was not sufficient force—and which followed more nearly straight-line paths.

FIGURE 5–13 Forces on a car rounding a curve on a flat road, Example 5–7. (a) Front view, (b) top view.



as the tires are not slipping.) If the friction force is not great enough, as under icy conditions, sufficient force cannot be applied and the car will skid out of a circular path into a more nearly straight path. See Fig. 5–12.

EXAMPLE 5–7 **Skidding on a curve.** A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 50 km/h (14 m/s). Will the car make the turn, or will it skid, if: (*a*) the pavement is dry and the coefficient of static friction is $\mu_s = 0.60$; (*b*) the pavement is icy and $\mu_s = 0.25$?

SOLUTION Figure 5–13 shows the free-body diagram for the car. The normal force, F_N, on the car is equal to the weight since the road is flat and there is no vertical acceleration:

$$F_N = mg = (1000\text{ kg})(9.8\text{ m/s}^2) = 9800\text{ N}.$$

In the horizontal direction the only force is friction, and we must compare it to the force needed to produce the centripetal acceleration to see if it is sufficient. The net horizontal force required to keep the car moving in a circle around the curve is

$$\Sigma F_R = ma_R = m\frac{v^2}{r} = (1000\text{ kg})\frac{(14\text{ m/s})^2}{(50\text{ m})} = 3900\text{ N}.$$

Naturally we hope the maximum total friction force (the sum of the friction forces acting on each of the four tires) will be at least this large. For (*a*), $\mu_s = 0.60$, and the maximum friction force attainable (recall from Section 4–8 that $F_{fr} \le \mu_s F_N$) is

$$(F_{fr})_{max} = \mu_s F_N = (0.60)(9800\text{ N}) = 5900\text{ N}.$$

Since a force of only 3900 N is needed, and that is, in fact, how much will be exerted by the road as a static friction force, the car can make the turn fine. But in (*b*) the maximum friction force possible is

$$(F_{fr})_{max} = \mu_s F_N = (0.25)(9800\text{ N}) = 2500\text{ N}.$$

The car will skid because the ground cannot exert sufficient force (3900 N is needed) to keep it moving in a curve of radius 50 m.

The situation is worse if the wheels lock (stop rotating) when the brakes are applied too hard. When the tires are rolling, the bottom of the tire is at rest against the road at each instant, so static friction exists. But if the wheels lock, the tires slide and the friction force, which is now kinetic friction, is less. Moreover, when the road is wet or icy, locking of the wheels occurs with less force on the brake pedal since there is less road friction to keep the wheels turning rather than sliding. Antilock brakes (ABS) are designed to limit brake pressure just before the point where sliding would occur, by means of delicate sensors and a fast computer.

The banking of curves can reduce the chance of skidding because the normal force of the road (acting perpendicular to the road) will have a com ponent toward the center of the circle (Fig. 5–14), thus reducing the reliance on friction. For a given banking angle, θ, there will be one speed for which no friction at all is required. This will be the case when the horizontal component of the normal force toward the center of the curve, $F_N \sin\theta$ (see Fig. 5–14), is just equal to the force required to give a vehicle its centripetal acceleration—that is, when

$$F_N \sin\theta = m\frac{v^2}{r}.$$

The banking angle of a road, θ, is chosen so that this condition holds for a particular speed, called the "design speed."

PHYSICS APPLIED

Banked curves

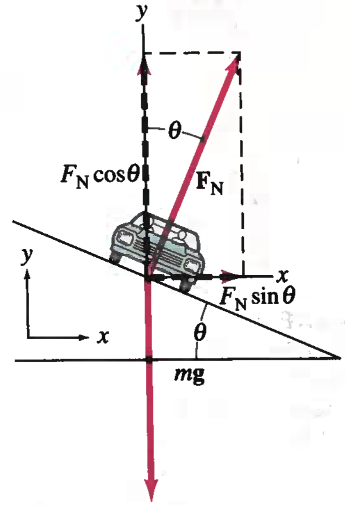


FIGURE 5–14 Normal force on a car rounding a banked curve, resolved into its horizontal and vertical components. Note that the centripetal acceleration is horizontal (and not parallel to the sloping road).

EXAMPLE 5–8 **Banking angle.** (*a*) For a car traveling with speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required. (*b*) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

SOLUTION We choose our x and y axes as horizontal and vertical so that a_R, which is horizontal, is along the x axis. The components of F_N are as shown in Fig. 5–14. (*a*) For the horizontal direction, $\Sigma F_R = ma_R$ gives

$$F_N \sin\theta = \frac{mv^2}{r}.$$

In the vertical direction, the forces are $F_N \cos\theta$ upward (Fig. 5–14) and the weight of the car (mg) downward. Since there is no vertical motion, the y component of the acceleration is zero, so $\Sigma F_y = ma_y$ gives us

$$F_N \cos\theta - mg = 0.$$

Thus,

$$F_N = \frac{mg}{\cos\theta}.$$

[Note in this case that $F_N \geq mg$ since $\cos\theta \leq 1$.] We substitute this relation for F_N into the equation for the horizontal motion,

Horizontal component of normal force alone acts to provide centripetal acceleration (friction is desired to be zero—otherwise it too would contribute)

$$F_N \sin\theta = m\frac{v^2}{r},$$

and obtain

$$\frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

or

$$mg\tan\theta = m\frac{v^2}{r}$$

so

$$\tan\theta = \frac{v^2}{rg}.$$

This is the formula for the banking angle θ.

(*b*) For $r = 50\,\text{m}$ and $v = 50\,\text{km/h}$ (or $14\,\text{m/s}$),

$$\tan\theta = \frac{(14\,\text{m/s})^2}{(50\,\text{m})(9.8\,\text{m/s}^2)} = 0.40,$$

so $\theta = 22°$.

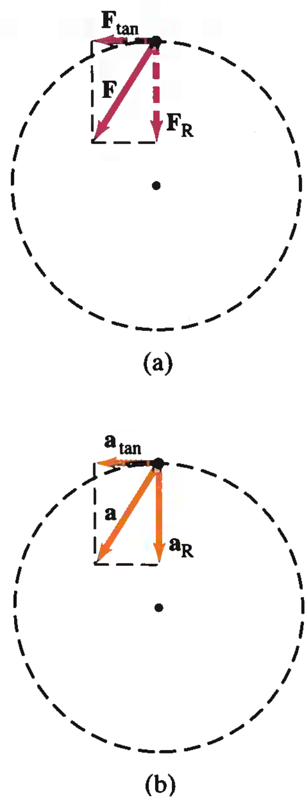


FIGURE 5–15 The speed of an object moving in a circle changes if the force on it has a tangential component, $F_{\tan}$. Part (a) shows the force **F** and its vector components; part (b) shows the acceleration vector and its vector components.

* 5–4 Nonuniform Circular Motion

Circular motion at constant speed occurs when the net force on an object is exerted toward the center of the circle. If the net force is not directed toward the center but is at an angle, as shown in Fig. 5–15a, the force has two components. The component directed toward the center of the circle, F_R, gives rise to the centripetal acceleration, a_R, and keeps the object moving in a circle. The component tangent to the circle, $F_{\tan}$, acts to increase (or decrease) the speed, and thus gives rise to a component of the acceleration tangent to the circle, $a_{\tan}$. When the speed of the object is changing, a tangential component of force is acting.

When you first start revolving a ball on the end of a string around your head, you must give it tangential acceleration. You do this by pulling on the string with your hand displaced from the center of the circle. In athletics, a hammer thrower accelerates the hammer tangentially in a similar way so that it reaches a high speed before release.

The tangential component of the acceleration, $a_{\tan}$, is equal to the rate of change of the *magnitude* of the velocity of the object:

$$a_{\tan} = \frac{\Delta v}{\Delta t}.$$

The radial (centripetal) acceleration arises from the change in *direction* of the velocity and, as we have seen (Eq. 5–1), is given by

$$a_R = \frac{v^2}{r}.$$

The tangential acceleration always points in a direction tangent to the circle, and is in the direction of motion (parallel to **v**) if the speed is increasing, as shown in Fig. 5–15b. If the speed is decreasing, $\mathbf{a}_{\tan}$ points antiparallel to **v**. In either case, $\mathbf{a}_{\tan}$ and $\mathbf{a}_R$ are always perpendicular to each other; and *their directions change* continually as the object moves along its circular path. The total vector acceleration, **a**, is the sum of these two:

$$\mathbf{a} = \mathbf{a}_{\tan} + \mathbf{a}_R.$$

Since $\mathbf{a}_R$ and $\mathbf{a}_{\tan}$ are always perpendicular to each other, the magnitude of **a** at any moment is

$$a = \sqrt{a_{\tan}^2 + a_R^2}.$$

EXAMPLE 5–9 **Two components of acceleration.** A racing car starts from rest in the pit area and accelerates at a uniform rate to a speed of 35 m/s in 11 s, moving on a circular track of radius 500 m. Assuming constant tangential acceleration, find (*a*) the tangential acceleration, and (*b*) the centripetal acceleration when the speed is 30 m/s.

SOLUTION (*a*) $a_{\tan}$ is constant, of magnitude

$$a_{\tan} = \frac{\Delta v}{\Delta t} = \frac{(35\text{ m/s} - 0\text{ m/s})}{11\text{ s}} = 3.2\text{ m/s}^2.$$

(*b*)

$$a_R = \frac{v^2}{r} = \frac{(30\text{ m/s})^2}{500\text{ m}} = 1.8\text{ m/s}^2.$$

*5–5 Centrifugation

A useful device that very nicely illuminates the dynamic aspects of circular motion is the centrifuge, or the very high speed ultracentrifuge. These devices are used to sediment materials quickly or to separate materials with slightly different characteristics. Test tubes or other containers are held in the centrifuge rotor, which is accelerated to very high rotational speeds: see Fig. 5–16, where one test tube is shown in two different positions as the rotor turns. The small green dot represents a small particle, perhaps a macromolecule, in a fluid-filled test tube. When the tube is at position A and the rotor is turning, the particle has a tendency to move in a straight line in the direction of the dashed arrow. But the fluid, resisting the motion of the particles, exerts a centripetal force that keeps the particles moving nearly in a circle. Usually, the resistance of the fluid (which could be a liquid, a gas, or a gel, depending on the application) does not quite equal mv^2/r, and the particles eventually reach the bottom of the tube. If the particles are sedimenting in a semi-rigid medium like a gel, and the rotation is stopped before the particles reach the bottom of the tube, the particles will be separated according to their size or other factors that influence their mobility. If the particles reach the bottom of the tube, then the bottom of the tube exerts a force that keeps the particles moving in a circle. In fact, the bottom of the tube must exert a force on the whole tube of fluid, keeping it moving in a circle. If the tube is not strong enough to exert this force, it will break.

The kinds of materials placed in a centrifuge are those that do not sediment or separate quickly under the action of gravity. The purpose of a centrifuge is to provide an "effective gravity" much larger than normal gravity because of the high rotational speeds, so that the particles move down the tube more rapidly.

