Acceleration at the start of a race. Can you describe the force that causes an athlete's acceleration? (Hint: note the force vector **F**.)

CHAPTER 4

MOTION AND FORCE: DYNAMICS

We have discussed how motion is described in terms of velocity and acceleration. Now we deal with the question of *why* objects move as they do: What makes an object at rest begin to move? What causes a body to accelerate or decelerate? What is involved when an object moves in a circle? We can answer in each case that a force is required. In this chapter, we will investigate the connection between force and motion. Before we delve into this subject of *dynamics*, we first discuss the concept of force in a qualitative way.

4–1 Force

Intuitively, we experience **force** as any kind of a push or a pull on an object. When you push a grocery cart or a stalled car (Fig. 4–1), you are exerting a force on it. When a motor lifts an elevator, or a hammer hits a nail, or the wind blows the leaves of a tree, a force is being exerted. We say that an object falls because of the *force of gravity*. Forces do not always give rise to motion. For example, you may push very hard on a heavy desk and it may not move.

FIGURE 4–1 Exerting a force on a stalled car.

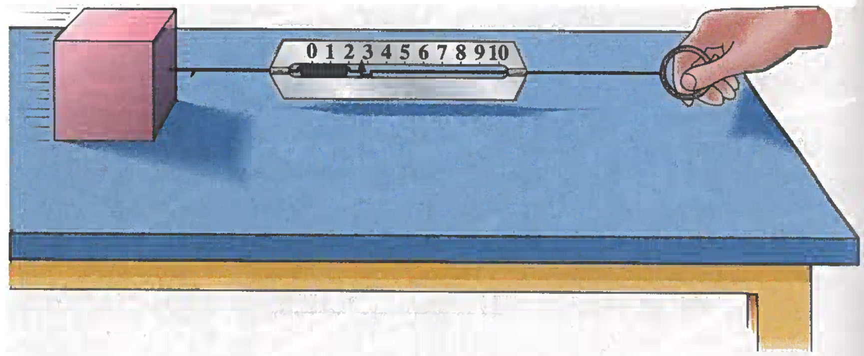


FIGURE 4–2 A spring scale used to measure a force.

Measuring force

One way to measure the magnitude (or strength) of a force is to make use of a spring scale (Fig. 4–2). Normally, such a spring scale is used to find the weight of an object; by weight we mean the force of gravity acting on the body (Section 4–6). The spring scale, once calibrated, can be used to measure other kinds of forces as well, such as the pulling force shown in Fig. 4–2.

A force has direction as well as magnitude, and is indeed a vector that follows the rules of vector addition discussed in Chapter 3. We can represent any force on a diagram by an arrow, just as we did with velocity. The direction of the arrow is the direction of the push or pull, and its length is drawn proportional to the magnitude of the force.

4–2 Newton's First Law of Motion

Galileo vs. Aristotle

What is the exact connection between force and motion? Aristotle (384–322 B.C.) believed that a force was required to keep an object moving along a horizontal plane. He would argue that to make a book move across the table, you would have to exert a force on it continuously. To Aristotle, the natural state of a body was at rest, and a force was believed necessary to keep a body in motion. Furthermore, Aristotle argued, the greater the force on the body, the greater its speed.

Some 2000 years later, Galileo questioned these Aristotelian views and came to a radically different conclusion. Galileo maintained that it is just as natural for an object to be in horizontal motion with a constant velocity as it is for it to be at rest!

To understand Galileo's idea, consider the following observations involving motion along a horizontal plane. To push an object with a rough surface along a tabletop at constant speed requires a certain amount of force. To push an equally heavy object with a very smooth surface across the table at the same speed will require less force. If a layer of oil or other lubricant is placed between the surface of the object and the table, then almost no force is required to move the object. Notice that in each successive step, less force is required. As the next step, we can imagine a situation in which the object does not rub against the table at all—or there is a perfect lubricant between the object and the table—and theorize that once started, the object would move across the table at constant speed with *no* force applied. A steel ball bearing rolling on a hard horizontal surface approaches this situation. So does a puck on an air table (Fig. 4–3), in which a thin layer of air reduces friction almost to zero.

FIGURE 4–3 Photo of an air table. Air issuing from many tiny holes forms a thin air layer between the table and a puck, which, when given an initial shove, will travel at constant speed in a straight line (until it hits a wall or another puck).

It was Galileo's genius to imagine such an idealized world—in this case, one where there is no friction—and to see that it could produce a more useful view of the real world. It was this idealization that led him to his remarkable conclusion that if no force is applied to a moving object, it will continue to move with constant speed in a straight line. An object slows down only if a force is exerted on it. Galileo thus interpreted friction as a force akin to ordinary pushes and pulls.

Friction as a force

To push an object across a table at constant speed requires a force from your hand only to balance out the force of friction (Fig. 4–4). When the object moves at constant speed, your pushing force is equal in magnitude to the friction force, but these two forces are in opposite directions, so the *net* force on the object (the vector sum of the two forces) is zero. This is consistent with Galileo's viewpoint, for the object moves with constant speed when no net force is exerted on it.

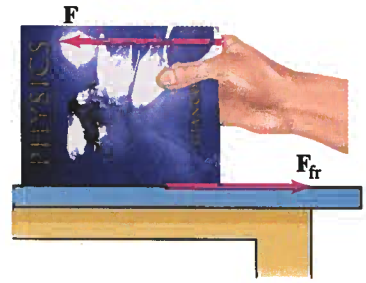


FIGURE 4–4 **F** represents the force applied by the person and $\mathbf{F}_{fr}$ represents the force of friction.

The difference between Aristotle's view and Galileo's is not simply one of right or wrong. Aristotle's view was not really wrong, for our everyday experience indicates that moving objects do tend to come to a stop if not continually pushed. The real difference lies in the fact that Aristotle's view about the "natural state" of a body was essentially a final statement—no further development was possible. Galileo's analysis, on the other hand, could be extended to explain a great many more phenomena, and it provided a quantitative theory allowing verifiable predictions. By making the creative leap of imagining the experimentally unattainable situation of no friction, and by interpreting friction as a force, Galileo was able to reach his conclusion that an object will continue moving with constant velocity if no force acts to change this motion.

Upon this foundation, Isaac Newton (Fig. 4–5) built his great theory of motion. Newton's analysis of motion is summarized in his famous "three laws of motion." In his great work, the *Principia* (published in 1687), Newton readily acknowledged his debt to Galileo. In fact, **Newton's first law of motion** is very close to Galileo's conclusions. It states that

NEWTON'S FIRST LAW OF MOTION

Every body continues in its state of rest or of uniform speed in a straight line unless acted on by a nonzero net force.

The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called **inertia**. As a result, Newton's first law is often called the **law of inertia.**

Inertia

[Newton's first law does not hold in every reference frame. For example, if your reference frame is fixed in an accelerating car, an object such as a cup resting on the dashboard may begin to move toward you (it stayed at rest as long as the car's velocity remained constant). The cup accelerated toward you but neither you nor anything else exerted a force on it in that direction. In such an accelerating reference frame, Newton's first law does not hold. Reference frames in which Newton's first law does hold are called **inertial reference frames** (the law of inertia is valid in them). For most purposes, we can usually assume that reference frames fixed on the Earth are inertial frames. (This is not precisely true, due to the Earth's rotation, but usually it is close enough.) Any reference frame that moves with constant velocity (say, a car or an airplane) relative to an inertial frame is also an inertial reference frame. Reference frames where the law of inertia does *not*

FIGURE 4–5 Isaac Newton (1642–1727).

hold, such as the accelerating reference frame discussed above, are called **noninertial** reference frames. How can we be sure a reference frame is inertial or not? By checking to see if Newton's first law holds. Thus Newton's first law serves as the definition of inertial reference frames.]

4–3 Mass

Newton's second law, which we come to in the next section, makes use of the concept of mass. Newton used the term *mass* as a synonym for *quantity of matter*. This intuitive notion of the mass of a body is not very precise because the concept "quantity of matter" is not very well defined. More precisely, we can say that **mass** is a *measure of the inertia of a body*. The more mass a body has, the harder it is to change its state of motion. It is harder to start it moving from rest, or to stop it when it is moving, or to change its motion sideways out of a straight-line path. A truck has much more inertia than a baseball, and it is much harder to speed it up or slow it down. It therefore has much more mass.

Mass as inertia

To quantify the concept of mass, we must define a standard. In SI units, the unit of mass is the **kilogram** (kg) as we discussed in Chapter 1, Section 1–5.

The terms *mass* and *weight* are often confused with one another, but it is important to distinguish between them. Mass is a property of a body itself (it is a measure of a body's inertia, or its "quantity of matter"). Weight, on the other hand, is a force, the force of gravity acting on a body. To see the difference, suppose we take an object to the Moon. The object will weigh only about one sixth as much as it did on Earth, since the force of gravity is weaker, but its mass will be the same. It will have the same amount of matter and it will have just as much inertia—for in the absence of friction, it will be just as hard to start it moving or to stop it once it is moving. (More on weight in Section 4–6.)

Mass vs. weight

4–4 Newton's Second Law of Motion

Newton's first law states that if no net force is acting on a body, it remains at rest, or if moving, it continues moving with constant speed in a straight line. But what happens if a net force is exerted on a body? Newton perceived that the velocity will change (Fig. 4–6). A net force exerted on an object may make its speed increase. Or, if the net force is in a direction opposite to the motion, it will reduce the speed. If the net force acts sideways on a moving object, the *direction* of the velocity changes (and the magnitude may as well). Since a change in speed or velocity is an acceleration (Chapter 2, Section 2–4), we can say that *a net force gives rise to acceleration.*

FIGURE 4–6 The bobsled accelerates because the team exerts a force.

What precisely is the relationship between acceleration and force? Everyday experience can answer this question. Consider the force required to push a cart whose friction is minimal. (If there is friction, consider the *net* force, which is the force you exert minus the force of friction.) Now if you push with a gentle but constant force for a certain period of time, you will make the cart accelerate from rest up to some speed, say

3 km/h. If you push with twice the force, you will find that the cart will reach 3 km/h in half the time. That is, the acceleration will be twice as great. If you double the force, the acceleration doubles. If you triple the force, the acceleration is tripled, and so on. Thus, the acceleration of a body is directly proportional to the net applied force. But the acceleration depends on the mass of the object as well. If you push an empty grocery cart with the same force as you push one that is filled with groceries, you will find that the latter accelerates more slowly. The greater the mass, the less the acceleration for the same net force. The mathematical relation, as Newton argued, is that the acceleration of a body is inversely proportional to its mass. These relationships are found to hold in general and can be summarized as follows:

NEWTON'S SECOND LAW OF MOTION

> **The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object.**

This is **Newton's second law of motion**. As an equation, it can be written

$$\mathbf{a} = \frac{\Sigma \mathbf{F}}{m},$$

where **a** stands for acceleration, m for the mass, and $\Sigma\mathbf{F}$ for the *net force*. The symbol Σ (Greek "sigma") stands for "sum of"; **F** stands for force, so $\Sigma\mathbf{F}$ means the *vector sum of all forces* acting on the body, which we define as the **net force**.

Net force

We rearrange this equation to obtain the familiar statement of Newton's second law:

NEWTON'S SECOND LAW OF MOTION

$$\Sigma \mathbf{F} = m\mathbf{a}. \qquad (4\text{–}1)$$

Newton's second law relates the description of motion to the cause of motion, force. It is one of the most fundamental relationships in physics. From Newton's second law we can make a more precise definition of **force** as *an action capable of accelerating an object.*

Force defined

Every force **F** is a vector, with magnitude and direction. Equation 4–1 is a vector equation valid in any inertial reference frame. It can be written in component form in rectangular coordinates as

$$\Sigma F_x = ma_x, \qquad \Sigma F_y = ma_y, \qquad \Sigma F_z = ma_z.$$

If the motion is all along a line (one dimensional), we can leave out the subscripts and simply write $\Sigma F = ma$.

In SI units, with the mass in kilograms, the unit of force is called the **newton** (N). One newton, then, is the force required to impart an acceleration of $1\ \text{m/s}^2$ to a mass of 1 kg. Thus $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$.

Unit of force; the newton

In cgs units, the unit of mass is the gram (g) as mentioned earlier.† The unit of force is the *dyne*, which is defined as the net force needed to impart an acceleration of $1\ \text{cm/s}^2$ to a mass of 1 g. Thus $1\ \text{dyne} = 1\ \text{g}\cdot\text{cm/s}^2$. It is easy to show that $1\ \text{dyne} = 10^{-5}\ \text{N}$.

In the British system, the unit of force is the *pound* (abbreviated lb), where $1\ \text{lb} \approx 4.45\ \text{N}$. The unit of mass is the *slug*, which is defined as that

† Be careful not to confuse g for gram with *g* for the acceleration due to gravity. The latter is always italicized (or bold face as a vector).

mass which will undergo an acceleration of 1 ft/s^2 when a force of 1 lb is applied to it. Thus 1 lb = 1 slug·ft/s^2. Table 4–1 summarizes the units in the different systems.

PROBLEM SOLVING
Use a consistent set of units

It is very important that only one set of units be used in a given calculation or problem, with the SI being preferred. If the force is given in, say, newtons, and the mass in grams, then before attempting to solve for the acceleration in SI units, the mass must be changed to kilograms. For example, if the force is given as 2.0 N along the x axis and the mass is 500 g, we change the latter to 0.50 kg, and the acceleration will then automatically come out in m/s^2 when Newton's second law is used:

$$a_x = \frac{\Sigma F_x}{m} = \frac{2.0\ \text{N}}{0.50\ \text{kg}} = \frac{2.0}{0.50}\ \frac{\text{kg}\cdot\text{m}}{\text{kg}\cdot\text{s}^2} = 4.0\ \text{m/s}^2.$$

TABLE 4–1
Units for Mass and Force

System	Mass	Force (including weight)
SI	kilogram (kg)	newton (N) (= kg·m/s^2)
cgs	gram (g)	dyne (= g·cm/s^2)
British	slug	pound (lb)

EXAMPLE 4–1 ESTIMATE **Force to accelerate a fast car.** Estimate the net force needed to accelerate a 1000-kg car at $\frac{1}{2}g$.

SOLUTION The car's acceleration is $a = \frac{1}{2}g = \frac{1}{2}(9.8\ \text{m/s}^2) \approx 5\ \text{m/s}^2$. We use Newton's second law to get the net force needed to achieve this acceleration:

$$\Sigma F = ma = (1000\ \text{kg})(5\ \text{m/s}^2) = 5000\ \text{N}.$$

(If you are used to the British units, to get an idea what a 5000N force is, you can divide by 4.45 N/lb and get a force of about 1000 lb.)

EXAMPLE 4–2 **Force to stop a car.** What net force is required to bring a 1500-kg car to rest from a speed of 100 km/h within a distance of 55 m?

SOLUTION We use Newton's second law, $\Sigma F = ma$, but first we must determine the acceleration a, which we assume is constant. We assume the motion is along the $+x$ axis (Fig. 4–7). We are given the initial velocity $v_0 = 100$ km/h = 28 m/s, the final velocity $v = 0$, and the distance traveled $x - x_0 = 55$ m. From Eq. 2–10c, we have

$$v^2 = v_0^2 + 2a(x - x_0)$$

so

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{0 - (28\ \text{m/s})^2}{2(55\ \text{m})} = -7.1\ \text{m/s}^2.$$

The net force required is then

$$\Sigma F = ma = (1500\ \text{kg})(-7.1\ \text{m/s}^2) = -1.1 \times 10^4\ \text{N}.$$

The force must be exerted in the direction *opposite* to the initial velocity, which is what the negative sign tells us.

FIGURE 4–7 Example 4–2.

4–5 Newton's Third Law of Motion

Newton's second law of motion describes quantitatively how forces affect motion. But where, we may ask, do forces come from? Observations suggest that a force applied to any object is always applied *by another object.* A horse pulls a wagon, a person pushes a grocery cart, a hammer pushes on a nail, a magnet attracts a paper clip. In each of these examples, a force is exerted *on* one body, and that force is exerted *by* another body. For example, the force exerted *on* the nail is exerted *by* the hammer.

A force is exerted **on** *an object and is exerted* **by** *another object*

But Newton realized that things are not so one-sided. True, the hammer exerts a force on the nail (Fig. 4–8). But the nail evidently exerts a force back on the hammer as well, for the hammer's speed is rapidly reduced to zero upon contact. Only a strong force could cause such a rapid change of velocity of the hammer. Thus, said Newton, the two bodies must be treated on an equal basis. The hammer exerts a force on the nail, and the nail exerts a force back on the hammer. This is the essence of **Newton's third law of motion:**

Whenever one object exerts a force on a second object, the second exerts an equal and opposite force on the first.

NEWTON'S THIRD LAW OF MOTION

This law is sometimes paraphrased as "to every action there is an equal and opposite reaction." This is perfectly valid. But to avoid confusion, it is very important to remember that the "action" force and the "reaction" force are acting on *different* objects.

Action and reaction act on different objects

As evidence for the validity of Newton's third law, look at your hand when you push against a grocery cart or against the edge of a desk, Fig. 4–9. Your hand's shape is distorted, clear evidence that a force is being exerted on it. You can *see* the edge of the desk pressing into your hand. You can even *feel* the desk exerting a force on your hand; it hurts! The harder you push against the desk, the harder the desk pushes back on your hand. (Note that you only feel forces exerted *on* you, not forces you exert on something else.)

FIGURE 4–8 Multiflash photo of a hammer striking a nail. In accordance with Newton's third law, the hammer exerts a force on the nail, and the nail exerts a force back on the hammer. The latter force decelerates the hammer and brings it to rest.

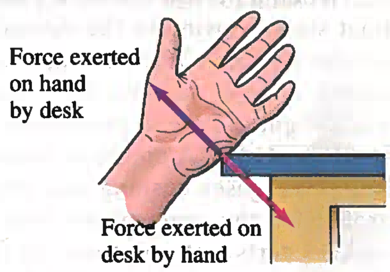


FIGURE 4–9 If your hand pushes against the edge of a desk (the force vector is shown in red), the desk pushes back against your hand (this force vector is shown in a different color, purple, to remind us that this force acts on a different object).

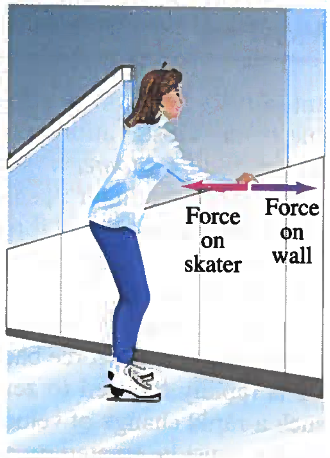


FIGURE 4–10 When an ice-skater pushes against the railing, the railing pushes back and this force causes her to move away.

FIGURE 4–11 The launch of a rocket. Newton's third law.

As another demonstration of Newton's third law, consider the ice-skater in Fig. 4–10. Since there is very little friction between her skates and the ice, she will move freely if a force is exerted on her. She pushes against the railing; and then *she* starts moving backward. Clearly, there had to be a force exerted on her to make her move. The force she exerts on the railing cannot make *her* move, for that force acts on the railing. Something had to exert a force on her to make her start moving, and that force could only have been exerted by the railing. The force with which the railing pushes on her is, by Newton's third law, equal and opposite to the force she exerts on the railing.

When a person throws a package out of a boat (initially at rest), the boat starts moving in the opposite direction. The person exerts a force on the package. The package exerts an equal and opposite force back on the person, and this force propels the person (and the boat) backward slightly. Rocket propulsion also is explained using Newton's third law (Fig. 4–11). A common misconception is that rockets accelerate because the gases rushing out the back of the engine push against the ground or the atmosphere. Not true. What happens, instead, is that a rocket exerts a strong force on the gases, expelling them; and the gases exert an equal and opposite force *on the rocket*. It is this latter force that propels the rocket forward. Thus, a space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate.

PHYSICS APPLIED

How does a rocket accelerate?

How we can walk

Consider how we walk. A person begins walking by pushing with the foot against the ground. The ground then exerts an equal and opposite force back on the person (Fig. 4–12) and it is this force, *on* the person, that moves the person forward. (If you doubt this, try walking on very smooth slippery ice.) In a similar way, a bird flies forward by exerting a force on the air, but it is the air pushing back on the bird's wings that propels the bird forward.

CONCEPTUAL EXAMPLE 4–3 **What exerts the force on a car?** What makes a car go forward?

RESPONSE A common answer is that the engine makes the car move forward. But it is not so simple. The engine makes the wheels go around. But what good is that if they are on slick ice or mud? They just spin. A car moves forward due to the friction force exerted by the ground on the tires, and this force is the reaction to the force exerted on the ground by the tires.

Inanimate objects can exert a force

We tend to associate forces with active bodies such as humans, animals, engines, or a moving object like a hammer. It is often difficult to see how an inanimate object at rest, such as a wall or a desk, can exert a force. The explanation lies in the fact that every material, no matter how hard, is elastic, at least to some degree. No one can deny that a stretched rubber band can exert a force on a wad of paper and send it flying across the room. Other materials may not stretch as easily as rubber, but they do stretch when a force is applied to them. And just as a stretched rubber band exerts a force, so does a stretched (or compressed) wall, desk, or car fender.

PROBLEM SOLVING

For each force, be clear ***on*** *which object it acts, and* ***by*** *which object it is exerted.* $\Sigma \mathbf{F} = m\mathbf{a}$ *applies only to forces acting* ***on*** *a body.*

From the examples discussed above, it is clear that it is quite important to remember *on* what object a given force is exerted and *by* what object that force is exerted. The point is that a force influences the motion of an object only when it is applied *on* that object. A force exerted *by* a body does not influence that body; it only influences the other body *on* which it is exerted. Thus, to avoid confusion, the two prepositions *on* and *by* must always be used—and used with care.

One way to keep clear which force acts on which object is to use double subscripts. For example the force exerted on the **P**erson by the **G**round in Fig. 4–13 can be labeled $\mathbf{F}_{PG}$. And the force exerted on the ground by the person is $\mathbf{F}_{GP}$, as shown in Fig. 4–13. Note that we have used different colors for the force vectors when they act on different objects. By Newton's third law

NEWTON'S THIRD LAW OF MOTION

$$\mathbf{F}_{GP} = -\mathbf{F}_{PG}. \quad (4\text{–}2)$$

$\mathbf{F}_{GP}$ and $\mathbf{F}_{PG}$ have the same magnitude, and the minus sign reminds us that these two forces are in opposite directions.

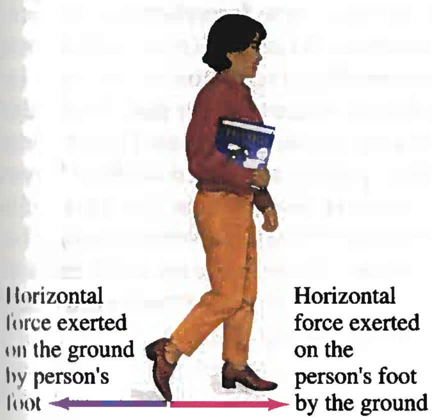
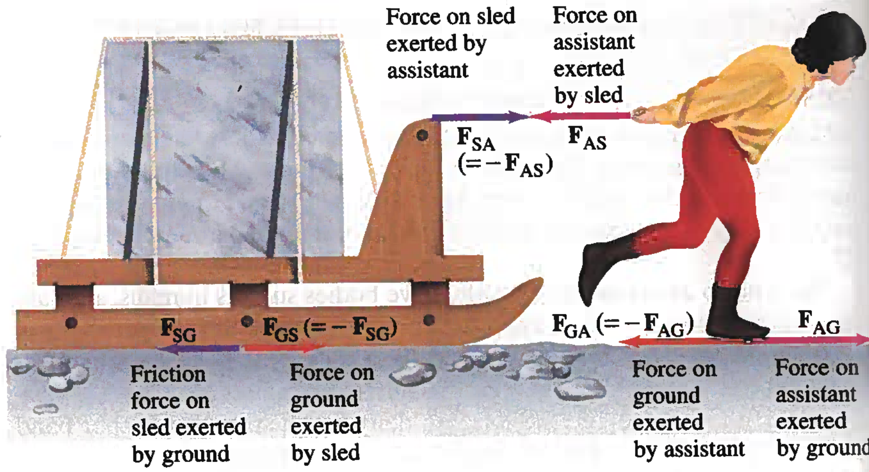


FIGURE 4–12 We can walk forward because, when one foot pushes backward against the ground, the ground pushes forward on that foot.

FIGURE 4–13 Newton's third law. Subscripts on forces remind us *on* which body a force acts and *by* which body it is exerted.

FIGURE 4–14 Seventy-year-old Michelangelo has selected a fine block of marble for his next sculpture. Shown here is his assistant pulling it on a sled away from the quarry. Forces on the assistant are shown as red (magenta) arrows. Forces on the sled are purple arrows. Forces acting on the ground are orange arrows. Action–reaction forces that are equal and opposite are labeled by the same subscripts but reversed (such as $\mathbf{F}_{GA}$ and $\mathbf{F}_{AG}$) and are of different colors because they act on different objects.



CONCEPTUAL EXAMPLE 4–4 **Third law clarification.** Michelangelo's assistant has been assigned the task of moving a block of marble using a sled (Fig. 4–14). He says to his boss, "When I exert a forward force on the sled, the sled exerts an equal and opposite force backward. So how can I ever start it moving? No matter how hard I pull, the backward reaction force always equals my forward force, so the net force must be zero. I'll never be able to move this load." Is this a case of a little knowledge being dangerous? Explain.

➥ **PROBLEM SOLVING**

A study of Newton's second and third laws

RESPONSE Yes. Although it is true that the action and reaction forces are equal in magnitude, the assistant has forgotten that they are exerted on different objects. The forward ("action") force is exerted by the assistant on the sled (Fig. 4–14), whereas the backward "reaction" force is exerted by the sled on the assistant. To determine if the *assistant* moves or not, we must consider only the forces *on the assistant* and then apply $\Sigma\mathbf{F} = m\mathbf{a}$, where $\Sigma\mathbf{F}$ is the net force *on the assistant*, $\mathbf{a}$ is the acceleration of the assistant, and m is the assistant's mass. There are two forces on the assistant that affect his forward motion and he seems to have forgotten one of them. The two forces on the assistant are shown as bright red (magenta) arrows in Fig. 4–14; they are (1) the horizontal force $\mathbf{F}_{AG}$ exerted on the assistant by the ground (the harder he pushes backward against the ground, the harder the ground pushes forward on him—Newton's third law), and (2) the force $\mathbf{F}_{AS}$ exerted on the assistant by the sled, pulling backward on him. When the ground pushes forward on the assistant harder than the sled pulls backward, the assistant accelerates forward (Newton's second law). The sled, on the other hand, accelerates forward when the force on it exerted by the assistant is greater than the frictional force acting backward (that is, when $\mathbf{F}_{SA}$ has greater magnitude than $\mathbf{F}_{SG}$ in Fig. 4–14).

Using double subscripts to clarify Newton's third law can become cumbersome, and we won't usually use them in this way. Nevertheless, if there is any confusion in your mind about a given force, go ahead and use them to identify *on* what object and *by* what object the force is exerted.

4–6 Weight—the Force of Gravity; and the Normal Force

Galileo claimed that objects dropped near the surface of the Earth will all fall with the same acceleration, **g**, if air resistance can be neglected. The force that gives rise to this acceleration is called the force of gravity. We now apply Newton's second law to the gravitational force; and for the acceleration, **a**, we use the downward acceleration due to gravity, **g**. Thus, the force of gravity on an object, $\mathbf{F}_G$, whose magnitude is commonly called its **weight**, can be written as

$$\mathbf{F}_G = m\mathbf{g}. \qquad \textbf{(4–3)}$$

Weight = force of gravity

The direction of this force is down toward the center of the Earth.