➥ **PHYSICS APPLIED**

Centrifuge

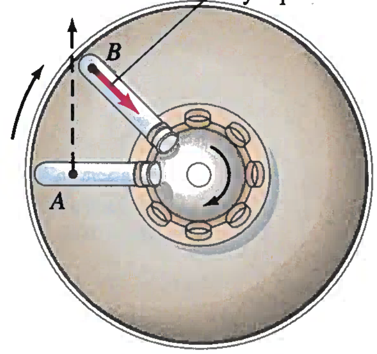


FIGURE 5–16 Rotating test tube in a centrifuge (top view). Tube is shown at two positions. At A, the green dot represents a macromolecule or other particle being sedimented. It would tend to go along the dashed line heading toward the bottom of the tube but the fluid resists this motion by exerting a force on the particle as shown at point B.

EXAMPLE 5–10 **Ultracentrifuge.** An ultracentrifuge rotor rotates at 50,000 rpm (revolutions per minute). The top of a 4.00-cm long test tube (Fig. 5–16) is 6.00 cm from the rotation axis and is perpendicular to it. The bottom of the tube is 10.00 cm, from the axis of rotation. (*a*) Calculate the centripetal acceleration, in "g's," at the top and the bottom of the tube. (*b*) If the contents of the tube have a total mass of 12.0 g, what force must the bottom of the tube withstand?

SOLUTION We can calculate the centripetal acceleration from $a_R = v^2/r$. (*a*) At the top of the tube, a particle revolves in a circle of circumference $2\pi r$, which is a distance

$$2\pi r = 2(3.14)(0.0600\text{ m}) = 0.377\text{ m per revolution.}$$

It makes 5.00×10^4 such revolutions each minute, or, dividing by 60 s/min, 833 rev/s; so the time to make one revolution, the period T, is

$$T = 1/(833\text{ rev/s}) = 1.20 \times 10^{-3}\text{ s/rev.}$$

The speed of the particle is then

$$v = \frac{2\pi r}{T} = \left(\frac{0.377\text{ m/rev}}{1.20 \times 10^{-3}\text{ s/rev}}\right) = 3.14 \times 10^2\text{ m/s.}$$

The centripetal acceleration is

$$a_R = \frac{v^2}{r} = \frac{(3.14 \times 10^2\ \text{m/s})^2}{0.0600\ \text{m}} = 1.64 \times 10^6\ \text{m/s}^2,$$

which, dividing by $g = 9.80\ \text{m/s}^2$, is 1.67×10^5 g's.

At the bottom of the tube ($r = 0.1000$ m), the speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi\,(0.1000\ \text{m})}{1.20 \times 10^{-3}\ \text{s/rev}} = 5.23 \times 10^2\ \text{m/s}.$$

Then

$$a_R = v^2/r = (5.23 \times 10^2\ \text{m/s})^2/(0.1000\ \text{m}) = 2.74 \times 10^6\ \text{m/s}^2$$
$$= 2.79 \times 10^5\ g\text{'s}.$$

(b) Since the acceleration varies with distance from the axis, we estimate the force using the average acceleration

$$\bar{a} = \frac{(1.64 \times 10^6\ \text{m/s}^2 + 2.74 \times 10^6\ \text{m/s}^2)}{2}$$
$$= 2.19 \times 10^6\ \text{m/s}^2.$$

Then

$$F = m\bar{a} = (0.0120\ \text{kg})(2.19 \times 10^6\ \text{m/s}^2) = 2.63 \times 10^4\ \text{N}.$$

which is equivalent to the weight of a 2680-kg mass [since $m = F/g = (2.63 \times 10^4\ \text{N})/9.80\ \text{m/s}^2 = 2.68 \times 10^3$ kg], or almost 3 tons!

5–6 Newton's Law of Universal Gravitation

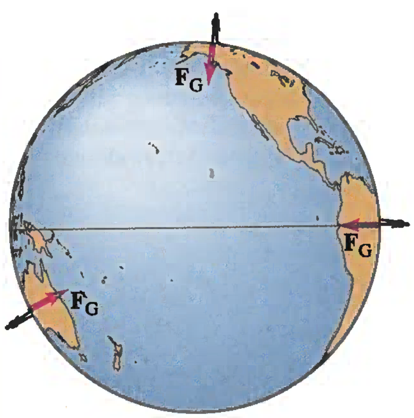


FIGURE 5–17 Anywhere on Earth, whether in Alaska, Peru, or Australia, the force of gravity acts downward toward the center.

Newton's apple

Besides developing the three laws of motion, Sir Isaac Newton also examined the motion of the planets and the Moon. In particular, he wondered about the nature of the force that must act to keep the Moon in its nearly circular orbit around the Earth.

Newton was also thinking about the problem of gravity. Since falling bodies accelerate, Newton had concluded that they must have a force exerted on them, a force we call the force of gravity. Whenever a body has a force exerted *on* it, that force is exerted *by* some other body. But what *exerts* the force of gravity? Every object on the surface of the Earth feels the force of gravity, and no matter where the object is, the force is directed toward the center of the Earth, Fig. 5–17. Newton concluded that it must be the Earth itself that exerts the gravitational force on objects at its surface.

According to an early account, Newton was sitting in his garden and noticed an apple drop from a tree. He is said to have been struck with a sudden inspiration: if gravity acts at the tops of trees, and even at the tops of mountains, then perhaps it acts all the way to the Moon! Whether this story is true or not, it does seem to capture something of Newton's reasoning and inspiration. With this idea that it is terrestrial gravity that holds the Moon in its orbit, Newton developed his great theory of gravitation. [But there was controversy at the time. Many thinkers had trouble accepting the idea of a force "acting at a distance." Typical forces act through contact—your hand pushes a cart and pulls a wagon, a bat hits a ball, and so on. But gravity acts without contact, said Newton: the Earth exerts a force on a falling apple and on the Moon, even though there is no contact, and the two objects may even be very far apart.]

Newton set about determining the magnitude of the gravitational force that the Earth exerts on the Moon as compared to the gravitational force on objects at the Earth's surface. At the surface of the Earth, the force of gravity accelerates objects at $9.80\,\mathrm{m/s^2}$. But what is the centripetal acceleration of the Moon? Since the Moon moves with nearly uniform circular motion, the acceleration can be calculated from $a_R = v^2/r$. We already performed this calculation in Example 5–2 and found that $a_R = 0.00272\,\mathrm{m/s^2}$. In terms of the acceleration of gravity at the Earth's surface, g, this is equivalent to

The Moon's acceleration toward Earth

$$a_R \approx \frac{1}{3600}\,g.$$

That is, the acceleration of the Moon toward the Earth is about $\frac{1}{3600}$ as great as the acceleration of objects at the Earth's surface. The Moon is 384,000 km from the Earth, which is about 60 times the Earth's radius of 6380 km. That is, the Moon is 60 times farther from the Earth's center than are objects at the Earth's surface. But $60 \times 60 = 60^2 = 3600$. Again that number 3600! Newton concluded that the gravitational force exerted by the Earth on any object decreases with the square of its distance, r, from the Earth's center:

$$\text{force of gravity} \propto \frac{1}{r^2}.$$

The Moon, being 60 Earth radii away, feels a gravitational force only $\frac{1}{60^2} = \frac{1}{3600}$ times as strong as it would if it were at the Earth's surface. Any object placed 384,000 km from the Earth would experience the same acceleration due to the Earth's gravity as the Moon experiences: $0.00272\,\mathrm{m/s^2}$.

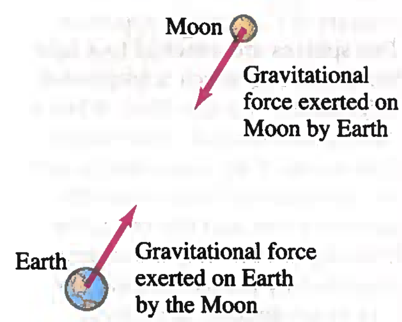


FIGURE 5–18 The gravitational force one body exerts on a second body is directed toward the first body, and is equal and opposite to the force exerted by the second body on the first.

Newton realized that the force of gravity on an object depends not only on distance but also on the object's mass. In fact, it is directly proportional to its mass, as we have seen. According to Newton's third law, when the Earth exerts its gravitational force on any body, such as the Moon, that other body exerts an equal and opposite force on the Earth (Fig. 5–18). Because of this symmetry, Newton reasoned, the magnitude of the force of gravity must be proportional to *both* the masses. Thus

$$F \propto \frac{m_E m_B}{r^2}$$

where m_E is the mass of the Earth, m_B the mass of the other body, and r the distance from the Earth's center to the center of the other body.

Newton went a step further in his analysis of gravity. In his examination of the orbits of the planets, he concluded that the force required to hold the different planets in their orbits around the Sun seems to diminish as the inverse square of their distance from the Sun. This led him to believe that it is also the gravitational force that acts between the Sun and each of the planets to keep them in their orbits. And if gravity acts between these objects, why not between all objects? Thus he proposed his famous **law of universal gravitation**, which we can state as follows:

NEWTON'S LAW OF UNIVERSAL GRAVITATION

> **Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.**

The magnitude of the gravitational force can be written as

$$F = G\frac{m_1 m_2}{r^2}, \tag{5-4}$$

where m_1 and m_2 are the masses of the two particles, r is the distance between them, and G is a universal constant which must be measured experimentally and has the same numerical value for all objects.

The value of G must be very small, since we are not aware of any force of attraction between ordinary-sized objects, such as between two baseballs. The force between two ordinary objects was first measured, over 100 years after Newton's publication of his law, by Henry Cavendish in 1798. To detect and measure the incredibly small force, he used an apparatus like that shown in Fig. 5–19. Cavendish confirmed Newton's hypothesis that two bodies attract one another, and that Eq. 5–4 accurately describes this force. In addition, because he could measure F, m_1, m_2, and r accurately, he was able to determine the value of the constant G as well. The accepted value today is

$$G = 6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}.$$

[Strictly speaking, Eq. 5–4 gives the magnitude of the gravitational force that one particle exerts on a second particle that is a distance r away. For an extended object (that is, not a point), we must consider how to measure the distance r. You might think that r would be the distance between the centers of the objects. This is often true and often a good approximation even when not quite true, but to do a calculation correctly, each extended body must be considered as a collection of tiny particles. The total force is the sum of the forces due to all the particles. The sum over all these particles is often best done using integral calculus, which Newton himself invented. Newton showed that for two uniform spheres, Eq. 5–4 gives the correct force where r is the distance between their centers. Also, when extended bodies are small compared to the distance between them (as for the Earth–Sun system), little inaccuracy results from considering them as point particles.]

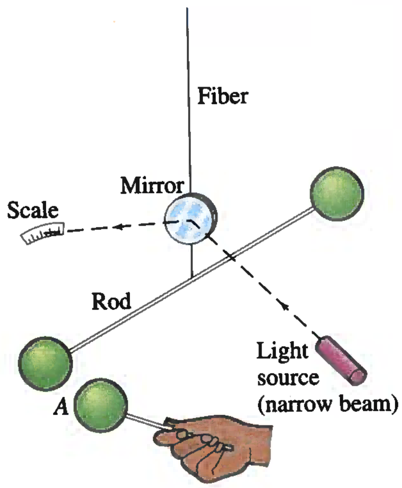


FIGURE 5–19 Schematic diagram of Cavendish's apparatus. Two spheres are attached to a light horizontal rod, which is suspended at its center by a thin fiber. When a third sphere labeled A is brought close to one of the suspended spheres, the gravitational force causes the latter to move, and this twists the fiber slightly. The tiny movement is magnified by the use of a narrow light beam directed at a mirror mounted on the fiber. The beam reflects onto a scale. Previous determination of how large a force will twist the fiber a given amount then allows one to determine the magnitude of the gravitational force between two objects.

EXAMPLE 5–11 ESTIMATE **Can you attract another person gravitationally?** A 50-kg person and a 75-kg person are sitting on a bench so that their centers are about 50 cm apart. Estimate the magnitude of the gravitational force each exerts on the other.

SOLUTION We use Eq. 5–4, which gives

$$F = \frac{(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(50\,\mathrm{kg})(75\,\mathrm{kg})}{(0.50\,\mathrm{m})^2} = 1.0 \times 10^{-6}\,\mathrm{N},$$

which is unnoticeably small unless very delicate instruments are used.

EXAMPLE 5–12 **Spacecraft at $2r_E$.** What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth's center (that is, a distance r_E = 6380 km above the Earth's surface, (Fig. 5–20)? The mass of the Earth is $M_E = 5.98 \times 10^{24}$ kg.

SOLUTION We could plug all the numbers into Eq. 5–4, but there is a simpler approach. The spacecraft is twice as far from the Earth's center as when at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance (and $\frac{1}{2^2} = \frac{1}{4}$), the force of gravity on it will be only one fourth its weight at the Earth's surface:

$$F_G = \tfrac{1}{4}mg = \tfrac{1}{4}(2000\text{ kg})(9.80\text{ m/s}^2) = 4900\text{ N}.$$

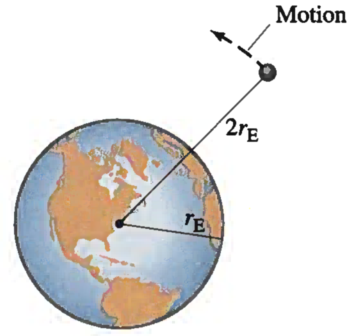


FIGURE 5–20 Example 5–12.

EXAMPLE 5–13 **Force on the moon.** Find the net force on the Moon ($m_M = 7.35 \times 10^{22}$ kg) due to the gravitational attraction of both the Earth ($m_E = 5.98 \times 10^{24}$ kg) and the Sun ($m_S = 1.99 \times 10^{30}$ kg), assuming they are at right angles to each other (Fig. 5–21).

SOLUTION We must add the two forces vectorially. First we calculate their magnitudes. The Earth is 3.84×10^5 km $= 3.84 \times 10^8$ m from the Moon, so F_{ME} (the force on the Moon due to the Earth) is

$$F_{ME} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\text{ kg})(5.98 \times 10^{24}\text{ kg})}{(3.84 \times 10^8\text{ m})^2}$$
$$= 1.99 \times 10^{20}\text{ N}.$$

The Sun is 1.50×10^8 km from the Earth and the Moon, so F_{MS} (the force on the Moon due to the Sun) is

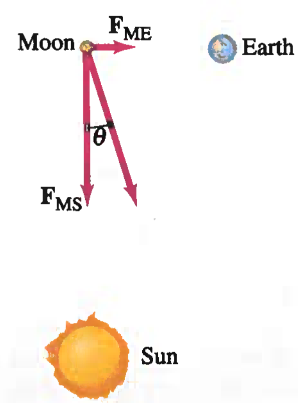
$$F_{MS} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\text{ kg})(1.99 \times 10^{30}\text{ kg})}{(1.50 \times 10^{11}\text{ m})^2}$$
$$= 4.34 \times 10^{20}\text{ N}.$$

Since the two forces act at right angles in the case we are considering (Fig. 5–21), the total force is

$$F = \sqrt{(1.99)^2 + (4.34)^2} \times 10^{20}\text{ N} = 4.77 \times 10^{20}\text{ N}$$

which acts at an angle $\theta = \tan^{-1}(1.99/4.34) = 24.6°$.

FIGURE 5–21 Orientation of Sun (S), Earth (E), and Moon (M) for Example 5–13 (not to scale).



The law of universal gravitation should not be confused with Newton's second law of motion, $\Sigma\mathbf{F} = m\mathbf{a}$. The former describes a particular force, gravity, and how its strength varies with the distance and masses involved. Newton's second law, on the other hand, relates the net force on a body (i.e., the vector sum of all the different forces acting on the body whatever their sources) to the mass and acceleration of that body.

Careful: Distinction between Newton's second law and the law of gravity

5–7 Gravity Near the Earth's Surface; Geophysical Applications

When Eq. 5–4 is applied to the gravitational force between the Earth and an object at its surface, m_1 becomes the mass of the Earth m_E, m_2 becomes the mass of the object, m, and r becomes the distance of the object from the Earth's center,[†] which is the radius of the Earth r_E. This force of gravity due to the Earth is the weight of the object, which we have been writing as mg. Thus,

$$mg = G\frac{mm_E}{r_E^2}.$$

Hence

g in terms of G

$$g = G\frac{m_E}{r_E^2}. \tag{5–5}$$

Thus, the acceleration of gravity at the surface of the Earth, g, is determined by m_E and r_E. (Be careful not to confuse G with g; they are very different quantities, but are related by Eq. 5–5.)

Until G was measured, the mass of the Earth was not known. But once G was measured, Eq. 5–5 could be used to calculate the Earth's mass, and Cavendish was the first to do so. Since $g = 9.80\,\text{m/s}^2$ and the radius of the Earth is $r_E = 6.38 \times 10^6\,\text{m}$, then, from Eq. 5–5, we obtain

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\,\text{m/s}^2)(6.38 \times 10^6\,\text{m})^2}{6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98 \times 10^{24}\,\text{kg}$$

for the mass of the Earth.

When dealing with the weight of objects at the surface of the Earth, we can continue to use simply mg. If we wish to calculate the force of gravity on an object some distance from the Earth, or the force due to some other heavenly body, such as that exerted by the Moon or a planet, we can calculate the effective value of g from Eq. 5–5, replacing r_E (and m_E) by the appropriate distance (and mass), or we can use Eq. 5–4 directly.

EXAMPLE 5–14 ESTIMATE **Gravity on Everest.** Estimate the effective value of g on the top of Mt. Everest, 8848 m (29,028 ft) above the Earth's surface. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

SOLUTION Let us call the acceleration of gravity at the given point g'. We use Eq. 5–5, with r_E replaced by $r = 6380\,\text{km} + 8.8\,\text{km} = 6389\,\text{km} = 6.389 \times 10^6\,\text{m}$:

$$g' = G\frac{m_E}{r^2} = \frac{(6.67 \times 10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\,\text{kg})}{(6.389 \times 10^6\,\text{m})^2} = 9.77\,\text{m/s}^2$$

which is a reduction of about 3 parts in a thousand (0.3%). Note that we ignored the mass accumulated under the mountain top, and we have used $1\,\text{N/kg} = 1\,\text{m/s}^2$.

Note that Eq. 5–5 does not give precise values for g at different locations because the earth is not a perfect sphere. The Earth not only has mountains and valleys, and bulges at the equator, but its mass is not distributed

[†]That the distance is measured from the Earth's center does not imply that the force of gravity somehow emanates from that one point. Rather, all parts of the Earth attract gravitationally, but the net effect is a force acting toward the Earth's center.

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The value of g can vary locally on the Earth's surface because of the presence of irregularities and rocks of different densities. Such variations in g, known as "gravity anomalies," are very small—on the order of 1 part per 10^6 or 10^7 in the value of g. But they can be measured ("gravimeters" today can detect variations in g to 1 part in 10^9). Geophysicists use such measurements as part of their investigations into the structure of the Earth's crust, and in mineral and oil exploration. Mineral deposits, for example, often have a greater density than surrounding material. Because of the greater mass in a given volume, g can have a slightly greater value on top of such a deposit than at its flanks. "Salt domes," under which petroleum is often found, have a lower than average density and searches for a slight reduction in the value of g in certain locales have led to the discovery of oil.

Some geophysics

➥ **PHYSICS APPLIED**

Geology—mineral and oil exploration

5–8 Satellites and "Weightlessness"

Artificial satellites circling the Earth are now commonplace (Fig. 5–22). A satellite is put into orbit by accelerating it to a sufficiently high tangential speed with the use of rockets, as shown in Fig. 5–23. If the speed is too high, the spacecraft will not be confined by the Earth's gravity and will escape, never to return. If the speed is too low, it will return to Earth. Satellites are usually put into circular (or nearly circular) orbits, because they require the least takeoff speed. It is sometimes asked: "What keeps a satellite up?" The answer is: its high speed. If a satellite stopped moving, it would, of course, fall directly to Earth. But at the very high speed a satellite has, it would quickly fly out into space (Fig. 5–24) if it weren't for the gravitational force of the Earth pulling it into orbit. In fact, a satellite *is* falling (accelerating toward Earth), but its high tangential speed keeps it from hitting Earth.

For satellites that move in a circle (at least approximately), the acceleration is v^2/r. The force that gives a satellite this acceleration is the force of gravity, and since a satellite may be at a considerable distance from the Earth, we must use Eq. 5–4 for the force acting on it. When we apply

➥ **PHYSICS APPLIED**

Artificial Earth satellites

FIGURE 5–22 A satellite circling the Earth.

FIGURE 5–23 Artificial satellites launched at different speeds.

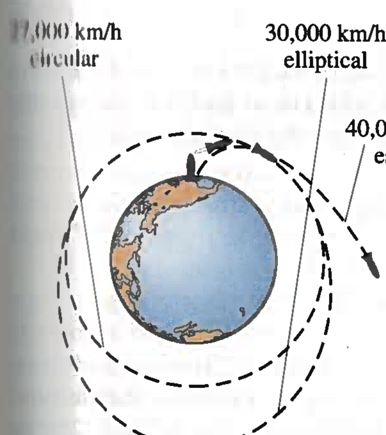
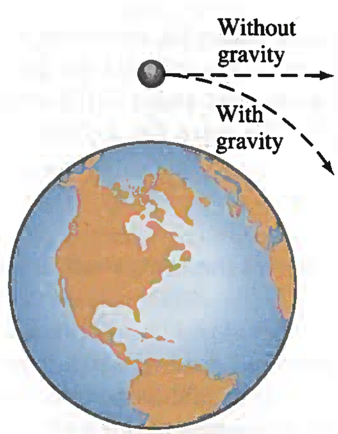


FIGURE 5–24 A moving satellite "falls" out of a straight-line path toward the Earth.



Newton's second law, $\Sigma F_R = ma_R$, we find

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r}, \quad (5\text{–}6)$$

where m is the mass of the satellite. This equation relates the distance of the satellite from the Earth's center, r, to its speed, v. Note that only one force—gravity—is acting on the satellite, and that r is the sum of the Earth's radius r_E plus the satellite's height h above the Earth: $r = r_E + h$.