In SI units, $g = 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N/kg}$,† so the weight of a 1.00-kg mass on Earth is $1.00\,\mathrm{kg} \times 9.80\,\mathrm{m/s^2} = 9.80\,\mathrm{N}$. We will mainly be concerned with the weight of objects on Earth, but we note that on the Moon, on other planets, or in space, the weight of a given mass will be different. For example, on the Moon g is about one sixth what it is on Earth, and a 1.0 kg mass weighs only 1.7 N. Although we will not have occasion to use British units, we note that for practical purposes on the Earth, a mass of 1 kg weighs about 2.2 lb. (On the Moon, 1 kg weighs only about 0.4 lb.)

The force of gravity acts on an object when it is falling. When an object is at rest on the Earth, the gravitational force on it does not disappear, as we know if we weigh it on a spring scale. The same force, given by Eq. 4–3, continues to act. Why, then, doesn't the object move? From Newton's second law, the net force on an object that remains at rest is zero. There must be another force on the object to balance the gravitational force. For an object resting on a table, the table exerts this upward force; see Fig. 4–15a. The table is compressed slightly beneath the object, and due to its elasticity, it pushes up on the object as shown. The force exerted by the table is often called a **contact force**, since it occurs when two objects are in contact. (The force of your hand pushing on a cart is also a contact force.) When a contact force acts *perpendicular* to the common surface of contact, it is usually referred to as the **normal force** ("normal" means perpendicular); hence it is labeled $\mathbf{F}_N$ in the diagram.

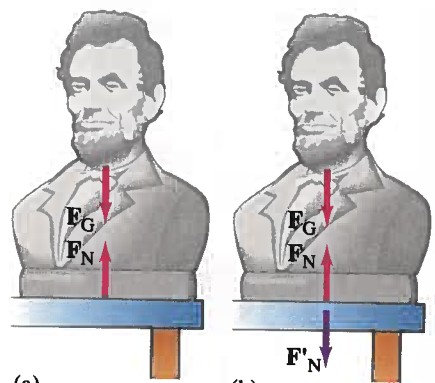


FIGURE 4–15 (a) The net force on an object at rest is zero according to Newton's second law. Therefore the downward force of gravity ($\mathbf{F}_G$) on an object must be balanced by an upward force (the normal force $\mathbf{F}_N$) exerted by the table in this case. (b) $\mathbf{F}'_N$ is the force exerted on the table by the statue and is the reaction force to $\mathbf{F}_N$ as per Newton's third law. ($\mathbf{F}'_N$ is shown in a different color to remind us it acts on a different body.) The reaction to $\mathbf{F}_G$ is not shown.

Careful:
Weight and normal force are **not** *action–reaction pairs*

The two forces shown in Fig. 4–15a are both acting on the statue, which remains at rest, so the vector sum of these two forces must be zero (Newton's second law). Hence $\mathbf{F}_G$ and $\mathbf{F}_N$ must be of equal magnitude and in opposite directions. But they are *not* the equal and opposite forces spoken of in Newton's third law. The action and reaction forces of Newton's third law act on *different objects*, whereas the two forces shown in Fig. 4–15a act on the *same* object. For each of the forces shown in Fig. 4–15a, we can ask, "What is the reaction force?" The upward force, $\mathbf{F}_N$, on the statue is exerted by the table. The reaction to this force is a force exerted by the statue on the table. It is shown in Fig. 4–15b, where it is labeled $\mathbf{F}'_N$. This force, $\mathbf{F}'_N$, exerted on the table by the statue, is the reaction force to $\mathbf{F}_N$ in accord with Newton's third law. (We could equally well say the reverse: the force $\mathbf{F}_N$ on the statue exerted by the table is the reaction to the force $\mathbf{F}'_N$ exerted on the table by the statue.) Now, what about the other force on the statue, the force of gravity $\mathbf{F}_G$? Can you guess what the reaction is to this force? [We will see in Chapter 5 that the reaction force is also a gravitational force exerted on the Earth by the statue, and can be considered to act at the Earth's center.]

†Since $1\,\mathrm{N} = 1\,\mathrm{kg{\cdot}m/s^2}$ (Section 4–4), $1\,\mathrm{m/s^2} = 1\,\mathrm{N/kg}$.

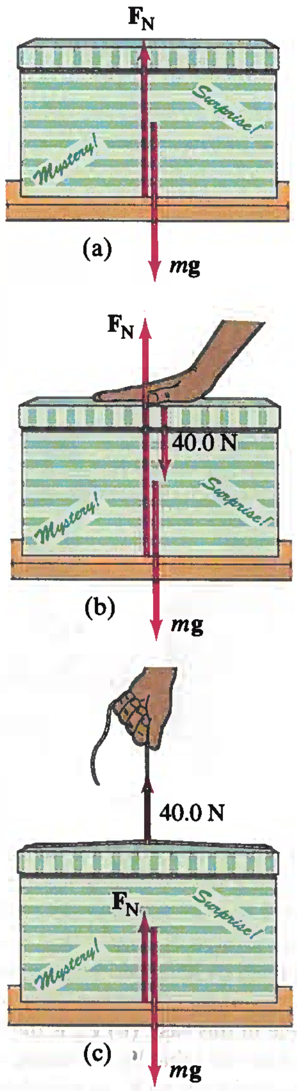


FIGURE 4–16 Example 4–5. (a) A 10-kg gift box is at rest on a table. (b) A person pushes down on the box with a force of 40.0 N. (c) A person pulls upward on the box with a force of 40.0 N. The forces are all assumed to act along a line; they are shown slightly displaced in order to be distinguishable on the diagram. Only forces acting on the box are shown.

EXAMPLE 4–5 **Weight, normal force, and a box.** A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. It's a reward for your fine showing on the physics final. The box is resting on the smooth (frictionless) horizontal surface of a table (Fig. 4–16a). (*a*) Determine the weight of the box and the normal force acting on it. (*b*) Now your friend pushes down on the box with a force of 40.0 N, as in Fig. 4–16b. Again determine the normal force acting on the box. (*c*) If your friend pulls upward on the box with a force of 40.0 N (Fig. 4–16c), what now is the normal force on the box?

SOLUTION (*a*) The box is resting on the table. The weight of the box is $mg = (10.0\,\text{kg})(9.80\,\text{m/s}^2) = 98.0\,\text{N}$, and this force acts downward. The only other force on the candy box is the normal force exerted upward on it by the table, as shown in Fig. 4–16a. We chose the upward direction as the positive y direction, and then the net force ΣF_y on the box is $\Sigma F_y = F_N - mg$. Since the box is at rest, the net force on it must be zero ($\Sigma F_y = ma_y$, and $a_y = 0$). Thus

$$\Sigma F_y = F_N - mg = 0,$$

so we have in this case

$$F_N = mg.$$

The normal force on the box, exerted by the table, is 98.0 N upward, and has magnitude equal to the box's weight.

(*b*) Your friend is pushing down on the box with a force of 40.0 N. So now there are three forces acting on the box, as shown in Fig. 4–16b. The weight of the box is still $mg = 98.0\,\text{N}$. The net force is $\Sigma F_y = F_N - mg - 40.0\,\text{N}$, and is equal to zero since the box remains at rest. Thus, since $a = 0$, Newton's second law gives

$$\Sigma F_y = F_N - mg - 40.0\,\text{N} = 0,$$

so the normal force is now

$$F_N = mg + 40.0\,\text{N} = 98.0\,\text{N} + 40.0\,\text{N} = 138.0\,\text{N},$$

which is greater than in (*a*). The table pushes back with more force.

(*c*) The box's weight is still 98.0 N and acts downward. The force exerted by your friend and the normal force both act upward (positive direction), as shown in Fig. 4–16c. The box doesn't move since your friend's upward force is less than the weight. The net force, again set to zero, is

$$\Sigma F_y = F_N - mg + 40.0\,\text{N} = 0,$$

so

$$F_N = mg - 40.0\,\text{N} = 98.0\,\text{N} - 40.0\,\text{N} = 58.0\,\text{N}.$$

The table does not push against the full weight of the box because of the upward pull exerted by your friend.

Notice that the normal force is elastic in origin (the table in Fig. 4–16 sags slightly under the weight of the box).

EXAMPLE 4–6 **Accelerating the box.** What happens when a person pulls upward on the box in Example 4–5(*c*) with a force equal to, or greater than, the box's weight, say $F_P = 100.0\text{ N}$ rather than the 40.0 N shown in Fig. 4–16c?

SOLUTION The net force is now

$$\Sigma F_y = F_N - mg + F_P = F_N - 98.0\text{ N} + 100.0\text{ N},$$

and if we set this equal to zero, we would get $F_N = -2.0\text{ N}$. This is nonsense, since the negative sign implies F_N points downward, and the table surely cannot pull down on the box (unless there's glue on the table). The least F_N can be is zero, which it will be in this case. What really happens here is clear: the box accelerates upward since the net force is not zero; it is

$$\Sigma F_y = F_P - mg = 100.0\text{ N} - 98.0\text{ N} = 2.0\text{ N}$$

upward. See Fig. 4–17. So the box moves upward with an acceleration of magnitude

$$a_y = \Sigma F_y/m = 2.0\text{ N}/10.0\text{ kg} = 0.20\text{ m/s}^2.$$

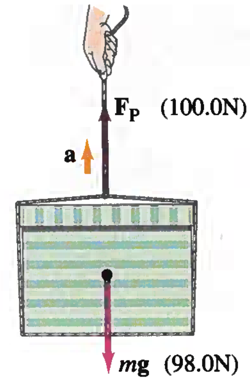


FIGURE 4–17 Example 4–6. The box accelerates upwards because $F_P > mg$.

4–7 Solving Problems with Newton's Laws: Vector Forces and Free-Body Diagrams

Newton's second law tells us that the acceleration of an object is proportional to the *net force* acting on the object. The **net force**, as mentioned earlier, is the *vector sum* of all forces acting on the object. Indeed, extensive experiments have shown that forces do add together as vectors precisely according to the rules we developed in Chapter 3. For example, in Fig. 4–18, two forces of equal magnitude (100 N each) are shown acting on an object at right angles to each other. Intuitively, we can see that the object will move at a 45° angle and thus the net force acts at a 45° angle. This is just what the rules of vector addition give. The Pythagorean theorem tells us that the magnitude of the resultant force is $F_R = \sqrt{(100\text{ N})^2 + (100\text{ N})^2} = 141\text{ N}$.

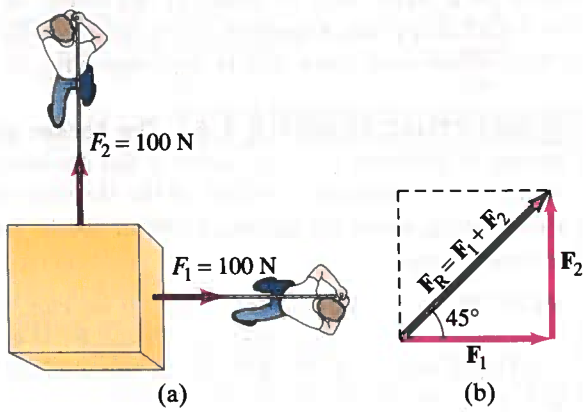


FIGURE 4–18 (a) Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, act on an object. (b) The sum, or resultant, of $\mathbf{F}_1$ and $\mathbf{F}_2$ is $\mathbf{F}_R$.

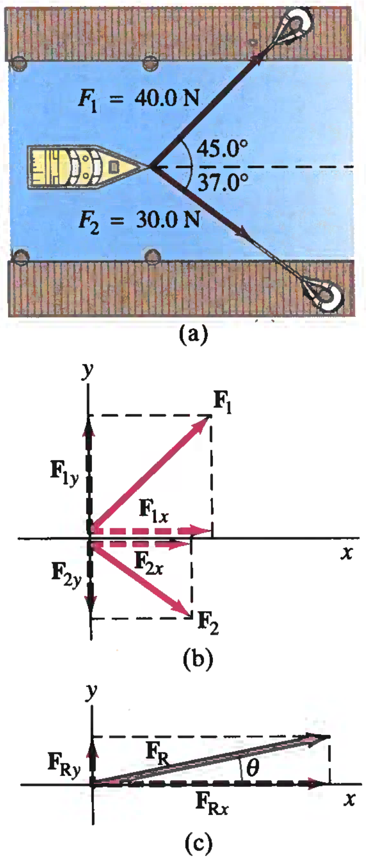


FIGURE 4–19 Two force vectors act on a boat (Example 4–7).

EXAMPLE 4–7 **Adding force vectors.** Calculate the sum of the two forces acting on the boat shown in Fig. 4–19a.

SOLUTION These two forces are shown resolved in Fig. 4–19b. We add the forces using the method of components. The components of $\mathbf{F}_1$ are

$$F_{1x} = F_1 \cos 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N},$$
$$F_{1y} = F_1 \sin 45.0° = (40.0\text{ N})(0.707) = 28.3\text{ N}.$$

The components of $\mathbf{F}_2$ are

$$F_{2x} = +F_2 \cos 37.0° = +(30.0\text{ N})(0.799) = +24.0\text{ N},$$
$$F_{2y} = -F_2 \sin 37.0° = -(30.0\text{ N})(0.602) = -18.1\text{ N}.$$

F_{2y} is negative because it points along the negative y axis. The components of the resultant force are (see Fig. 4–19c)

$$F_{Rx} = F_{1x} + F_{2x} = 28.3\text{ N} + 24.0\text{ N} = 52.3\text{ N},$$
$$F_{Ry} = F_{1y} + F_{2y} = 28.3\text{ N} - 18.1\text{ N} = 10.2\text{ N}.$$

To find the magnitude of the resultant force, we use the Pythagorean theorem:

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(52.3)^2 + (10.2)^2} = 53.3\text{ N}.$$

The only remaining question is the angle θ that the net force $\mathbf{F}_R$ makes with the x axis. We use:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} = \frac{10.2\text{ N}}{52.3\text{ N}} = 0.195,$$

and $\tan^{-1}(0.195) = 11.0°$.

PROBLEM SOLVING

Free-body diagram

When solving problems involving Newton's laws and force, it is very important to draw a diagram showing all the forces acting *on* each object involved. Such a diagram is called a **free-body diagram**, or **force diagram**: we draw an arrow to represent each force acting on a given body, being sure to include *every* force acting on that body.