PHYSICS APPLIED
Geosynchronous satellites

EXAMPLE 5–15 **Geosynchronous satellite.** A *geosynchronous* satellite is one that stays above the same point on the equator of the Earth. Such satellites are used for such purposes as cable TV transmission, for weather forecasting, and as communication relays. Determine (*a*) the height above the Earth's surface such a satellite must orbit and (*b*) such a satellite's speed.

SOLUTION (*a*) The only force on the satellite is gravity, so we apply Eq. 5–6 assuming the satellite moves in a circle:

$$G\frac{m_{Sat}m_E}{r^2} = m_{Sat}\frac{v^2}{r}.$$

This equation seems to have two unknowns, r and v. But we know that v must be such that the satellite revolves around the Earth with the same period that the Earth rotates on its axis, namely once in 24 hours. Thus the speed of the satellite must be

$$v = \frac{2\pi r}{T}$$

where $T = 1\text{ day} = (24\text{ h})(3600\text{ s/h}) = 86{,}400\text{ s}$. We put this into the first equation above and obtain (after canceling m_{Sat} on both sides):

$$G\frac{m_E}{r^2} = \frac{(2\pi r)^2}{rT^2}.$$

We solve for r:

$$r^3 = \frac{Gm_ET^2}{4\pi^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(86{,}400\text{ s})^2}{4\pi^2}$$
$$= 7.54\times10^{22}\text{ m}^3,$$

and, taking the cube root, $r = 4.23\times10^7$ m, or 42,300 km from the Earth's center. We subtract the Earth's radius of 6380 km to find that the satellite must orbit about 36,000 km (about 6 r_E) above the Earth's surface.
(*b*) We solve Eq. 5–6 for v:

$$v = \sqrt{\frac{Gm_E}{r}} = \sqrt{\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(4.23\times10^7\text{ m})}} = 3070\text{ m/s}.$$

We get the same result if we use $v = 2\pi r/T$.

Apparent weight and weightlessness

People and other objects in a satellite circling the Earth are said to experience apparent weightlessness. Before tackling the case of a satellite, however, let us first look at the simpler case of a falling elevator. In Fig. 5–25a, we see an elevator at rest with a bag hanging from a spring scale. The scale reading

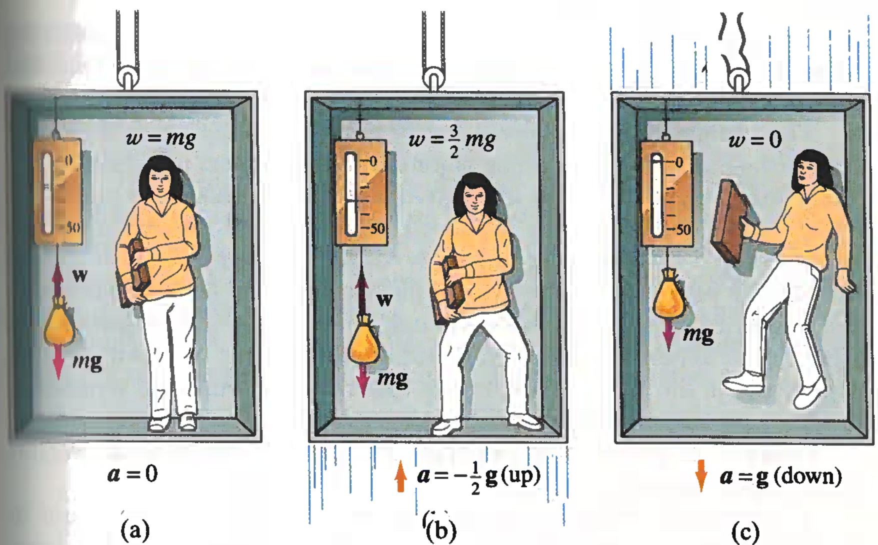


FIGURE 5–25 (a) An object in an elevator at rest exerts a force on a spring scale equal to its weight. (b) In an elevator accelerating upward at $\frac{1}{2}g$, the object's apparent weight is $1\frac{1}{2}$ times larger. (c) In a freely falling elevator, the object experiences "weightlessness."

indicates the downward force exerted on it by the bag. This force, exerted *on* the scale, is equal and opposite to the force exerted *by* the scale upward on the bag. We call this force **w**. (Similarly, if you were standing on a scale in an elevator, the normal force the scale exerts on you would be its reading.) Since the mass, m, is not accelerating, we apply $\Sigma F = ma$ to the bag and obtain

$$w - mg = 0,$$

where mg is the weight of the bag. Thus, $w = mg$, and since the scale indicates the force w exerted on it by the bag, it registers a force equal to the weight of the bag as we expect. If, now, the elevator has an acceleration, a, then applying $\Sigma F = ma$ to the bag, we have

$$w - mg = ma.$$

Solving for w, we have

$$w = mg + ma.$$

We have chosen the positive direction up. Thus, if the acceleration a is up, a is positive; and the scale, which measures w, will read more than mg. We call w the *apparent weight* of the bag, which here is greater than its actual weight (mg). If the elevator accelerates downward, a will be negative and w, the apparent weight, will be less than mg.

For example, if the elevator's acceleration is $\frac{1}{2}g$ upward, then we find $w = mg + m(\frac{1}{2}g) = \frac{3}{2}mg$. That is, the scale reads $1\frac{1}{2}$ times the actual weight (Fig. 5–25b). The apparent weight of the bag is $1\frac{1}{2}$ times its real weight. The same is true of the person: her apparent weight (equal to the normal force exerted on her by the elevator floor) is $1\frac{1}{2}$ times her real weight. We can say that she is experiencing $1\frac{1}{2}$ g's, just as astronauts experience so many g's at a rocket's launch.

If, instead, the elevator's acceleration is $-\frac{1}{2}g$ (downward), then $w = mg - \frac{1}{2}mg = \frac{1}{2}mg$. That is, the scale reads one half the actual weight. If the elevator is in *free fall* (for example, if the cables break), then $a = -g$ and $w = mg - mg = 0$. The scale reads zero! (See Fig. 5–25c.) The bag seems weightless. If the person in the elevator let go of a pencil, say, it would not fall to the floor. True, the pencil would be falling with acceleration g. But so would the floor of the elevator and the person. The pencil would hover right

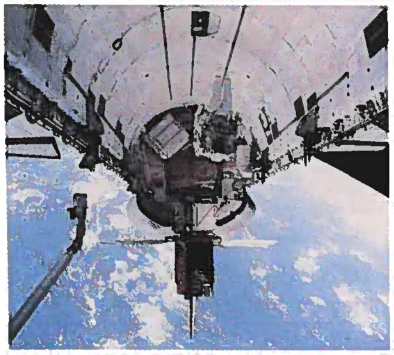


FIGURE 5–26 The astronaut seems not to know which end is up. Note edge of Earth on the right. (Turn photo upside down for another view.)

in front of the person. This phenomenon is called *apparent weightlessness* because, in fact, gravity is still acting on the object and its weight is still mg. The objects seem weightless only because the elevator is in free fall.

The "weightlessness" experienced by people in a satellite orbit close to the Earth (Fig. 5–26) is the same apparent weightlessness experienced in a freely falling elevator. It may seem strange, at first, to think of a satellite as freely falling. But a satellite is indeed falling toward the Earth, as was shown in Fig. 5–24. The force of gravity causes it to "fall" out of its natural straight-line path. The acceleration of the satellite must be the acceleration due to gravity at that point, since the only force acting on it is gravity. (We used this to obtain Eq. 5–6.) Thus, although the force of gravity acts on objects within the satellite, the objects experience an apparent weightlessness because they, and the satellite, are accelerating as in free fall.

Figure 5–27 shows some examples of "free fall," or apparent weightlessness, experienced by people on Earth for brief moments.

A completely different situation occurs when a spacecraft is out in space far from the Earth, the Moon, and other attracting bodies. The force of gravity due to the Earth and other heavenly bodies will then be quite small because of the distances involved, and persons in such a spacecraft will experience real weightlessness.

The effects on human beings of weightlessness (whether real or apparent makes no difference) are interesting. In ordinary circumstances, for example, people can become quite tired holding out their arms horizontally. But for a person experiencing weightlessness, no effort is needed. The arms will just "float" there, since there is no sensation of weight. This effect has many applications in athletics (Fig. 5–27). During a jump or a dive, while on a trampoline, and even between strides while running, a person is experiencing apparent weightlessness or free fall, although only for a short time. During these brief periods, limbs can be moved much more easily, since only inertia needs to be overcome. The loss of control because of lack of contact with the ground is compensated for by the increased mobility. Prolonged weightlessness in space, however, can have harmful effects on health. Red blood cells diminish in number, blood collects in the thorax, bones lose calcium and become brittle, and muscles lose their tone. These effects are being carefully studied.

FIGURE 5–27 Experiencing weightlessness on Earth.

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*5–9 Kepler's Laws and Newton's Synthesis

More than a half century before Newton proposed his three laws of motion and his law of universal gravitation, the German astronomer Johannes Kepler (1571–1630) had written a number of astronomical works in which we can find a detailed description of the motion of the planets about the Sun. Kepler's work resulted in part from the many years he spent examining data collected by Tycho Brahe (1546–1601) on the positions of the planets in their motion through the heavens. Among Kepler's writings were three findings that we now refer to as **Kepler's laws of planetary motion.** These are summarized as follows, with additional explanation in Fig. 5–28:

Kepler's first law: The path of each planet about the Sun is an ellipse (Fig. 5–28a) with the Sun at one focus.

Kepler's second law: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal periods of time (Fig. 5–28b).

Kepler's third law: The ratio of the squares of the periods (the time needed for one revolution about the Sun) of any two planets revolving about the Sun is equal to the ratio of the cubes of their mean distances from the Sun. That is, if T_1 and T_2 represent the periods for any two planets, and r_1 and r_2 represent their mean distances from the Sun, then

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3.$$

We can rewrite this as

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_2^2}$$

meaning that r^3/T^2 should be the same for each planet. (Present-day data are given in Table 5–1; see the last column.)

Newton was able to show that Kepler's laws could be derived mathematically from the law of universal gravitation and the laws of motion. He also showed that among the reasonable possibilities for the gravitational force law, only one that depends on the inverse square of the distance is fully

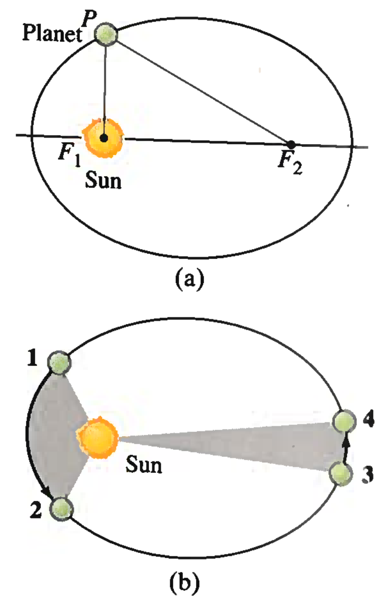

FIGURE 5–28 (a) Kepler's first law. An ellipse is a closed curve such that the sum of the distances from any point P on the curve to two fixed points (called the foci, F_1 and F_2) remains constant. That is, the sum of the distances $F_1P + F_2P$ is the same for all points on the curve. A circle is a special case of an ellipse in which the two foci coincide, at the center of the circle. (b) Kepler's second law. The two shaded regions have equal areas. The planet moves from point 1 to point 2 in the same time as it takes to move from point 3 to point 4. Planets move fastest in that part of their orbit where they are closest to the Sun.

TABLE 5–1 Planetary Data Applied to Kepler's Third Law

Planet	Mean distance from Sun, r (10^6 km)	Period, T (Earth years)	r^3/T^2 (10^{24} km^3/yr^2)
Mercury	57.9	0.241	3.34
Venus	108.2	0.615	3.35
Earth	149.6	1.0	3.35
Mars	227.9	1.88	3.35
Jupiter	778.3	11.86	3.35
Saturn	1427	29.5	3.34
Uranus	2870	84.0	3.35
Neptune	4497	165	3.34
Pluto	5900	248	3.33

consistent with all three of Kepler's laws. He thus used Kepler's laws as evidence for his law of universal gravitation, Eq. 5–4.

Derivation of Kepler's third law

Kepler's third law is the easiest to derive, and we do it here for the special case of a circular orbit. (Most of the planetary orbits are fairly close to a circle, which is a special case of an ellipse.) First, we write down Newton's second law of motion, $\Sigma F = ma$. Then for ΣF we substitute the law of universal gravitation, Eq. 5–4, and for a the centripetal acceleration, v^2/r:

$$\Sigma F = ma$$

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{v_1^2}{r_1}.$$

Here m_1 is the mass of a particular planet, r_1 its mean distance from the Sun, and v_1 its average speed in orbit. Then M_2 in Eq. 5–4 is taken to be M_S, the mass of the Sun, since it is the gravitational attraction of the Sun that keeps each planet in its orbit. Now the period T_1 of the planet is the time required for one complete orbit, a distance equal to $2\pi r_1$, the circumference of a circle. Thus,

$$v_1 = \frac{2\pi r_1}{T_1}.$$

We substitute this formula for v_1 into the equation above:

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{4\pi^2 r_1}{T_1^2}.$$

We rearrange to get

$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S}. \qquad \textbf{(5–7a)}$$

We derived this for planet 1 (say, Mars). The same derivation would apply for a second planet (say, Saturn):

$$\frac{T_2^2}{r_2^3} = \frac{4\pi^2}{GM_S},$$

where T_2 and r_2 are the period and orbit radius, respectively, for the second planet. Since the right sides of the two previous equations are equal, we have $T_1^2/r_1^3 = T_2^2/r_2^3$ or, rearranging,

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3, \qquad \textbf{(5–7b)}$$

which is Kepler's third law.

The derivations of Eqs. 5–7a and b (Kepler's third law) are general enough to be applied to other systems. For example, we could determine the mass of the Earth from Eq. 5–7a using the period of the Moon about the Earth and the Moon's distance from the Earth, or the mass of Jupiter from the period and distance of one of its moons (this is indeed how masses are determined; see the Problems). We can also use Eqs. 5–7a and b to compare objects that orbit other attracting centers, such as the Moon and a weather satellite orbiting Earth. But be careful not to use Eq. 5–7 to compare, say, the Moon's orbit to the orbit of Mars because they depend on different attracting centers.

Careful: A common error using Kepler's third law

In the following examples, we assume the orbits are circles, although it is not quite true in general.

EXAMPLE 5–16 **Where is Mars?** Mars' period (its "year") was first noted by Kepler to be about 684 days (Earth-days), which is (687 d/365 d) = 1.88 yr. Determine the distance of Mars from the Sun using the Earth as a reference.

SOLUTION The period of the Earth is $T_E = 1$ yr, and the distance of Earth from the Sun is $r_{ES} = 1.50 \times 10^{11}$ m. From Kepler's third law (Eq. 5–7b):

$$\frac{r_{MS}}{r_{ES}} = \left(\frac{T_M}{T_E}\right)^{2/3} = \left(\frac{1.88\text{ yr}}{1\text{ yr}}\right)^{2/3} = 1.52$$

So Mars is 1.52 the Earth's distance from the Sun, or 2.28×10^{11} m.

EXAMPLE 5–17 **The Sun's mass determined.** Determine the mass of the Sun given the Earth's distance from the Sun as $r_{ES} = 1.5 \times 10^{11}$ m.

➥ **PHYSICS APPLIED**

Determining the Sun's mass

SOLUTION We can use Eq. 5–7a and solve for M_S:

$$M_S = \frac{4\pi^2 r_{ES}^3}{GT_E^2} = \frac{4\pi^2(1.5 \times 10^{11}\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(3.16 \times 10^7\text{ s})^2}$$
$$= 2.0 \times 10^{30}\text{ kg}$$

where we used the fact that

$$T_E = 1\text{ yr} = (365\tfrac{1}{4}\text{ d})(24\text{ h/d})(3600\text{ s/h}) = 3.16 \times 10^7\text{ s}.$$

EXAMPLE 5–18 **ESTIMATE** **Geosynchronous satellite, simplified.** A *geosynchronous* satellite of the Earth (as mentioned in Example 5–15) is one that stays above the same point on the equator of the Earth. Estimate the height above the Earth's surface needed for a geosynchronous weather satellite. (This is to be a "lunchtime" calculation done on a napkin, without calculator, as compared to our earlier calculation in Example 5–15.)

SOLUTION To use Kepler's third law we must compare the satellite to some other object that orbits Earth. The simplest choice is the Moon because we know its period and distance. The Moon's period is about $T_M \approx 27$ d and its distance from the earth about $r_{ME} \approx 380{,}000$ km. The period of the weather satellite needs to be $T_{Sat} = 1$ d so that it stays above the same place on the Earth. Hence,

$$r_{Sat} = r_{ME}\left(\frac{T_{Sat}}{T_M}\right)^{2/3} = r_{ME}\left(\frac{1\text{ d}}{27\text{ d}}\right)^{2/3} = r_{ME}\left(\frac{1}{3}\right)^2 = \frac{r_{ME}}{9}.$$

(How nice the Moon's approximate period turns out to be a perfect cube.) A geosynchronous satellite must be $\frac{1}{9}$ the distance to the Moon, which is 42,000 km from the center of the Earth or 36,000 km above the Earth's surface. This is about 6 Earth radii high.

Accurate measurements on the orbits of the planets indicated that they did not precisely follow Kepler's laws. For example, slight deviations from perfectly elliptical orbits were observed. Newton was aware that this was to be expected from the law of universal gravitation ("every body in the universe attracts every other body…") because each planet exerts a gravitational force on the other planets. Since the mass of the Sun is much greater than that of any planet, the force on one planet due to any other planet will be small in comparison to the force on it due to the Sun. (The derivation of perfectly elliptical orbits ignores the forces due to other planets.) But because of this small force, each planetary orbit should depart from a perfect ellipse, especially when a second planet is fairly close to it. Such deviations, or **perturbations**, as they are called, from perfect ellipses are indeed observed. In fact, Newton's recognition of perturbations in the orbit of Saturn was a hint that helped him formulate the law of universal gravitation, that all bodies attract gravitationally. Observation of other perturbations later led to the discovery of Neptune and Pluto. Deviations in the orbit of Uranus, for example, could not be accounted for by perturbations due to the other known planets. Careful calculation in the nineteenth century indicated that these deviations could be accounted for if there were another planet farther out in the solar system. The position of this planet was predicted from the deviations in the orbit of Uranus, and telescopes focused on that region of the sky quickly found it; the new planet was called Neptune. Similar but much smaller perturbations of Neptune's orbit led to the discovery of Pluto in 1930.

PHYSICS APPLIED

Perturbations and discovery of planets

More recently, in 1996, planets revolving about distant stars (Fig. 5–29) were inferred from the regular "wobble" of each star due to the gravitational attraction of the revolving planet.

PHYSICS APPLIED

Planets around other stars

Newton's synthesis

The development by Newton of the law of universal gravitation and the three laws of motion was a major intellectual achievement. For with these laws, Newton was able to describe the motion of objects on Earth and in the heavens. The motions of heavenly bodies and bodies on Earth were seen to follow the same laws (something not previously recognized generally, although Galileo and Descartes had argued in its favor). For this reason (and also because Newton integrated the results of earlier workers into his system), we sometimes speak of Newton's "synthesis."

Causality

Newton's work was so encompassing that it constituted a theory of the universe, and influenced philosophy and other fields. The laws formulated by Newton are referred to as **causal laws**. By **causality** we mean the idea that one occurrence can cause another. We have repeatedly observed, for example, that when a rock strikes a window, the window almost immediately breaks. We infer that the rock *caused* the window to break. This idea of "cause and effect" took on more forceful meaning with Newton's laws. For the motion—or rather the acceleration—of any object was seen to be *caused* by the net force acting on it. As a result, the universe came to be pictured by many scientists and philosophers as a big machine whose parts

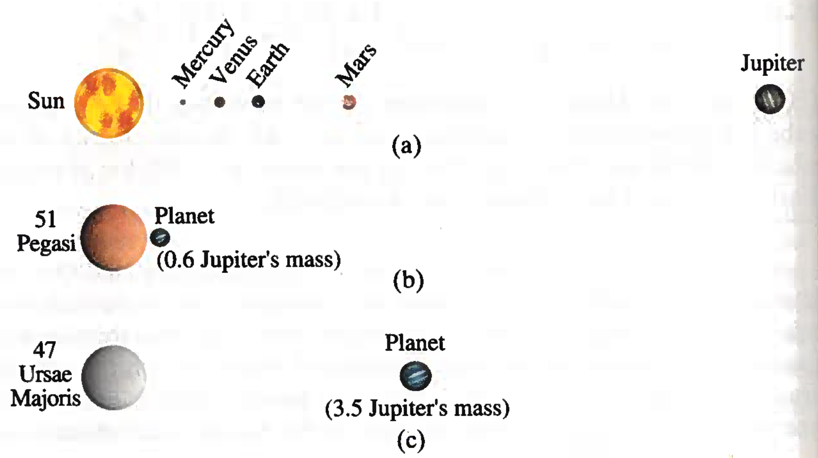

FIGURE 5–29 Our solar system (a), compared to recently discovered planets orbitting (b) the star 51 Pegasi, and (c) the star 47 Ursae Majoris.

move in a predetermined way—according to natural laws. However, this *deterministic* view of the universe had to be modified by scientists in the twentieth century, as we shall see in Chapters 27 and 28.

5–10 Types of Forces in Nature

We have already discussed that Newton's law of universal gravitation, Eq. 5–4, describes how a particular type of force—gravity—depends on the distance between, and masses of, the objects involved. Newton's second law, $\mathbf{F} = m\mathbf{a}$, on the other hand, tells how a body will accelerate due to *any* type of force. But what are the types of forces that occur in nature besides gravity?