[When concerned only about translational motion, we can draw all the forces on a given body as acting at the center of the object, thus treating the object as a point. However, when doing problems involving rotation or statics, *where* each force acts is also important, as we shall see.]

CONCEPTUAL EXAMPLE 4–8 **The hockey puck.** A hockey puck is sliding at constant velocity across a flat horizontal ice surface that is assumed to be frictionless. Which of the sketches in Fig. 4–20 is the correct free-body diagram for this puck? What would your answer be if the puck slowed down?

RESPONSE Did you choose (a)? If so, can you answer the question: what exerts the horizontal force labeled **F**? If you say that it is the force needed to maintain the motion (as the ancient Greeks said), ask yourself: what exerts this force? Remember that another object must exert any force—and there simply isn't any possibility here. Therefore, (a) is wrong. Besides, the force **F** in Fig. 4–20a would give rise to an accelera-

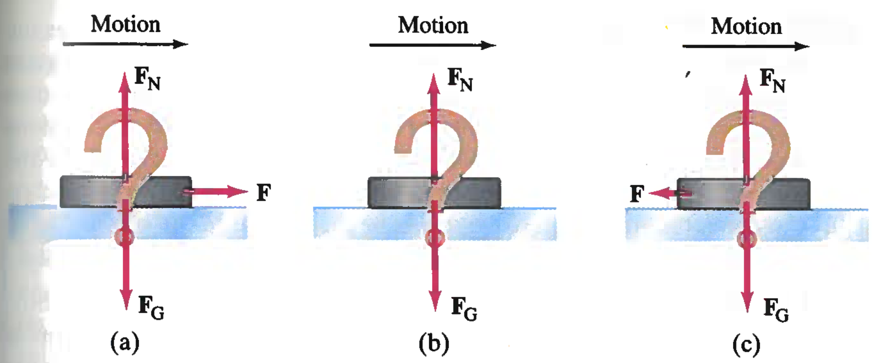


FIGURE 4–20 Which is the correct free-body diagram for a hockey puck sliding across frictionless ice (Example 4–8)?

tion by Newton's second law. It is (b) that is correct, as long as there is no friction. No net force acts on the puck, and the puck slides at constant velocity across the ice. But if someone insists that we come down from the ivory tower of idealized frictionless surfaces, down to the real world where even smooth ice exerts at least a tiny friction force, then (c) is the correct answer. The tiny friction force is in the direction opposite to the motion (it ought to be labeled $\mathbf{F}_{fr}$, not simply $\mathbf{F}$), and the puck's velocity decreases, even if very slowly.

Here now is a brief summary of how to approach solving problems involving Newton's laws:

➡ PROBLEM SOLVING Newton's Laws; Free-Body Diagrams

1. Draw a sketch of the situation.
2. Consider only one object (at a time), and draw a **free-body diagram** for that body, showing *all* the forces acting *on* that body, including any unknown forces that you have to solve for. Do not show any forces that the body exerts on other bodies. Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force, including forces you must solve for, as to its source (gravity, person, friction, and so on). If several bodies are involved, draw a free-body diagram for each body *separately*, showing all the forces acting *on that body* (and *only* forces acting on that body). For each (and every) force, you must be clear about: *on* what object that force acts; and *by* what object that force is exerted. Only forces acting *on* a given body can be included in $\Sigma\mathbf{F} = m\mathbf{a}$ for that body.
3. Newton's second law involves vectors, and it is usually important to resolve vectors into components. Choose an x and a y axis in a way that simplifies the calculation.
4. For each body, Newton's second law can be applied to the x and y components separately. That is, the x component of the net force on that body will be related to the x component of that body's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. Solve the equation or equations for the unknown(s).

This problem-solving box should not be considered a prescription. Rather it is a summary of things to do that will start your mind thinking and getting involved in the problem at hand.

In the Examples that follow, we assume that all surfaces are very smooth so that friction can be ignored. (Friction, and Examples using it, are discussed in the next Section.) In some of the following Examples, we will deal again with the large gift box on the table, which we first encountered in Example 4–5 (Fig. 4–16). In each successive Example, we add an additional complication so that, step by step, you can see how to approach solving problems.

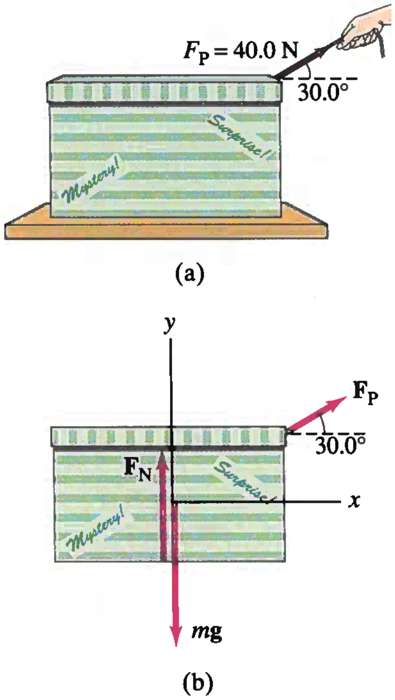


FIGURE 4–21 Example 4–9: (b) is the free-body diagram.

EXAMPLE 4–9 **Pulling the mystery box.** Suppose a friend asks to examine the 10.0-kg box you were given (Example 4–5, Fig. 4–16), hoping to guess what is inside; and you respond, "Sure, pull the box over to you." She then pulls the box by the attached ribbon (or string), as shown in Fig. 4–21a, along the smooth surface of the table. The magnitude of the force exerted by the person is $F_P = 40.0$ N, and it is exerted at a 30.0° angle as shown. Calculate (*a*) the acceleration of the box, and (*b*) the magnitude of the upward force F_N exerted by the table on the box. Assume that friction can be neglected.

SOLUTION Figure 4–21b shows the free-body diagram of the box, which means we show *all* the forces acting on the box and *only* the forces acting on the box. They are: the force of gravity $m\mathbf{g}$; the normal force exerted by the table $\mathbf{F}_N$; and the force exerted by the person $\mathbf{F}_P$. With the y axis vertical and the x axis horizontal, the pull of 40.0 N has components

$$F_{Px} = (40.0\ \text{N})(\cos 30.0°) = (40.0\ \text{N})(0.866) = 34.6\ \text{N},$$

$$F_{Py} = (40.0\ \text{N})(\sin 30.0°) = (40.0\ \text{N})(0.500) = 20.0\ \text{N}.$$

(*a*) In the horizontal (x) direction, $\mathbf{F}_N$ and $m\mathbf{g}$ have zero components. Thus the horizontal component of the net force is F_{Px}. From Newton's second law, $\Sigma F_x = ma_x$, we have

$$F_{Px} = ma_x,$$

so

$$a_x = \frac{F_{Px}}{m} = \frac{(34.6\ \text{N})}{(10.0\ \text{kg})} = 3.46\ \text{m/s}^2.$$

The acceleration of the box is thus 3.46 m/s² to the right.

(*b*) In the vertical (y) direction, with upward as positive, again using Newton's second law we have

$$\Sigma F_y = ma_y$$

$$F_N - mg + F_{Py} = ma_y.$$

Now $mg = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0$ N and $F_{Py} = 20.0$ N as we calculated above. Furthermore, we know $a_y = 0$ since the box does not even move vertically. Thus

$$F_N - 98.0\ \text{N} + 20.0\ \text{N} = 0$$

which tells us that the normal force is

$$F_N = 78.0\ \text{N}.$$

Notice that F_N is less than mg. The table does not push against the full weight of the box since part of the pull exerted by the person is in the upward direction. Compare this to Example 4–5, part *c*.

Tension in a cord.

When a flexible cord pulls on an object, the cord is said to be under **tension**, and the force it exerts on the object is the tension $\mathbf{F}_T$. If the cord has negligible mass, the force exerted at one end is transmitted undiminished to each adjacent piece of cord along the entire length to the other end. Note that flexible ropes and cords can only pull. They can't push because they bend.

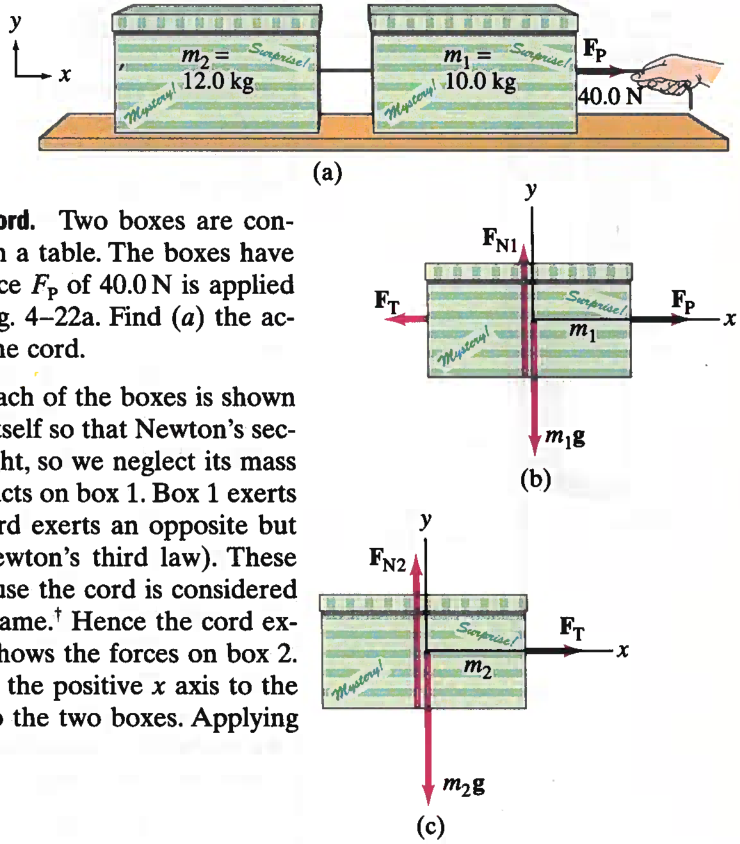


FIGURE 4–22 Example 4–10. (a) Two boxes are connected by a cord. A person pulls horizontally on box 1 with force F_P = 40.0 N. (b) Free-body diagram for box 1. (c) Free-body diagram for box 2.

EXAMPLE 4–10 **Two boxes connected by a cord.** Two boxes are connected by a lightweight cord and are resting on a table. The boxes have masses of 12.0 kg and 10.0 kg. A horizontal force F_P of 40.0 N is applied by a person to the 10.0-kg box, as shown in Fig. 4–22a. Find (*a*) the acceleration of each box, and (*b*) the tension in the cord.

SOLUTION (*a*) The free-body diagram for each of the boxes is shown in Figs. 4–22b and c. We consider each box by itself so that Newton's second law can be applied to each. The cord is light, so we neglect its mass relative to the mass of the boxes. The force F_P acts on box 1. Box 1 exerts a force F_T on the connecting cord, and the cord exerts an opposite but equal magnitude force F_T back on box 1 (Newton's third law). These forces on box 1 are shown in Fig. 4–22b. Because the cord is considered to be massless, the tension at each end is the same.† Hence the cord exerts a force F_T on the second box; Fig. 4–22c shows the forces on box 2. There will be only horizontal motion. We take the positive x axis to the right, and we use subscripts 1 and 2 to refer to the two boxes. Applying $\Sigma F_x = ma_x$ to box 1, we have:

$$\Sigma F_x = F_P - F_T = m_1 a_1.$$

For box 2, the only horizontal force is F_T, so

$$\Sigma F_x = F_T = m_2 a_2.$$

The boxes are connected, and if the cord remains taut and doesn't stretch, then the two boxes will have the same acceleration a. Thus $a_1 = a_2 = a$, and we are given m_1 = 10.0 kg and m_2 = 12.0 kg. We add the two equations above and obtain

$$(m_1 + m_2)a = F_P - F_T + F_T = F_P$$

or

$$a = \frac{F_P}{m_1 + m_2} = \frac{40.0\ \text{N}}{22.0\ \text{kg}} = 1.82\ \text{m/s}^2.$$

This is what we sought. Notice that we would have obtained the same result had we considered a single system, of mass $m_1 + m_2$, acted on by a net horizontal force equal to F_P. (The tension forces F_T would then be considered internal to the system as a whole, and summed together would make zero contribution to the net force on the *whole* system.)

➡ **PROBLEM SOLVING**

An alternate analysis

(*b*) From the equation above for box 2 ($F_T = m_2 a_2$), the tension in the cord is

$$F_T = m_2 a = (12.0\ \text{kg})(1.82\ \text{m/s}^2) = 21.8\ \text{N}.$$

†Since the mass m of the cord is zero, the net force on the cord is $\Sigma F = ma = 0$ no matter what a is. Hence the forces pulling on the cord at its two ends must add up to zero.

PHYSICS APPLIED

Elevator (as Atwood's machine)

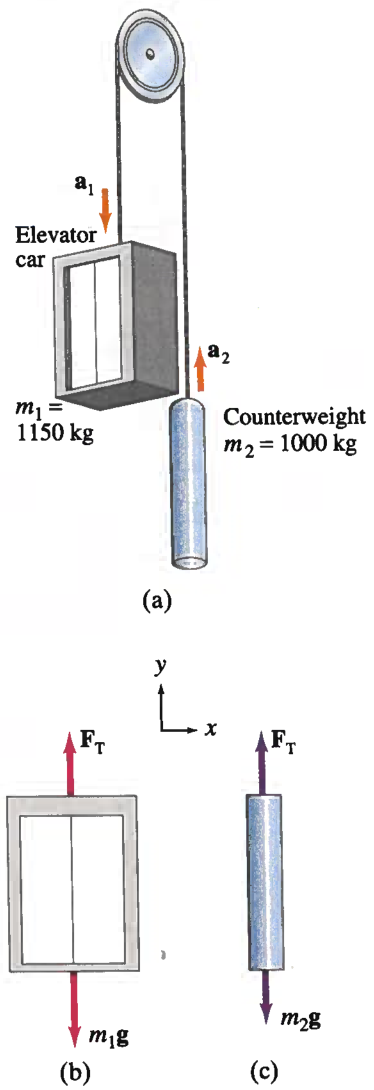


FIGURE 4–23 Example 4–11. (a) Atwood's machine in the form of an elevator-counterweight system. (b) and (c) Free-body diagrams for the two masses.