In the twentieth century, physicists came to recognize four different fundamental forces in nature: (1) the gravitational force; (2) the electromagnetic force (we shall see later that electric and magnetic forces are intimately related); (3) the strong nuclear force; and (4) the weak nuclear force. In this chapter, we discussed the gravitational force in detail. The nature of the electromagnetic force will be discussed in detail in Chapters 16 to 22. The strong and weak nuclear forces, which are discussed in Chapters 30 to 32, operate at the level of the atomic nucleus, and although they manifest themselves in such phenomena as radioactivity and nuclear energy, they are much less obvious in our daily lives.

Physicists have been working on theories that would unify these four forces—that is, to consider some or all of these forces as different manifestations of the same basic force. So far, the electromagnetic and weak nuclear forces have been theoretically united to form *electroweak* theory, in which the electromagnetic and weak forces are seen as two different manifestations of a single *electroweak force*. Attempts to further unify the forces, such as in *grand unified theories* (GUT), are hot research topics today.

Electroweak and GUT

But where do everyday forces fit into this scheme? Ordinary forces, other than gravity, such as pushes, pulls, and other contact forces like the normal force and friction, are today considered to be due to the electromagnetic force acting at the atomic level. For example, the force your fingers exert on a pencil is the result of electrical repulsion between the outer electrons of the atoms of your finger and those of the pencil.

Everyday forces are gravity and electromagnetic

PROBLEM SOLVING Uniform Circular Motion and Gravity

1. Draw a free-body diagram, showing all the forces acting on the object under consideration. If more than one object is involved, draw a separate free-body diagram for each. Be sure you can identify the source of each force (tension in a cord, Earth's gravity, friction, normal force, and so on), so you don't put in something that isn't there.
2. Determine which of these forces, or which of their components, act to provide the centripetal (radial) acceleration—that is, all the forces or components that act radially, toward or away from the center of the circular path. The sum of these forces (or components) provides the centripetal acceleration, $a_R = v^2/r$.
3. Choose a coordinate system, and positive and negative directions, and apply Newton's second law to the radial direction:
$$\Sigma F_R = ma_R = m\frac{v^2}{r}.$$
4. For the gravitational force, use Newton's law of universal gravitation (if the object is close to the Earth's surface, you can simply use $m\mathbf{g}$); be sure to use the correct value for r. Remember that for large objects, such as the Earth or Moon, r is measured from the *center* of the sphere, not from the surface.

SUMMARY

An object moving in a circle of radius r with constant speed v is said to be in **uniform circular motion**. It has a **centripetal acceleration** a_R that is directed radially toward the center of the circle (also called **radial acceleration**), whose magnitude is

$$a_R = \frac{v^2}{r}.$$

The direction of the velocity vector and that of the acceleration $\mathbf{a}_R$ are continually changing in direction, but are perpendicular to each other at each moment.

A force is needed to keep a particle revolving in a circle, and the direction of this force is toward the center of the circle. This force may be due to gravity, to tension in a cord, to a component of the normal force, or other type of force.

Newton's **law of universal gravitation** states that every particle in the universe attracts every other particle with a force proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$F = G\frac{m_1 m_2}{r^2}.$$

The direction of this force is along the line joining the two particles. It is this gravitational force that keeps the Moon revolving around the Earth, and the planets revolving around the Sun.

Satellites revolving around the Earth are acted on by gravity, but "stay up" because of their high tangential speed.

QUESTIONS

1. It is sometimes said that water is removed from clothes in a spin dryer by centrifugal force throwing the water outward. Is this correct?
2. Will the acceleration of a car be the same when it travels around a sharp curve at 60 km/h as when it travels around a gentle curve at the same speed? Explain.
3. Suppose a car moves at constant speed along a mountain road. At what places does it exert the greatest and least forces on the road: (*a*) at the top of a hill, (*b*) at a dip between two hills, (*c*) on a level stretch near the bottom of a hill?
4. Describe all the forces acting on a child riding a horse on a merry-go-round. Which of these forces provides the centripetal acceleration of the child?
5. A bucket of water can be whirled in a vertical circle without the water spilling out, even at the top of the circle when the bucket is upside down. Explain.
6. Does an apple exert a gravitational force on the Earth? If so, how large a force? Consider an apple (*a*) attached to a tree, and (*b*) falling.
7. If the Earth's mass were double what it is, in what ways would the Moon's orbit be different?
8. Describe how careful measurements of the variation in g in the vicinity of an ore deposit might be used to estimate the amount of ore present.
9. When will your apparent weight be the greatest, as measured by a scale in a moving elevator: when the elevator (*a*) accelerates downward, (*b*) accelerates upward, (*c*) is in free fall, (*d*) moves upward at constant speed? In which case would your weight be the least? When would it be the same as when you are on the ground?
10. The Earth is not completely spherical but bulges outward at the equator. Why?
11. An antenna loosens and becomes detached from a satellite in a circular orbit around the Earth. Describe the antenna's motion subsequently. If it will land on Earth, describe where; if not, describe how it could be made to land on Earth.
12. Astronauts who spend long periods in outer space could be adversely affected by weightlessness. One way to simulate gravity is to shape the spaceship like a cylindrical shell that rotates, with the astronauts walking on the inside surface (Fig. 5–30). Explain how this simulates gravity. Consider (*a*) how objects fall, (*b*) the force we feel on our feet, and (*c*) any other aspects of gravity you can think of.

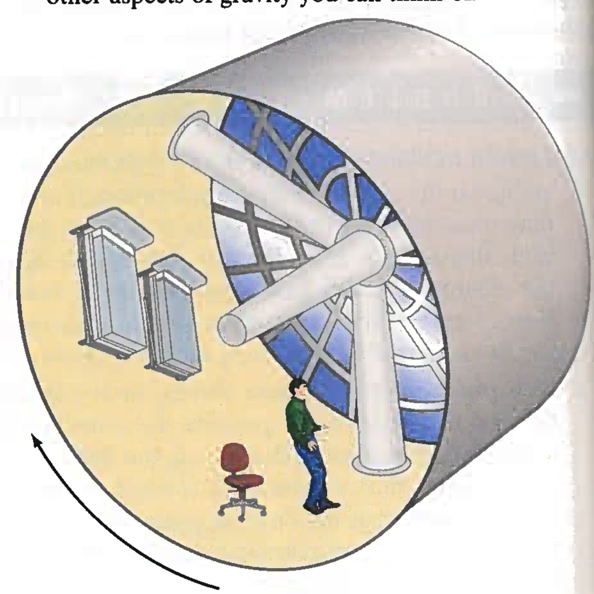

FIGURE 5–30 Question 12 and Problem 41.

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Problem 41.

13. Which pulls harder gravitationally, the Earth on the Moon, or the Moon on the Earth? Which accelerates more?

14. How many "accelerators" do you have in your car? There are at least three controls in the car which can be used to cause the car to accelerate. What are they? What accelerations do they produce?

15. A child on a sled comes flying over the crest of a small hill, as shown in Fig. 5–31. His sled does not leave the ground (he does not achieve "air"), but he feels the normal force between his chest and the sled decrease as he goes over the hill. Explain this decrease using Newton's second law.

16. People sometimes ask, "What keeps a satellite up in its orbit around the Earth?" How would you respond?

17. Explain how a runner experiences "free fall" or "apparent weightlessness" between steps.

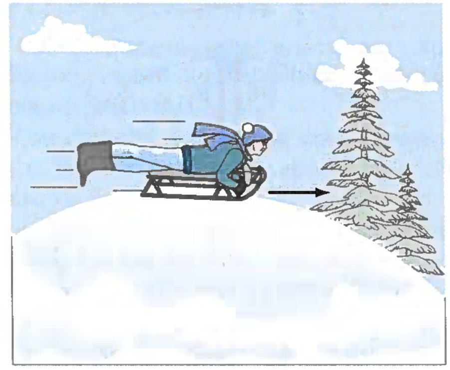


FIGURE 5–31 Question 15.

18. The Earth moves faster in its orbit around the Sun in winter than in summer. Is it closer to the Sun in summer or in winter? Does this affect the seasons? Explain. [*Note*: This is not much of a factor in explaining the seasons—the main factor is the tilt of the Earth's axis relative to the plane of its orbit.]

PROBLEMS

SECTIONS 5–1 TO 5–3

1. (I) A jet plane traveling 1800 km/h (500 m/s) pulls out of a dive by moving in an arc of radius 6.00 km. What is the plane's acceleration in *g*'s?

2. (I) A child on a merry-go-round is moving with a speed of 1.35 m/s when 1.20 m from the center of the merry-go-round. Calculate (*a*) the centripetal acceleration of the child, and (*b*) the net horizontal force exerted on the child (mass = 25.0 kg).

3. (I) Calculate the centripetal acceleration of the Earth in its orbit around the Sun and the net force exerted on the Earth. What exerts this force on the Earth? Assume that the Earth's orbit is a circle of radius 1.50×10^{11} m.

4. (I) A horizontal force of 280 N is exerted on a 2.0-kg discus as it is rotated uniformly in a horizontal circle (at arms length) of radius 1.00 m. Calculate the speed of the discus.

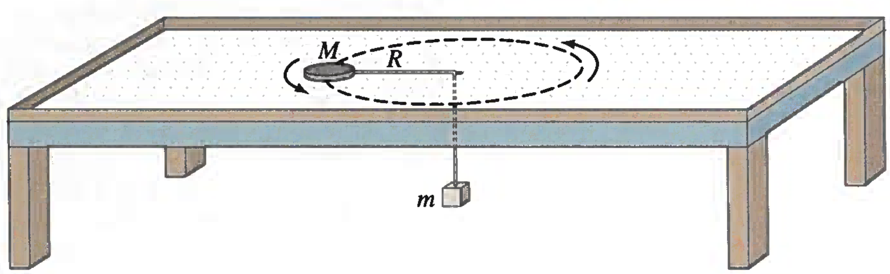
5. (II) A flat puck (mass *M*) is rotated in a circle on a frictionless air hockey tabletop, and is held in this orbit by a light cord which is connected to a dangling mass (mass *m*) through the central hole as shown in Fig. 5–32. Show that the speed of the puck is given by

$$v = \sqrt{\frac{mgR}{M}}.$$

6. (II) A 0.40-kg ball, attached to the end of a horizontal cord, is rotated in a circle of radius 1.3 m on a frictionless horizontal surface. If the cord will break when the tension in it exceeds 60 N, what is the maximum speed the ball can have? How would your answer be affected if there were friction?

7. (II) What is the maximum speed with which a 1050-kg car can round a turn of radius 70 m on a flat road if the coefficient of friction between tires and road is 0.80? Is this result independent of the mass of the car?

FIGURE 5–32 Problem 5.



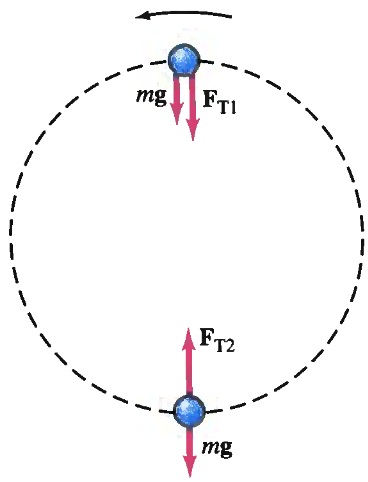


FIGURE 5–33 Problem 8.

FIGURE 5–34 Problem 12.

8. (II) A ball on the end of a string is cleverly revolved at a uniform rate in a vertical circle of radius 85.0 cm, as shown in Fig. 5–33. If its speed is 4.15 m/s and its mass is 0.300 kg, calculate the tension in the string when the ball is (*a*) at the top of its path, and (*b*) at the bottom of its path.

9. (II) How large must the coefficient of friction be between the tires and the road if a car is to round a level curve of radius 85 m at a speed of 95 km/h?

10. (II) A device for training astronauts and jet fighter pilots is designed to rotate the trainee in a horizontal circle of radius 10.0 m. If the force felt by the trainee is 7.75 times her own weight, how fast is she rotating? Express your answer in both m/s and rev/s.

11. (II) A coin is placed 11.0 cm from the axis of a rotating turntable of variable speed. When the speed of the turntable is slowly increased, the coin remains fixed on the turntable until a rate of 36 rpm is reached, at which point the coin slides off. What is the coefficient of static friction between the coin and the turntable?

12. (II) At what minimum speed must a roller coaster be traveling when upside down at the top of a circle (Fig. 5–34) if the passengers are not to fall out? Assume a radius of curvature of 8.6 m.

13. (II) A 1000-kg sports car moving at 20 m/s crosses the rounded top of a hill (radius = 100 m). Determine (*a*) the normal force on the car, (*b*) the normal force on the 70-kg driver, and (*c*) the car speed at which the normal force equals zero.

14. (II) How many revolutions per minute would a 15-m-diameter Ferris wheel need to make for the passengers to feel "weightless" at the topmost point of the trip?

15. (II) Use dimensional analysis (see Appendix B) to check the form for the centripetal acceleration, $a_R = v^2/r$.

16. (II) Two masses, m_1 and m_2, connected to each other and to a central post by cords, as shown in Fig. 5–35, rotate about the post at a frequency f (revolutions per second) on a frictionless horizontal surface at distances r_1 and r_2, respectively, from the post. Derive an algebraic expression for the tension in each segment of the cord.

17. (II) A 1200-kg car rounds a curve of radius 70 m banked at an angle of 12°. If the car is traveling at 90 km/h, will a friction force be required? If so, how much and in what direction?

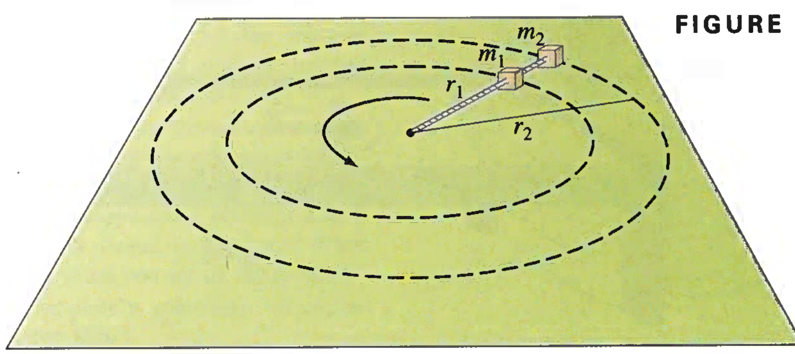


FIGURE 5–35 Problem 16.

FIGURE 5–36 Problem 18.

18. (II) In a "Rotor-ride" at a carnival, people pay money to be rotated in a vertical cylindrically walled "room." (See Fig. 5–36.) If the room radius is 5.0 m, and the rotation frequency is 0.50 revolutions per second when the floor drops out, what is the minimum coefficient of static friction so that the people will not slip down? People describe this ride by saying they were being "pressed against the wall." Is this true? Is there really an outward force pressing them against the wall? If so, what is its source? If not, what is the proper description of their situation (besides "scary")? [*Hint*: First draw the free-body diagram for a person.]

19. (III) Redo Example 5–3, precisely this time, by not ignoring the weight of the ball. In particular, find the magnitude of $\mathbf{F}_T$, and the angle it makes with the horizontal. [*Hint*: Set the horizontal component of $\mathbf{F}_T$ equal to ma_R; also, since there is no vertical motion, what can you say about the vertical component of $\mathbf{F}_T$?]

20. (III) If a curve with a radius of 80 m is perfectly banked for a car traveling 70 km/h, what must be the coefficient of static friction for a car not to skid when traveling at 90 km/h?

21. (III) A pilot performs an evasive maneuver by diving vertically at 310 m/s. If he can withstand an acceleration of 9.0 g's without blacking out, at what altitude must he begin to pull out of the dive to avoid crashing into the sea?

SECTION 5–4

22. (I) Determine the tangential and centripetal components of the net force exerted on the car (by the ground) in Example 5–9 when its speed is 30 m/s. The car's mass is 1000 kg.

23. (II) A car at the Indianapolis-500 accelerates uniformly from the pit area, going from rest to 320 km/h in a semicircular arc with a radius of 200 m. Determine the tangential and radial acceleration of the car when it is halfway through the turn, assuming constant tangential acceleration. If the curve were flat, what would the coefficient of static friction have to be between the tires and the roadbed to provide this acceleration with no slipping or skidding?

24. (III) A particle revolves in a horizontal circle of radius 2.70 m. At a particular instant, its acceleration is 1.05 m/s², in a direction that makes an angle of 32.0° to its direction of motion. Determine its speed (*a*) at this moment, and (*b*) 2.00 s later, assuming constant tangential acceleration.

SECTIONS 5–6 AND 5–7

25. (I) Calculate the force of gravity on a spacecraft 12,800 km (2 earth radii) above the Earth's surface if its mass is 1400 kg.

26. (I) Calculate the acceleration due to gravity on the Moon. The Moon's radius is about 1.74×10^6 m and its mass is 7.35×10^{22} kg.

27. (I) A hypothetical planet has a radius 2.5 times that of Earth, but has the same mass. What is the acceleration due to gravity near its surface?

28. (I) A hypothetical planet has a mass 2.5 times that of Earth, but the same radius. What is g near its surface?

29. (I) At the surface of a certain planet, the gravitational acceleration g has a magnitude of 12.0 m/s². A 2.10-kg brass ball is transported to this planet. What is (*a*) the mass of the brass ball on the Earth and on the planet, and (*b*) the weight of the brass ball on the Earth and on the planet?

30. (II) You are explaining to friends why astronauts feel weightless orbiting in the space shuttle, and they respond that they thought gravity was just a lot weaker up there. Convince them and yourself that it isn't so by calculating how much weaker gravity is 300 km above the Earth's surface.

31. (II) An exotic finish to massive stars is that of a neutron star, which might have as much as five times the mass of our Sun packed into a sphere about 10 km in radius! Estimate the surface gravity on this monster.

32. (II) What is the distance from the Earth's center to a point outside the Earth where the gravitational acceleration due to the Earth is $\frac{1}{10}$ of its value at the Earth's surface?

33. (II) A typical white dwarf star, which once was an average star like our Sun but is now in the last stage of its evolution, is the size of our Moon but has the mass of our Sun. What is the surface gravity on this star?

34. (II) Calculate the effective value of g, the acceleration of gravity, at (*a*) 3200 m, and (*b*) 3200 km, above the Earth's surface.

35. (II) Four 7.5-kg spheres are located at the corners of a square of side 0.60 m. Calculate the magnitude and direction of the gravitational force on one sphere due to the other three.

36. (II) Every few hundred years most of the planets line up on the same side of the Sun. Calculate the total force on the Earth due to Venus, Jupiter, and Saturn, assuming all four planets are in a line, Fig. 5–37. The masses are respectively $M_V = 0.815\,M_E$, $M_J = 318\,M_E$, $M_S = 95.1\,M_E$, and their mean distances from the Sun are 108, 150, 778, and 1430 million km.

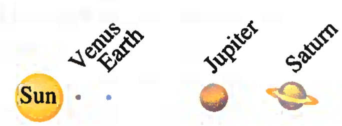


FIGURE 5–37 Problem 36 (not to scale).

37. (II) Given that the acceleration of gravity at the surface of Mars is 0.38 of what it is on Earth, and that Mars' radius is 3400 km, determine the mass of Mars.,

38. (III) Determine the mass of the Sun using the known value for the period of the Earth and its distance from the Sun. [*Note*: Compare your answer to that obtained using Kepler's laws, Example 5–17.]

SECTION 5–8

39. (I) Calculate the velocity of a satellite moving in a stable circular orbit about the Earth at a height of 3600 km.

40. (II) A 17.0-kg monkey hangs from a cord suspended from the ceiling of an elevator. The cord can withstand a tension of 220 N and breaks as the elevator accelerates. What was the elevator's minimum acceleration (magnitude and direction)?

41. (II) At what rate must the cylindrical spaceship of Fig. 5–30 rotate (see Question 12), if occupants are to experience simulated gravity of $\frac{1}{2}g$? Assume the spaceship's diameter is 32 m, and give your answer as the time needed for one revolution.

42. (II) Determine the time it takes for a satellite to orbit the Earth in a circular "near-Earth" orbit. The definition of "near-Earth" orbit is one which is at a height above the surface of the Earth which is small compared to the radius of the Earth, so that you may take the acceleration due to gravity as essentially the same as that on the surface. Does your result depend on the mass of the satellite?

43. (II) During an *Apollo* lunar landing mission, the command module continued to orbit the Moon at an altitude of about 100 km. How long did it take to go around the Moon once?

44. (II) What will a spring scale read for the weight of a 58-kg woman in an elevator that moves (*a*) with constant upward speed of 6.0 m/s, (*b*) with constant downward speed of 6.0 m/s, (*c*) with upward acceleration of 0.33 *g*, (*d*) with downward acceleration 0.33 *g*, and (*e*) in free fall?

45. (II) The rings of Saturn are composed of chunks of ice that orbit the planet. The inner radius of the rings is 73,000 km, while the outer radius is 170,000 km. Find the period of an orbiting chunk of ice at the inner radius and the period of a chunk at the outer radius. Compare your numbers with Saturn's mean rotation period of 10 hours and 39 minutes. The mass of Saturn is 5.69×10^{26} kg.

46. (II) A Ferris wheel, 24.0 m in diameter, rotates once every 12.5 s (see Fig. 5–10). What is the fractional change in a person's apparent weight (*a*) at the top, and (*b*) at the bottom, as compared to her weight at rest?

47. (II) What is the apparent weight of a 70-kg astronaut 4200 km from the center of the Earth's Moon in a space vehicle (*a*) moving at constant velocity, and (*b*) accelerating toward the Moon at 2.9 m/s^2? State "direction" in each case.

48. (II) Describe a general procedure to determine the mass of a planet from observations on the orbit of one of its satellites.

49. (II) Suppose that a binary star system consists of two stars of equal mass. They are observed to be separated by 360 million km and to take 5.0 Earth years to orbit about a point midway between them. What is the mass of each?