EXAMPLE 4–11 **Elevator and counterweight (Atwood's machine).** Two masses suspended over a pulley by a cable, as shown in Fig. 4–23a, is sometimes referred to generically as an *Atwood's machine*. Consider the real-life application of an elevator (m_1) and its counterweight (m_2). To minimize the work done by the motor to raise and lower the elevator safely, m_1 and m_2 are similar in mass. We leave the motor out of the system for this calculation, and assume the cable's mass is negligible and the pulley is frictionless and massless,[†] which assures that the tension $\mathbf{F}_\mathrm{T}$ in the cord has the same magnitude on both sides of the pulley. Let the mass of the counterweight be $m_2 = 1000\,\mathrm{kg}$. Assume the mass of the empty elevator is 850 kg, and its mass when carrying four passengers is $m_1 = 1150\,\mathrm{kg}$. For the latter case ($m_1 = 1150\,\mathrm{kg}$), calculate (*a*) the acceleration of the elevator and (*b*) the tension in the cable.

SOLUTION (*a*) Figures 4–23b and c show the free-body diagrams for the two masses. It is clear that m_1, being the heavier, will accelerate downward, and m_2 will accelerate upward. The magnitudes of their accelerations will be equal (we assume the cable doesn't stretch). For the counterweight, $m_2 g = (1000\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 9800\,\mathrm{N}$, so F_T must be greater than 9800 N (in order that m_2 will accelerate upward). For the elevator, $m_1 g = (1150\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 11{,}300\,\mathrm{N}$, which must have greater magnitude than F_T so that m_1 accelerates downward. Thus our calculation must give F_T between 9800 N and 11,300 N. To find F_T as well as the acceleration a, we apply $\Sigma F = ma$ to each box, where we take upward as the positive y direction for both boxes. With this choice of axes, $a_2 = a$, and $a_1 = -a$. Thus

$$F_\mathrm{T} - m_1 g = m_1 a_1 = -m_1 a$$

$$F_\mathrm{T} - m_2 g = m_2 a_2 = +m_2 a.$$

We subtract the first equation from the second to get

$$(m_1 - m_2)g = (m_1 + m_2)a.$$

We solve this for a:

$$a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{1150\,\mathrm{kg} - 1000\,\mathrm{kg}}{1150\,\mathrm{kg} + 1000\,\mathrm{kg}} g = 0.070g = 0.68\,\mathrm{m/s^2}.$$

The elevator (m_1) accelerates downward (and the counterweight m_2 upward) at $a = 0.070\,g = 0.68\,\mathrm{m/s^2}$.

(*b*) The tension in the cord, F_T, can be obtained from either of the two $\Sigma F = ma$ equations, setting $a = 0.070\,g$:

$$\begin{aligned} F_\mathrm{T} &= m_1 g - m_1 a = m_1 (g - a) \\ &= 1150\,\mathrm{kg}\,(9.80\,\mathrm{m/s^2} - 0.68\,\mathrm{m/s^2}) = 10500\,\mathrm{N}, \\ F_\mathrm{T} &= m_2 g + m_2 a = m_2 (g + a) \\ &= 1000\,\mathrm{kg}\,(9.80\,\mathrm{m/s^2} + 0.68\,\mathrm{m/s^2}) = 10500\,\mathrm{N}, \end{aligned}$$

which are consistent.

We can check our equation for the acceleration a in this Example by noting that if the masses were equal ($m_1 = m_2$), then our equation above for a would give $a = 0$, as we should expect. Also, if one of the masses is zero (say, $m_1 = 0$), then the other mass ($m_2 \neq 0$) would be predicted by our equation to accelerate at $a = g$, again as expected.

PROBLEM SOLVING

Check your result by seeing if it works in situations where the answer is easily guessed

[†]We'll see how to deal with a rotating pulley with mass in Chapter 8.

CONCEPTUAL EXAMPLE 4–12 **The advantage of a pulley.** Muscleman is trying to lift a piano (slowly) up to a second-story apartment (Fig. 4–24). He is using a rope looped over two pulleys as shown. How much of the piano's 2000 N weight does he have to pull on the rope?

RESPONSE Look at the forces acting on the lower pulley at the piano. The weight of the piano is pulling down. The tension in the rope, looped through this pulley, pulls up *twice*, once on each side of the pulley. Thus, Newton's second law gives

$$2F_T - mg = ma.$$

To move the piano with constant speed ($a = 0$) requires a tension in the cord, and hence a pull on the cord, of $F_T = mg/2$. Muscleman exerts a force equal to half the piano's weight. We say the pulley has given a **mechanical advantage** of 2, since without the pulley Muscleman would have to exert twice the force.

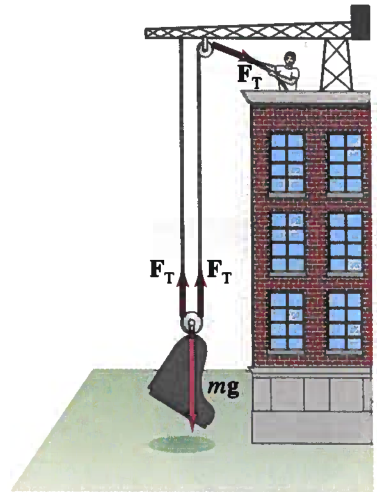


FIGURE 4–24 Conceptual Example 4–12.

EXAMPLE 4–13 **Getting the car out of the mud.** Finding her car stuck in the mud, a bright graduate of a good physics course ties a strong rope to the back bumper of the car, and the other end to a tree, as shown in Fig. 4–25a. She pushes at the midpoint of the rope with her maximum effort, which she estimates to be a force $F_P \approx 300$ N. The car just begins to budge with the rope at an angle θ (see the figure) which she estimates to be 5°. With what force is the rope pulling on the car? Neglect the mass of the rope.

How to get out of the mud

SOLUTION First, note that the tension in a rope is always along the rope. Any component perpendicular to the rope would cause the rope to bend or buckle (as it does here where $\mathbf{F}_P$ acts)—in other words, a rope can support a tension force only along its length. Let $\mathbf{F}_{T1}$ and $\mathbf{F}_{T2}$ be the forces the rope exerts on the tree and on the car, as shown in Fig. 4–25a. As our "free body," we choose the tiny section of rope where she pushes. The free-body diagram is shown in Fig. 4–25b, which shows $\mathbf{F}_P$ as well as the tensions in the rope (note that we have used Newton's third law). At the moment the car budges, the acceleration is still essentially zero, so $\mathbf{a} = 0$. For the x component of $\Sigma\mathbf{F} = m\mathbf{a} = 0$ on that small section of rope, we have

$$\Sigma F_x = F_{T2x} - F_{T1x} = 0, \quad \text{or} \quad F_{T1}\cos\theta - F_{T2}\cos\theta = 0.$$

Hence $F_{T1} = F_{T2}$, and we can write $F_T = F_{T1} = F_{T2}$. In the y direction, the forces acting are F_P, and the components of $\mathbf{F}_{T1}$ and $\mathbf{F}_{T2}$ that point in the negative y direction (each equal to $F_T \sin\theta$). So for the y component of $\Sigma\mathbf{F} = m\mathbf{a}$, we have

$$\Sigma F_y = F_P - 2F_T \sin\theta = 0.$$

We solve this for F_T and insert $F_P \approx 300$ N, which was given:

$$F_T = \frac{F_P}{2\sin\theta} = \frac{300\text{ N}}{2\sin 5°} \approx 1700\text{ N}.$$

She was able to magnify her effort almost six times using this technique! Notice the symmetry of the problem, which ensures that $F_{T1} = F_{T2}$.

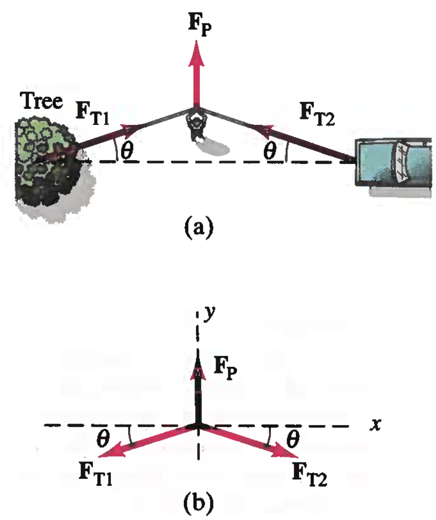


FIGURE 4–25 Example 4–13. Getting a car out of the mud.

PROBLEM SOLVING

Use any symmetry present to simplify a problem

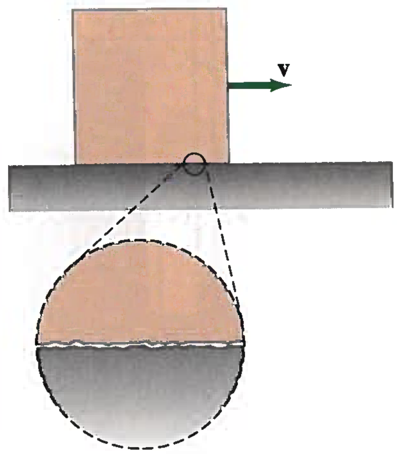


FIGURE 4–26 An object moving to the right on a table or floor. The two surfaces in contact are rough, at least on a microscopic scale.

4–8 Applications Involving Friction, Inclines

Until now we have ignored friction, but it must be taken into account in most practical situations. Friction exists between two solid surfaces because even the smoothest looking surface is quite rough on a microscopic scale, Fig. 4–26. When we try to slide an object across another surface, these microscopic bumps impede the motion. In addition, at the atomic level, the atoms on a bump of one surface come so close to the atoms of the other surface that electric forces between the atoms can form chemical bonds, as a tiny weld between the two surfaces. Sliding an object across a surface is often jerky due to the making and breaking of these bonds. Even when a body rolls across a surface, there is still some friction, called *rolling friction*, although it is generally much less than when a body slides across a surface. We will be concerned mainly with sliding friction in this section, and it is usually called **kinetic friction** (*kinetic* is from the Greek for "moving").

When a body is in motion along a rough surface, the force of kinetic friction acts opposite to the direction of the body's velocity. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, experiment shows that the friction force is approximately proportional to the *normal force* between the two surfaces, which is the force that either object exerts on the other, perpendicular to their common surface of contact (see Fig. 4–27). The force of friction between hard surfaces depends very little on the total surface area of contact; that is, the friction force on this book is roughly the same whether it is being slid on its wide face or on its spine, assuming the surfaces have the same smoothness. We can write the proportionality as an equation by inserting a constant of proportionality, μ_k:

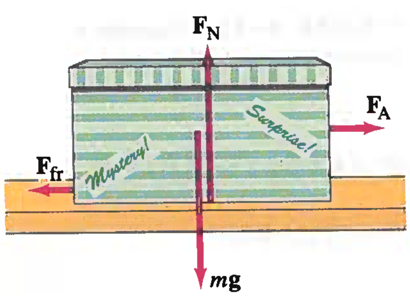
Kinetic friction

$$F_{fr} = \mu_k F_N.$$

This relation is not a fundamental law; it is an experimental relation between the magnitude of the friction force $\mathbf{F}_{fr}$ which acts parallel to the two surfaces, and the magnitude of the normal force $\mathbf{F}_N$ which acts perpendicular to the surfaces. It is *not* a vector equation since the two forces are perpendicular to one another. The term μ_k is called the *coefficient of kinetic friction*, and its value depends on the nature of the two surfaces. Measured values for a variety of surfaces are given in Table 4–2. These are only approximate, however, since μ depends on whether the surfaces are wet or dry, on how much they have been sanded or rubbed, if any burrs remain, and other such factors. But μ_k is roughly independent of the sliding speed.

$\mathbf{F}_{fr} \perp \mathbf{F}_N$

FIGURE 4–27 When an object is pulled by an applied force ($\mathbf{F}_A$) along a surface, the force of friction $\mathbf{F}_{fr}$ opposes the motion. The magnitude of $\mathbf{F}_{fr}$ is proportional to the magnitude of the normal force (F_N).



What we have been discussing up to now is *kinetic friction*, when one object slides over another. There is also **static friction**, which refers to a force parallel to the two surfaces that can arise even when they are not sliding. Suppose an object such as a desk is resting on a horizontal floor. If no horizontal force is exerted on the desk, there also is no friction force. But now, suppose you try to push the desk, but it doesn't move. You are exerting a horizontal force, but the desk isn't moving, so there must be another force on the desk keeping it from moving (the net force is zero on an object that doesn't move). This is the force of *static friction* exerted by the floor on the desk. If you push with a greater force without moving the desk, the force of static friction also has increased. If you push hard enough, the desk will

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TABLE 4–2 Coefficients of Friction†

Surfaces	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Wood on wood	0.4	0.2
Ice on ice	0.1	0.03
Metal on metal (lubricated)	0.15	0.07
Steel on steel (unlubricated)	0.7	0.6
Rubber on dry concrete	1.0	0.8
Rubber on wet concrete	0.7	0.5
Rubber on other solid surfaces	1–4	1
Teflon® on Teflon in air	0.04	0.04
Teflon on steel in air	0.04	0.04
Lubricated ball bearings	<0.01	<0.01
Synovial joints (in human limbs)	0.01	0.01

† Values are approximate and are intended only as a guide.

eventually start to move, and kinetic friction takes over. At this point, you have exceeded the maximum force of static friction, which is given by $F_{max} = \mu_s F_N$, where μ_s is the *coefficient of static friction* (Table 4–2). Since the force of static friction can vary from zero to this maximum value, we write

Static friction

$$F_{fr} \leq \mu_s F_N.$$

You may have noticed that it is often easier to keep a heavy object moving, such as pushing a table, than it is to start it moving in the first place. This is consistent with the fact (see Table 4–2) that μ_s is generally greater than μ_k. (It can never be less. Why?)

EXAMPLE 4–14 **Friction: static and kinetic.** Our 10.0-kg mystery box rests on a horizontal floor. The coefficient of static friction is $\mu_s = 0.40$ and the coefficient of kinetic friction is $\mu_k = 0.30$. Determine the force of friction, F_{fr}, acting on the box if a horizontal external applied force F_A is exerted on it of magnitude: (*a*) 0, (*b*) 10 N, (*c*) 20 N, (*d*) 38 N, and (*e*) 40 N.

SOLUTION The free-body diagram of the box is shown in Fig. 4–27. Examine it carefully. In the vertical direction there is no motion, so $\Sigma F_y = ma_y = 0$ yields $F_N - mg = 0$. Hence the normal force for all cases is

$$F_N = mg = (10.0\,\text{kg})(9.8\,\text{m/s}^2) = 98\,\text{N}.$$

(*a*) Since no force is applied in this first case, the box doesn't move, and $F_{fr} = 0$.
(*b*) The force of static friction will oppose any applied force up to a maximum of

$$\mu_s F_N = (0.40)(98\,\text{N}) = 39\,\text{N}.$$

The applied force is $F_A = 10\,\text{N}$. Thus the box will not move; since $\Sigma F_x = F_A - F_{fr} = 0$ then $F_{fr} = 10\,\text{N}$.
(*c*) An applied force of 20 N is also not sufficient to move the box. Thus $F_{fr} = 20\,\text{N}$ to balance the applied force.
(*d*) The applied force of 38 N is still not quite large enough to move the box; so the friction force has now increased to 38 N to keep the box at rest.

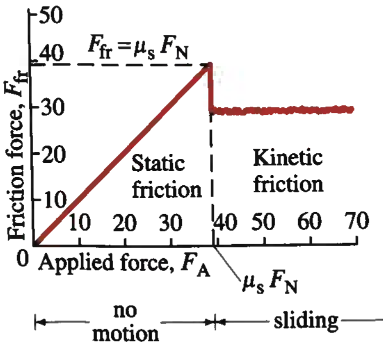


FIGURE 4–28 Magnitude of the force of friction as a function of the external force applied to a body initially at rest. As the applied force is increased in magnitude, the force of static friction increases linearly to just match it, until the applied force equals $\mu_s F_N$. If the applied force increases further, the body will begin to move, and the friction force drops to a roughly constant value characteristic of kinetic friction.

(*e*) A force of 40 N will start the box moving since it exceeds the maximum force of static friction, $\mu_s F_N = (0.40)(98\text{ N}) = 39\text{ N}$. Instead of static friction, we now have kinetic friction, and its magnitude is

$$F_{fr} = \mu_k F_N = (0.30)(98\text{ N}) = 29\text{ N}.$$

There is now a net (horizontal) force on the box of magnitude $F = 40\text{ N} - 29\text{ N} = 11\text{ N}$, so the box will accelerate at a rate

$$a_x = \Sigma F/m = 11\text{ N}/10\text{ kg} = 1.1\text{ m/s}^2$$

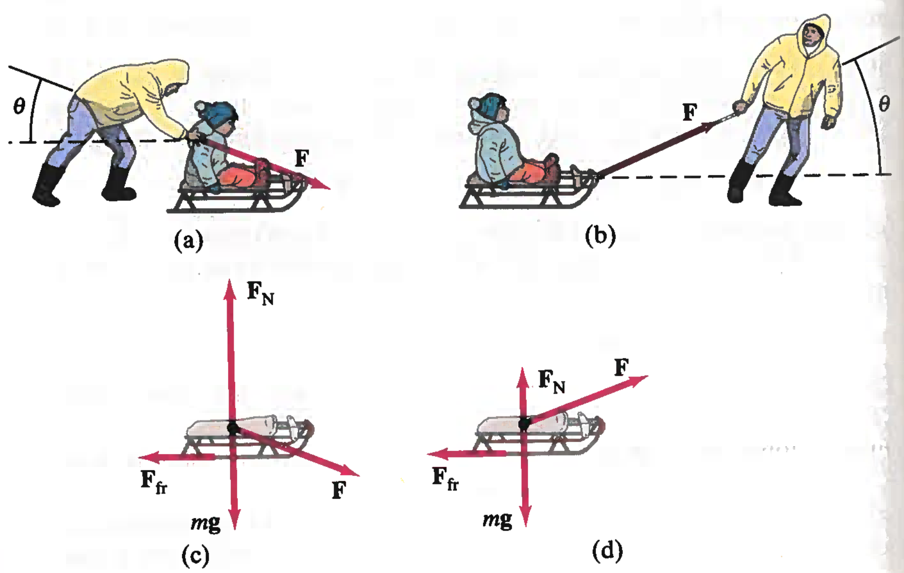
as long as the applied force is 40 N. Figure 4–28 shows a graph that summarizes this Example.

Now we look at some Examples involving kinetic friction in a variety of situations. Note that both the normal force and the friction force are forces exerted by one surface on another; one is perpendicular to the contact surfaces (the normal force), and the other is parallel (the friction force).

CONCEPTUAL EXAMPLE 4–15 **To push or to pull a sled?** Your little sister wants a ride on her sled. If you are on flat ground, will you exert less force if you push her or pull her? See Figs. 4–29a and b. Assume the same angle θ in each case.

RESPONSE Free-body diagrams are shown in Figs. 4-29c and d. If you push her, and $\theta > 0$, there is a vertically downward component to your force. Hence the normal force upward exerted by the ground will be larger than mg (where m is the mass of sister plus sled). If you pull her, your force has a vertically upward component, so the normal force F_N can be less than mg. Because the friction force is proportional to the normal force, it will be less if you pull her. So you exert less force if you pull her.

FIGURE 4–29 Conceptual Example 4–15.



EXAMPLE 4–16 **Two boxes and a pulley.** In Fig. 4–30a, two boxes are connected by a cord running over a pulley. The coefficient of kinetic friction between box I and the table is 0.20. We ignore the mass of the cord and pulley and any friction in the pulley, which means we can assume that a force applied to one end of the cord will have the same magnitude at the other end. We wish to find the acceleration, a, of the system, which will have the same magnitude for both boxes assuming the cord doesn't stretch. As box II moves down, box I moves to the right.

SOLUTION Free-body diagrams are shown for each box in Fig. 4–30b and c. Box I does not move vertically, so the normal force just balances the weight,

$$F_N = m_I g = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N}.$$

In the horizontal direction, there are two forces on box I (Fig. 4–30b): F_T, the tension in the cord (whose value we don't know), and the force of friction

$$F_{fr} = \mu_k F_N = (0.20)(49\,\text{N}) = 9.8\,\text{N}.$$

The horizontal acceleration is what we wish to find; we use Newton's second law in the x direction, $\Sigma F_{Ix} = m_I a_x$, which becomes (taking the positive direction to the right and setting $a_{Ix} = a$):

$$\Sigma F_{Ix} = F_T - F_{fr} = m_I a. \qquad \text{[box I]}$$

Next consider box II. The force of gravity $F_G = m_{II} g = 19.6\,\text{N}$ pulls downward; and the cord pulls upward with a force F_T. So we can write Newton's second law for box II (taking the downward direction as positive):

$$\Sigma F_{IIy} = m_{II} g - F_T = m_{II} a. \qquad \text{[box II]}$$

[Note here that if $a \neq 0$, then F_T is not equal to $m_{II} g$.] We have two unknowns, a and F_T, and we also have two equations. We solve the box I equation for F_T:

$$F_T = F_{fr} + m_I a,$$

and substitute this into the box II equation:

$$m_{II} g - F_{fr} - m_I a = m_{II} a.$$

Now we solve for a and put in numerical values:

$$a = \frac{m_{II} g - F_{fr}}{m_I + m_{II}} = \frac{19.6\,\text{N} - 9.8\,\text{N}}{5.0\,\text{kg} + 2.0\,\text{kg}} = 1.4\,\text{m/s}^2,$$

which is the acceleration of box I to the right, and of box II down.
If we wish, we can calculate F_T using the first equation:

$$F_T = F_{fr} + m_I a = 9.8\,\text{N} + (5.0\,\text{kg})(1.4\,\text{m/s}^2) = 17\,\text{N}.$$

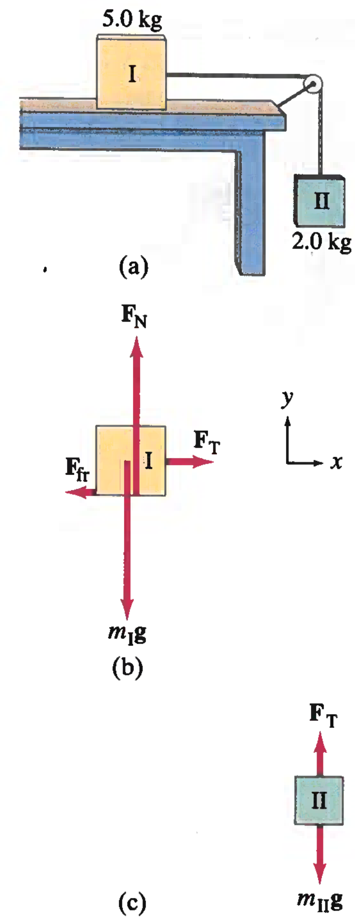
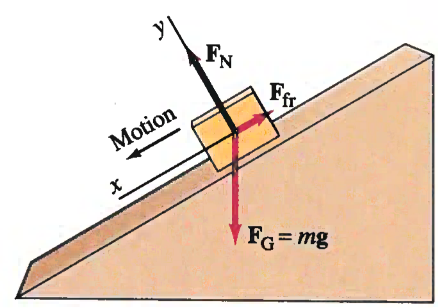


FIGURE 4–30 Example 4–16.

We now discuss some examples of objects moving on an incline such as a hill or a ramp. Solving problems is usually easier if we choose the xy coordinate system so that the x axis points along the incline (either up the incline, or down the incline), and the y axis perpendicular to the incline, as shown in Fig. 4–31. This helps because then **a** has only one component, and if friction is present, two of the forces will have only one component: $\mathbf{F}_{fr}$ along the plane, opposite to the object's velocity, and $\mathbf{F}_N$ which is not vertical but is perpendicular to the plane.

FIGURE 4–31 Forces on an object sliding down an incline.



PHYSICS APPLIED

Sport

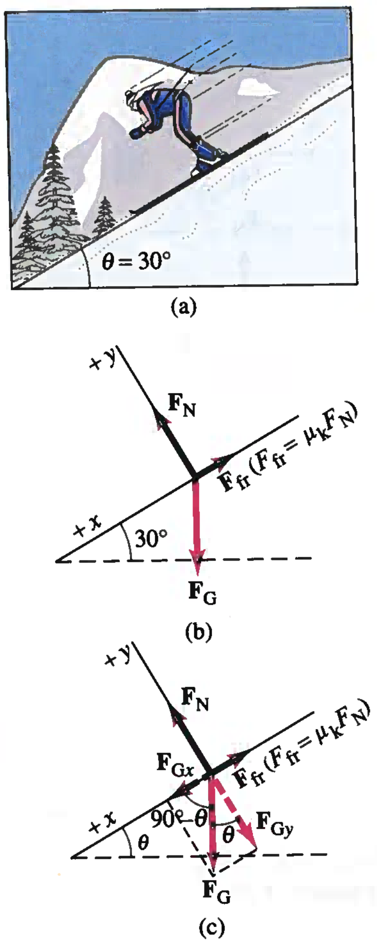


FIGURE 4–32 Example 4–17. A skier descending a slope.

EXAMPLE 4–17 **The skier.** The skier in Fig. 4–32a has just begun descending the 30° slope. Assuming the coefficient of kinetic friction is 0.10, (*a*) first draw the free-body diagram, then calculate (*b*) her acceleration and (*c*) the speed she will reach after 4.0 s.

SOLUTION (*a*) The free-body diagram in Fig. 4–32b shows all the forces acting on the skier: her weight ($\mathbf{F}_G = m\mathbf{g}$) downward, and the two forces exerted on her skis by the snow—the normal force perpendicular to the snow's surface, and the friction force parallel to the surface. These three forces are shown acting at one point in Fig. 4–32b, for convenience. Also for convenience, we choose the x axis parallel to the snow surface with positive direction downhill, and the y axis perpendicular to the surface. With this choice, we only have to resolve one vector into components, the weight. The components of the weight are shown as dashed lines in Fig. 4–32c. They are given by

$$F_{Gx} = mg\sin\theta,$$

$$F_{Gy} = mg\cos\theta,$$

where we have stayed general, using θ rather than 30° for now.

(*b*) To calculate her acceleration down the hill, a_x, we apply Newton's second law to the x direction:

$$\Sigma F_x = ma_x$$

$$mg\sin\theta - \mu_k F_N = ma_x$$

where the two forces are the component of the gravity force ($+x$ direction) and the friction force ($-x$ direction). We want to find the value of a_x, but we don't yet know F_N in the last equation. Let's see if we can get F_N from the y component of Newton's second law:

$$\Sigma F_y = ma_y$$

$$F_N - mg\cos\theta = ma_y = 0$$

where we set $a_y = 0$ because there is no motion in the y direction (perpendicular to the slope). Thus we can solve for F_N:

$$F_N = mg\cos\theta$$

and we can substitute this into our equation above for ma_x:

$$mg\sin\theta - \mu_k(mg\cos\theta) = ma_x.$$

There is an m in each term which can be canceled out. Thus (setting $\theta = 30°$ and $\mu_k = 0.10$):

$$a_x = g\sin 30° - \mu_k g\cos 30° = 0.50g - (0.10)(0.866)g = 0.41g$$

The skier's acceleration is 0.41 times the acceleration of gravity, which in numbers is $a = (0.41)(9.8\,\mathrm{m/s^2}) = 4.0\,\mathrm{m/s^2}$. It is interesting that the mass canceled out and so we have the useful conclusion that *the acceleration doesn't depend on the mass.* That such a cancellation sometimes occurs, and thus may give a useful conclusion as well as saving calculation, is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

(*c*) The speed after 4.0 s is found by using Eq. 2–10a:

$$v = v_0 + at = 0 + (4.0\,\mathrm{m/s^2})(4.0\,\mathrm{s}) = 16\,\mathrm{m/s},$$

where we assumed a start from rest.