50. (III) (*a*) Show that if a satellite orbits very near the surface of a planet with period T, the density (mass/volume) of the planet is $\rho = 3\pi/GT^2$. (*b*) Estimate the density of the Earth, given that a satellite near the surface orbits with a period of about 90 minutes.

*SECTION 5–9

*51. (I) Use Kepler's laws and the period of the Moon (27.4 d) to determine the period of an artificial satellite orbiting very near the Earth's surface.

*52. (I) The asteroid Icarus, though only a few hundred meters across, orbits the Sun like the other planets. Its period is about 410 d. What is its mean distance from the Sun?

*53. (I) Neptune is an average distance of 4.5×10^9 km from the Sun. Estimate the length of the Neptunian year given that the Earth is 1.50×10^8 km from the Sun on the average.

*54. (II) Determine the mass of the Earth from the known period and distance of the Moon.

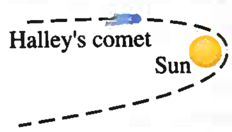


FIGURE 5–38 Problem 55.

*55. (II) Halley's comet orbits the Sun roughly once every 76 years. It comes very close to the surface of the Sun on its closest approach (Fig. 5–38). Roughly how far out from the Sun is it at its farthest? Is it still "in" the Solar System? What planet's orbit is nearest when it is out there? [*Hint*: The "mean distance" in Kepler's third law is $\frac{1}{2}$ the sum of the nearest and farthest distance from the Sun.]

*56. (II) The Sun rotates about the center of the Milky Way Galaxy (Fig. 5–39) at a distance of about 30,000 light years from the center (1 ly = 9.5×10^{15} m). If it takes about 200 million years to make one rotation, estimate the mass of our galaxy. Assume that the mass distribution of our galaxy is concentrated mostly in a central uniform sphere. If all the stars had about the mass of our Sun (2×10^{30} kg), how many stars would there be in our galaxy?

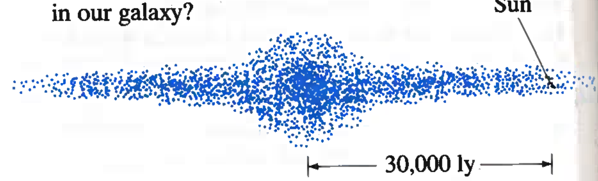


FIGURE 5–39 Edge-on view of our galaxy. Problem 56.

TABLE 5–2 Principal Moons of Jupiter (Problems 57 and 58)

Moon	Mass (kg)	Period (Earth days)	Mean distance from Jupiter (km)
Io	8.9×10^{22}	1.77	422×10^3
Europa	4.9	3.55	671
Ganymede	15	7.16	1070
Callisto	11	16.7	1883

*57. (II) Table 5–2 gives the mean distance, period, and mass for the four largest moons of Jupiter (those discovered by Galileo in 1609). (*a*) Determine the mass of Jupiter using the data for Io. (*b*) Determine the mass of Jupiter using data for each of the other three moons. Are the results consistent?

*58. (II) Determine the mean distance from Jupiter for each of Jupiter's moons, using the distance of Io and the periods given in Table 5–2. Compare to the values in the Table.

*59. (II) The asteroid belt between Mars and Jupiter consists of many fragments (which some space scientists think came from a planet that once orbited the Sun but was destroyed). (*a*) If the center of mass of the asteroid belt is about 3 times farther from the Sun than the Earth is, how long would it have taken this hypothetical planet to orbit the Sun? (*b*) Can we use this data to deduce the mass of this planet?

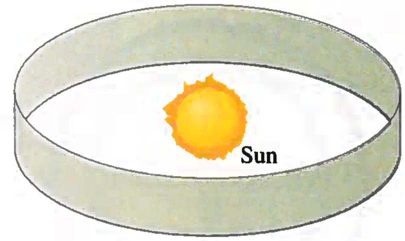


FIGURE 5–40 Problem 61.

*60. (III) (*a*) Use Kepler's second law to show that the ratio of the speeds of a planet at its nearest and farthest points from the Sun is equal to the inverse ratio of the near and far distances: $v_N/v_F = d_F/d_N$. (*b*) Given that the Earth's distance from the Sun varies from 1.47 to 1.52×10^{11} m, determine the minimum and maximum velocities of the Earth in its orbit around the Sun.

*61. (III) A science fiction tale describes an artificial "planet" in the form of a band completely encircling a sun, the inhabitants living on the inside surface (Fig. 5–40) (where it is always noon). Imagine the sun is exactly like our own, that the distance to the band is the same as the Earth–Sun distance (to make the climate temperate), and that the ring rotates quickly enough to produce an apparent gravity of one *g* as on Earth. What will be the period of revolution, this planet's year, in Earth days?

GENERAL PROBLEMS

62. How far above the Earth's surface will the acceleration of gravity be half what it is on the surface?

63. Tarzan plans to cross a gorge by swinging in an arc from a hanging vine (Fig. 5–41). If his arms are capable of exerting a force of 1400 N on the rope, what is the maximum speed he can tolerate at the lowest point of his swing? His mass is 80 kg and the vine is 4.8 m long.

64. Is it possible to whirl a bucket of water fast enough in a vertical circle so the water won't fall out? If so, what is the minimum speed?

65. On an ice rink two skaters of equal mass grab hands and spin in a mutual circle once every three seconds. If we assume their arms are each 0.80 m long, how hard are they pulling on one another, assuming their individual masses are 60.0 kg?

66. Because the Earth rotates once per day, the effective acceleration of gravity at the equator is slightly less than it would be if the Earth didn't rotate. Estimate the magnitude of this effect. What fraction of *g* is this?

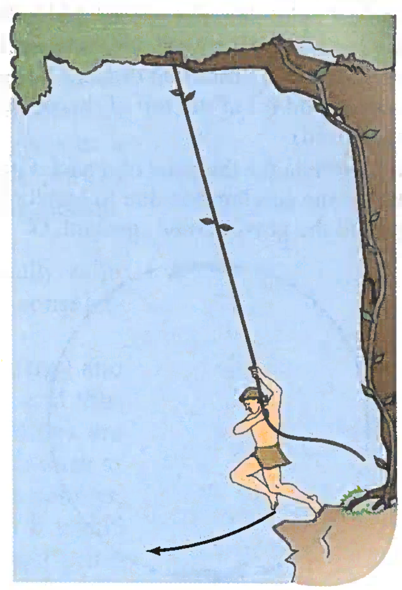


FIGURE 5–41 Problem 63.

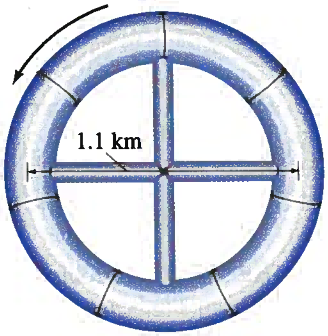


FIGURE 5–42 Problem 68.

67. At what distance from the Earth will a spacecraft traveling directly from the Earth to the Moon experience zero net force because the Earth and Moon pull with equal and opposite forces?

68. A projected space station consists of a circular tube that is set rotating about its center (like a tubular bicycle tire) (Fig. 5–42). The circle formed by the tube has a diameter of about 1.1 km. (*a*) On which wall inside the tube will people be able to walk? (*b*) What must be the rotation speed (revolutions per day) if an effect equal to gravity at the surface of the Earth (1.0 *g*) is to be felt?

69. You know your mass is 60 kg, but when you stand on a bathroom scale in an elevator, it says your mass is 80 kg. What is the acceleration of the elevator, and in which direction?

70. A jet pilot takes his aircraft in a vertical loop (Fig. 5–43). (*a*) If the jet is moving at a speed of 1500 km/h at the lowest point of the loop, determine the minimum radius of the circle so that the centripetal acceleration at the lowest point does not exceed 6.0 *g*'s. (*b*) Calculate also the 80-kg pilot's effective weight (the force with which the seat pushes up on him) at the bottom of the circle, and (*c*) at the top of the circle (assume the same speed).

71. Derive a formula for the mass of a planet in terms of its radius, *r*, the acceleration due to gravity at its surface, g_P, and the gravitational constant, *G*.

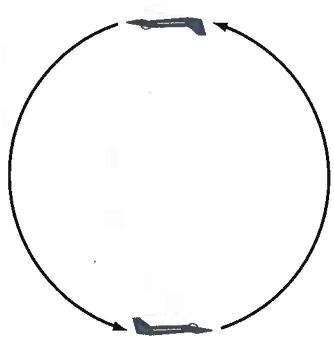


FIGURE 5–43 Problem 70.

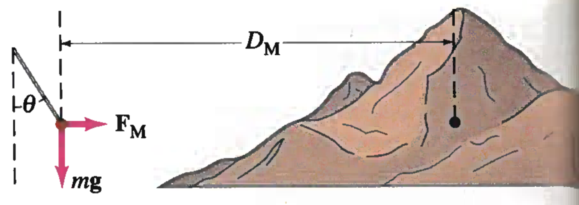


FIGURE 5–44 Problem 72.

72. A plumb bob is deflected from the vertical by an angle θ due to a massive mountain nearby (Fig. 5–44). (*a*) Find an approximate formula for θ in terms of the mass of the mountain, M_M, the distance to its center, D_M, and the radius and mass of the Earth. (*b*) Make a rough estimate of the mass of Mt. Everest, assuming it has the shape, say, of an equilateral pyramid (or cone) 4000 m high above its base, and then (*c*) estimate the angle θ of the pendulum bob if it is 5 km from the center of Mt. Everest.

73. A curve of radius 60 m is banked for a design speed of 100 km/h. If the coefficient of static friction is 0.30 (wet pavement), at what range of speeds can a car safely make the curve?

74. How long would a day be if the Earth were rotating so fast that objects at the equator were apparently weightless?

75. Two equal-mass stars maintain a constant distance apart of 8.0×10^{10} m and rotate about a point midway between them at a rate of one revolution every 12.6 yr. (*a*) Why don't the two stars crash into one another due to the gravitational force between them? (*b*) What must be the mass of each star?

76. A train traveling at a constant speed rounds a curve of radius 275 m. A chandelier suspended from the ceiling swings out to an angle of 17.5° throughout the turn. What is the speed of the train?

77. The planet Jupiter is about 320 times as massive as the Earth. Thus, it has been claimed that a person would be crushed by the force of gravity on Jupiter since people can't survive more than a few *g*'s. Calculate the number of *g*'s a person would experience if she could stand on the equator of Jupiter. Use the following astronomical data for Jupiter: mass = 1.9×10^{27} kg, equatorial radius = 7.1×10^{4} km, rotation period 9 hr 55 min. Take the centripetal acceleration into account.

78. Astronomers using the Hubble Space Telescope have recently deduced the presence of an extremely massive core in the distant galaxy M87, so dense that it could well be a black hole (from which no light escapes). They did this by measuring the speed of gas clouds orbiting the core to be 780 m/s at a distance of 60 light-years (5.7×10^{17} m) from the core. Deduce the mass of the core, and compare it to the mass of our Sun.