PROBLEM SOLVING

It is often helpful to put in numbers only at the end

EXAMPLE 4–18 **Measuring μ_k.** Suppose in Example 4–17 that the snow is slushy and the skier moves down the 30° slope at constant speed. What can you say about the coefficient of friction, μ_k?

SOLUTION Now the skier moves down the slope at constant speed, and we want to find μ_k. The free-body diagram and the $\Sigma F = ma$ equations for the x and y components will be the same as above, except that now we are given $a_x = 0$. Thus

$$\Sigma F_y = F_N - mg\cos\theta = ma_y = 0$$

$$\Sigma F_x = mg\sin\theta - \mu_k F_N = ma_x = 0.$$

From the first equation, we have $F_N = mg\cos\theta$; we substitute this into the second equation:

$$mg\sin\theta - \mu_k(mg\cos\theta) = 0.$$

Now we solve for μ_k:

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

which for $\theta = 30°$ is

$$\mu_k = \tan\theta = \tan 30° = 0.58.$$

Notice that we could use the equation

$$\mu_k = \tan\theta$$

to determine μ_k under a variety of conditions. All we need to do is observe at what slope angle the skier descends at constant speed. Here is another reason why it is often useful to plug in numbers only at the end: we obtained a general result useful for other situations as well.

A method for determining μ_k

4–9 Problem Solving—A General Approach

A basic part of a physics course is solving problems effectively. Problem solving is important not only because it is valuable in itself, but also because it makes you think about the ideas and concepts, and by applying the concepts, you come to understand them better. We have already studied a range of Examples in these first few chapters. It is so important to learn problem solving that we will spend a little time now summarizing how to approach problems in general, even though much of this has been discussed before. The approach discussed here, though emphasizing Newton's laws, can be applied generally for other topics discussed throughout this book.

➥ PROBLEM SOLVING In General

1. **Read** and reread written problems carefully. A common error is to leave out a word or two when reading, which can completely change the meaning of a problem.
2. **Draw** an accurate picture or diagram of the situation. (This is probably the most overlooked, yet most crucial, part of solving a problem.) Use arrows to represent vectors such as velocity or force, and label the vectors with appropriate symbols. When dealing with forces and applying Newton's laws, make sure to include all forces on a given body, including unknown ones, and make clear what forces act on what body (otherwise you may make an error in determining the *net force* on a particular body). A separate *free-body diagram* needs to be drawn for each body involved, and it must show *all* the forces acting on a given body (and only on that body). Do not show forces that the body exerts on other bodies.
3. Choose a convenient *xy* **coordinate system** (choose one that makes your calculations easier). Vectors are to be resolved into components along these axes. When using Newton's second law, apply $\Sigma \mathbf{F} = m\mathbf{a}$ separately to x and y components, remembering that x direction forces are related to a_x, and similarly for y.
4. Note what the unknowns are—that is, what you are trying to determine—and decide what you need in order to find the unknowns. For problems in the present chapter, we use Newton's laws. More generally, it may help to see if there are one or more **relationships** (or **equations**) that relate the unknowns to the knowns. But be sure each relationship is applicable in the given case. It is very important to know the limitations of each formula or relationship—when it is valid and when not. In this book, the more general equations have been given numbers, but even these can have a limited range of validity (often stated briefly, in brackets, to the right of the equation).
5. Try to solve the problem approximately, to see if it is doable (to check if enough information has been given) and reasonable. Use your intuition, and make **rough calculations**—see "Order of Magnitude Estimating" in Section 1–7. A rough calculation, or a reasonable guess about what the range of final answers might be, is very useful. And a rough calculation can be checked against the final answer to catch errors in calculation (such as in a decimal point or the powers of 10).
6. **Solve** the problem, which may include algebraic manipulation of equations and/or numerical calculations. Substituting numbers into the equations only at the end can give you greater insight into the problem and to related ones.
7. Be sure to keep track of **units**, for they can serve as a check (they must balance on both sides of any equation).
8. Again consider if your answer is **reasonable**. The use of dimensional analysis, described in Appendix B, can also serve as a check for many problems.

SUMMARY

Newton's three laws of motion are the basic classical laws describing motion.

Newton's first law (the law of inertia) states that if the net force on an object is zero, an object originally at rest remains at rest, and an object in motion remains in motion in a straight line with constant velocity.

Newton's second law states that the acceleration of a body is directly proportional to the net force acting on it, and inversely proportional to its mass:

$$\Sigma \mathbf{F} = m\mathbf{a}.$$

Newton's second law is one of the most important and fundamental laws in classical physics.

Newton's third law states that whenever one body exerts a force on a second body, the second body always exerts a force on the first body which is equal in magnitude but opposite in direction:

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

The tendency of a body to resist a change in its motion is called **inertia**. **Mass** is a measure of the inertia of a body.

Weight refers to the force of gravity on a body, and is equal to the product of the body's mass m and the acceleration of gravity $\mathbf{g}$:

$$\mathbf{F}_G = m\mathbf{g}.$$

Force, which is a vector, can be considered as a push or pull; or, from Newton's second law, force can be defined as an action capable of giving rise to acceleration. The **net force** on an object is the vector sum of all forces acting on it.

When two bodies slide over one another, the force of friction that each body exerts on the other can be written approximately as $F_{fr} = \mu_k F_N$, where F_N is the **normal force** (the force each body exerts on the other perpendicular to their contact surfaces), and μ_k is the coefficient of **kinetic friction**. If the bodies are at rest relative to each other, then F_{fr} is just large enough to hold them at rest and satisfies the inequality $F_{fr} < \mu_s F_N$, where μ_s is the coefficient of **static friction**.

For solving problems involving the forces on one or more bodies, it is essential to draw a **free-body diagram** for each body, showing all the forces acting on only that body. Newton's second law can be applied to the vector components for each body.

QUESTIONS

1. Why does a child in a wagon seem to fall backward when you give the wagon a sharp pull?
2. What, roughly, does an apple weigh in newtons?
3. If the acceleration of a body is zero, are no forces acting on it?
4. Why do you push harder on the pedals of a bicycle when first starting out than when moving at constant speed?
5. Only one force acts on an object. Can the object have zero acceleration? Can it have zero velocity?
6. When a golf ball is dropped to the pavement, it bounces back up. (*a*) Is a force needed to make it bounce back up? (*b*) If so, what exerts the force?
7. Examine, in the light of Newton's first and second laws, the motion of your leg during one stride while walking.
8. Why might your foot hurt if you kick a heavy desk or a wall?
9. When you are running and want to stop quickly, you must decelerate quickly. (*a*) What is the origin of the force that causes you to stop? (*b*) Estimate (using your own experience) the maximum rate of deceleration of a person running at top speed to come to rest.

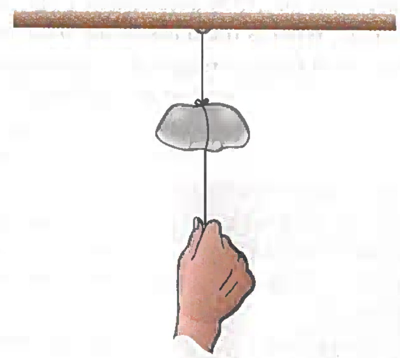


FIGURE 4–33 Question 10.

10. A stone hangs by a fine thread from the ceiling, and a section of the same thread dangles from the bottom of the stone (Fig. 4–33). If a person gives a sharp pull on the dangling thread, where is the thread likely to break: below the stone or above it? What if the person gives a slow and steady pull? Explain your answers.
11. The force of gravity on a 2-kg rock is twice as great as that on a 1-kg rock. Why then doesn't the heavier rock fall faster?

12. Compare the effort (or force) needed to lift a 10-kg object when you are on the Moon as compared to lifting it on Earth. Compare the force needed to throw a 2-kg object horizontally with a given speed when on the Moon as compared to on Earth.
13. Whiplash sometimes results from an automobile accident when the victim's car is struck violently from the rear. Explain why the head of the victim seems to be thrown backward in this situation. Is it really?
14. A person exerts an upward force of 40 N to hold a bag of groceries. Describe the "reaction" force (Newton's third law) by stating (*a*) its magnitude, (*b*) its direction, (*c*) *on* what body it is exerted, and (*d*) *by* what body it is exerted.
15. When you stand still on the ground, how large a force does the ground exert on you? Why doesn't this force make you rise up into the air?
16. According to Newton's third law, each team in a tug of war (Fig. 4–34) pulls with equal force on the other team. What, then, determines which team will win?
17. When driving on slick roads, why is it advisable to apply the brakes slowly?
18. Why is the stopping distance of a truck much shorter than for a train going the same speed?
19. Can a coefficient of friction exceed 1.0?

FIGURE 4–34 A tug of war. Describe the forces on each of the teams and on the rope. Question 16.

20. A block is given a push so that it slides up a ramp. After the block reaches its highest point, it slides back down. Why is the magnitude of its acceleration less on the descent than on the ascent?
21. A heavy crate rests on the bed of a flatbed truck. When the truck accelerates, the crate remains where it is on the truck, so it, too, accelerates. What force causes the crate to accelerate?
22. You can hold a heavy box against a rough wall and prevent it from slipping down by pressing only horizontally. How can the application of a *horizontal* force keep an object from moving vertically?

PROBLEMS

SECTIONS 4–4 TO 4–6

1. (I) What force is needed to accelerate a child on a sled (total mass = 60.0 kg) at $1.15\ m/s^2$?
2. (I) A net force of 255 N accelerates a bike and rider at $2.20\ m/s^2$. What is the mass of the bike and rider?
3. (I) How much force is required to accelerate a 9.0-g object at 10,000 "*g*'s" (say, in a centrifuge)?
4. (I) How much tension must a rope withstand if it is used to accelerate a 1050-kg car horizontally at $1.20\ m/s^2$? Ignore friction.
5. (I) What is the weight of a 66-kg astronaut (*a*) on Earth, (*b*) on the Moon ($g = 1.7\ m/s^2$), (*c*) on Mars ($g = 3.7\ m/s^2$), (*d*) in outer space traveling with constant velocity?
6. (II) A 20.0-kg box rests on a table. (*a*) What is the weight of the box and the normal force acting on it? (*b*) A 10.0-kg box is placed on top of the 20.0-kg box, as shown in Fig. 4–35. Determine the normal force that the table exerts on the 20.0-kg box and the normal force that the 20.0-kg box exerts on the 10.0-kg box.
7. (II) What average force is required to stop an 1100-kg car in 8.0 s if it is traveling at 90 km/h?
8. (II) What average force is needed to accelerate a 7.00-gram pellet from rest to 175 m/s over a distance of 0.700 m along the barrel of a rifle?

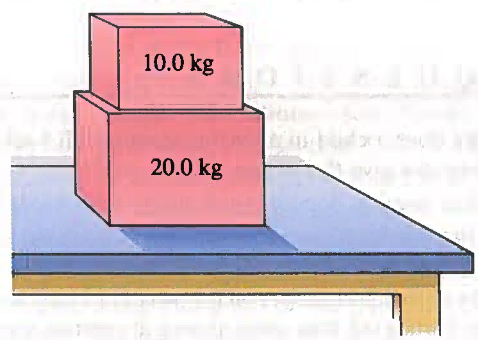


FIGURE 4–35 Problem 6.

9. (II) A fisherman yanks a fish out of the water with an acceleration of $4.5\ m/s^2$ using very light fishing line that has a "test" value of 22 N. The fisherman unfortunately loses the fish as the line snaps. What can you say about the mass of the fish?
10. (II) A 0.140-kg baseball traveling 45.0 m/s strikes the catcher's mitt, which, in bringing the ball to rest, recoils backward 11.0 cm. What was the average force applied by the ball on the glove?
11. (II) What is the average force exerted by a shot-putter on a 7.0-kg shot if the shot is moved through a distance of 2.8 m and is released with a speed of 13 m/s?
12. (II) How much tension must a rope withstand if it is used to accelerate a 1200-kg car vertically upward at $0.80\ m/s^2$? Ignore friction.

13. (II) A 10-kg bucket is lowered by a rope in which there is 63 N of tension. What is the acceleration of the bucket? Is it up or down?

14. (II) An elevator (mass 4850 kg) is to be designed so that the maximum acceleration is 0.0600 g. What are the maximum and minimum forces the motor should exert on the supporting cable?

15. (II) A 75-kg petty thief wants to escape from a third-story jail window. Unfortunately, a makeshift rope made of sheets tied together can support a mass of only 58 kg. How might the thief use this "rope" to escape? Give quantitative answer.

16. (II) A person stands on a bathroom scale in a motionless elevator. When the elevator begins to move, the scale briefly reads only 0.75 of the person's regular weight. Calculate the acceleration of the elevator, and find the direction of acceleration.

17. (II) The cable supporting a 2100-kg elevator has a maximum strength of 21,750 N. What maximum upward acceleration can it give the elevator without breaking?

18. (II) (*a*) What is the acceleration of two falling sky divers (mass 120.0 kg including parachute) when the upward force of air resistance is equal to one fourth of their weight? (*b*) After popping open the parachute, the divers descend leisurely to the ground at constant speed. What now is the force of air resistance on the sky divers and their parachute? See Fig. 4–36.

FIGURE 4–36 Problem 18.

19. (II) A Saturn V rocket has a mass of 2.75×10^6 kg and exerts a force of 33×10^6 N on the gases it expels. Determine (*a*) the initial vertical acceleration of the rocket, (*b*) its velocity after 8.0 s, and (*c*) how long it takes to reach an altitude of 9500 m. Ignore mass of gas expelled and assume g remains constant.

20. (III) An exceptional standing jump would raise a person 0.80 m off the ground. To do this, what force must a 66-kg person exert against the ground? Assume the person crouches a distance of 0.20 m prior to jumping, and thus the upward force has this distance to act over before he leaves the ground.

21. (III) A person jumps from the roof of a house 4.5-m high. When he strikes the ground below, he bends his knees so that his torso decelerates over an approximate distance of 0.70 m. If the mass of his torso (excluding legs) is 45 kg, find (*a*) his velocity just before his feet strike the ground, and (*b*) the average force exerted on his torso by his legs during deceleration.

22. (III) The 100-m dash can be run by the best sprinters in 10.0 s. A 65-kg sprinter accelerates uniformly for the first 50 m to reach top speed, which he maintains for the remaining 50 m. (*a*) What is the average horizontal component of force exerted on his feet by the ground during acceleration? (*b*) What is the speed of the sprinter over the last 50 m of the race (i.e., his top speed)?

SECTION 4–7

23. (I) A box weighing 70 N rests on a table. A rope tied to the box runs vertically upward over a pulley and a weight is hung from the other end (Fig. 4–37). Determine the force that the table exerts on the box if the weight hanging on the other side of the pulley weighs (*a*) 30 N, (*b*) 60 N, and (*c*) 90 N.

24. (I) A 650-N force acts in a northwesterly direction. A second 650-N force must be exerted in what direction so that the resultant of the two forces points westward?

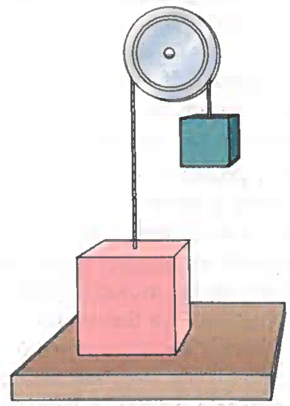
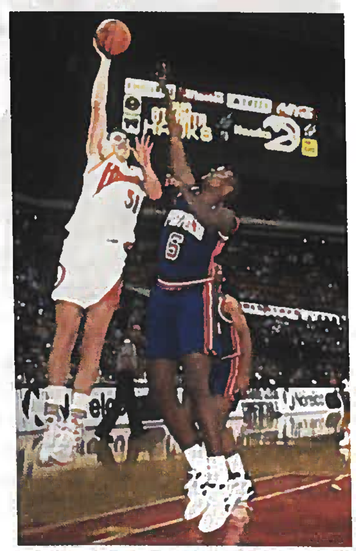


FIGURE 4–37 Problem 23.

25. (I) Draw the free-body diagram for a basketball player (*a*) just before leaving the ground on a jump, and (*b*) while in the air. (Fig. 4–38)

FIGURE 4–38
Problem 25.



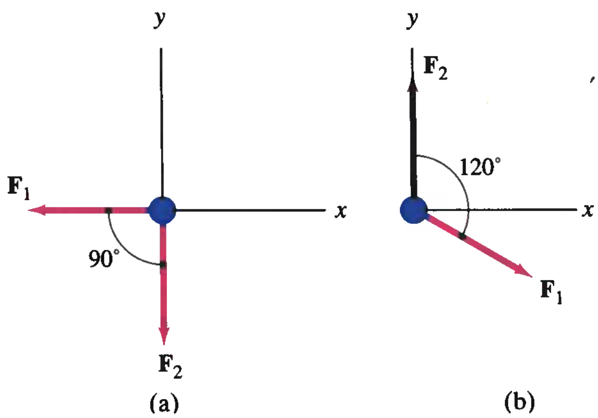


FIGURE 4–39 Problem 27.

26. (I) Sketch the free-body diagram of a baseball (*a*) at the moment it is hit by the bat, and again (*b*) after it has left the bat and is flying toward the outfield.

27. (II) The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Fig. 4–39a and b (looking down) act on a 27.0-kg object on a frictionless tabletop. If $F_1 = 10.2\text{ N}$ and $F_2 = 16.0\text{ N}$, find the net force on the object and its acceleration for each situation, (*a*) and (*b*).

28. (II) A person pushes a 14.5-kg lawn mower at constant speed with a force of 88.0 N directed along the handle, which is at an angle of 45.0° to the horizontal (Fig. 4–40). (*a*) Draw the free-body diagram showing all forces acting on the mower. Calculate (*b*) the horizontal retarding force on the mower, then (*c*) the normal force exerted vertically upward on the mower by the ground, and (*d*) the force the person must exert on the lawn mower to accelerate it from rest to 1.5 m/s in 2.5 seconds (assuming the same retarding force).

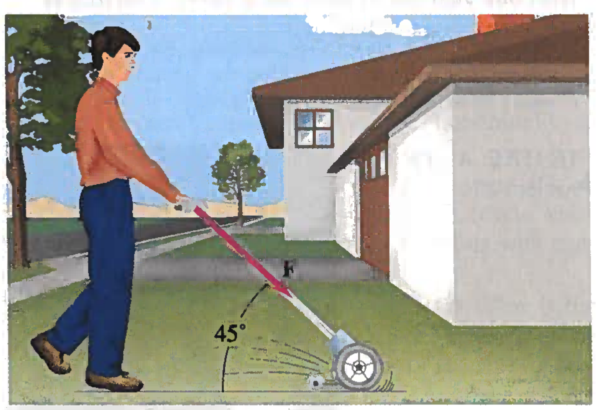


FIGURE 4–40 Problem 28.

29. (II) At the instant a race began, a 65-kg sprinter was found to exert a force of 800 N on the starting block at a 22° angle with respect to the ground (see chapter opening photo). (*a*) What was the horizontal acceleration of the sprinter? (*b*) If the force was exerted for 0.38 s, with what speed did the sprinter leave the starting block?

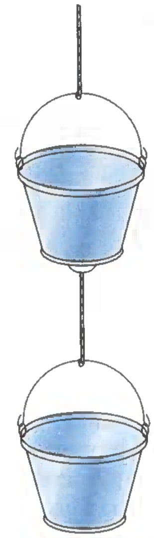


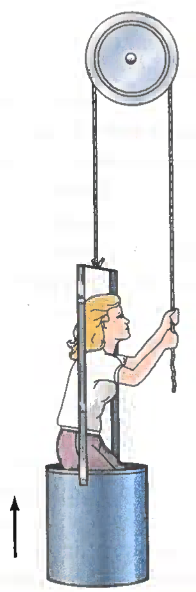
FIGURE 4–41 Problem 30.

30. (II) One 3.0-kg paint bucket is hanging by a massless cord from another 3.0-kg paint bucket, also hanging by a massless cord, as shown in Fig. 4–41. (*a*) If the buckets are at rest, what is the tension in each cord? (*b*) If the two buckets are pulled upward with an acceleration of 1.60 m/s^2 by the upper cord, calculate the tension in each cord.

31. (II) A 6500-kg helicopter accelerates upward at 0.60 m/s^2 while lifting a 1200-kg car. (*a*) What is the lift force exerted by the air on the rotors? (*b*) What is the tension in the cable (ignore its mass) that connects the car to the helicopter?

32. (II) A window washer pulls herself upward using the bucket-pulley apparatus shown in Fig. 4–42. (*a*) How hard must she pull downward to raise herself slowly at constant speed? (*b*) If she increases this force by 10 percent, what will her acceleration be? The mass of the person plus the bucket is 65 kg.

FIGURE 4–42
Problem 32.



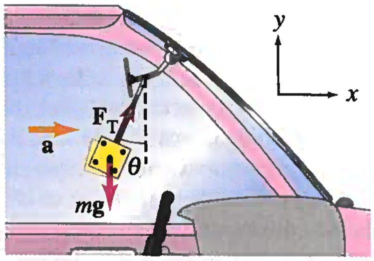


FIGURE 4–43 Problem 34.

33. (II) A train locomotive is pulling two cars of the same mass behind it. Show that the tension in the coupling between the locomotive and the first car is twice that between the first car and the second car, for any nonzero acceleration of the train.

34. (II) A pair of fuzzy dice is hanging by a string from your rearview mirror. While you are accelerating from a stoplight to 20 m/s (in 5.0 seconds), what angle θ does the string make with the vertical? See Fig. 4–43.

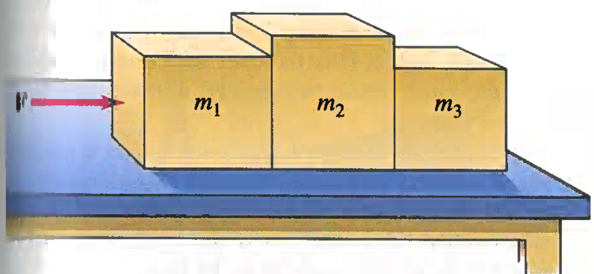


FIGURE 4–44 Problem 35.

35. (III) Three blocks on a frictionless horizontal surface are in contact with each other, as shown in Fig. 4–44. A force **F** is applied to block 1 (mass m_1). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_1, m_2, and m_3), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_1 = m_2 = m_3 = 12.0$ kg and $F = 96.0$ N, give numerical answers to (*b*), (*c*), and (*d*). Do your answers make sense intuitively?

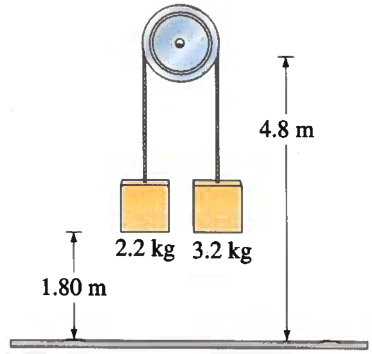


FIGURE 4–45 Problem 36.

36. (III) The two masses shown in Fig. 4–45 are each initially 1.80 m above the ground, and the massless frictionless pulley is 4.8 m above the ground. What maximum height does the lighter object reach after the system is released? [*Hint:* First determine the acceleration of the lighter mass and then its velocity at the moment the heavier one hits the ground. This is its "launch" speed.]

37. (III) Suppose the cord in Example 4–10 and Fig. 4–22 is a heavy rope of mass 1.0 kg. Calculate the acceleration of each box and the tension at each end of the cord, using the free-body diagrams shown in Fig. 4–46. Assume the cord doesn't sag.

SECTION 4–8

38. (I) If the coefficient of kinetic friction between a 35-kg crate and the floor is 0.30, what horizontal force is required to move the crate at a steady speed across the floor? What horizontal force is required if μ_k is zero?

39. (I) A force of 40.0 N is required to start a 5.0-kg box moving across a horizontal concrete floor. (*a*) What is the coefficient of static friction between the box and the floor? (*b*) If the 40.0-N force continues, the box accelerates at 0.70 m/s^2. What is the coefficient of kinetic friction?

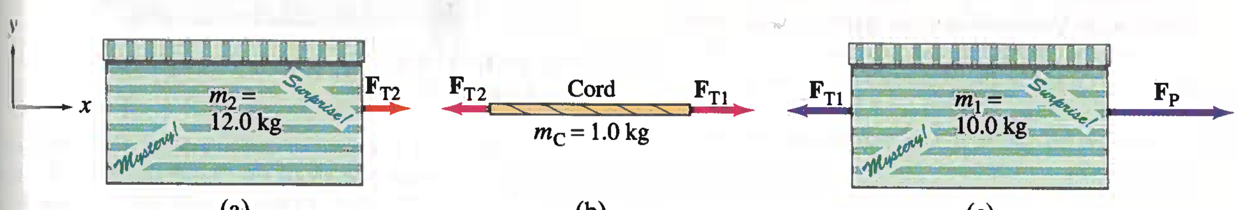


FIGURE 4–46 Problem 37. Free-body diagrams for each of the objects of the system shown in Fig. 4–22a. Vertical forces, F_N and F_G, are not shown.

40. (I) (*a*) A box sits at rest on a rough 30° inclined plane. Draw the free-body diagram, showing all the forces acting on the box. (*b*) How would the diagram change if the box were sliding down the plane? (*c*) How would it change if the box were sliding up the plane after an initial shove?

41. (I) A 2.0-kg silverware drawer becomes stuck, so the owner gradually pulls with more and more force. When the applied force reaches 8.0 N, the drawer suddenly opens, throwing all the utensils to the floor. Find the coefficient of static friction between the drawer and the cabinet.

42. (II) Drag race tires in contact with an asphalt surface probably have one of the highest coefficients of static friction in the everyday world. Assuming a constant acceleration and no slipping of tires, estimate the coefficient of static friction for a drag racer that covers the quarter mile in 6.0 s.

43. (II) For the system of Fig. 4–30 (Example 4–16), how large a mass would body I have to have to prevent any motion from occurring? Assume $\mu_s = 0.30$.

44. (II) A box is given a push so that it slides across the floor. How far will it go, given that the coefficient of kinetic friction is 0.20 and the push imparts an initial speed of 4.0 m/s?

45. (II) Two crates, of mass 75 kg and 110 kg, are in contact and at rest on a horizontal surface (Fig. 4–47). A 730-N force is exerted on the 75-kg crate. If the coefficient of kinetic friction is 0.15, calculate (*a*) the acceleration of the system, and (*b*) the force that each crate exerts on the other.

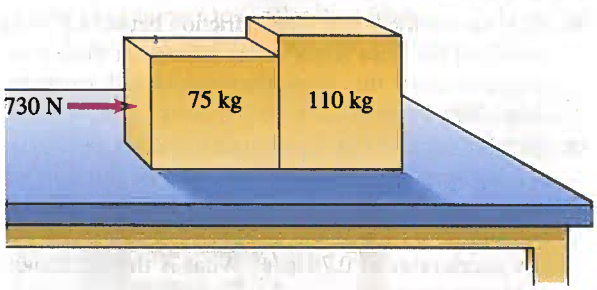


FIGURE 4–47 Problem 45.

46. (II) (*a*) Show that the minimum stopping distance for an automobile traveling at speed v is equal to $v^2/2\mu_s g$, where μ_s is the coefficient of static friction between the tires and the road, and g is the acceleration of gravity. (*b*) What is this distance for a 1200-kg car traveling 95 km/h if $\mu_s = 0.75$? (*c*) What would it be if the car were on the Moon but all else stayed the same?

47. (II) A flatbed truck is carrying a heavy crate. The coefficient of static friction between the crate and the bed of the truck is 0.75. What is the maximum rate at which the driver can decelerate and still avoid having the crate slide against the cab?

48. (II) On an icy day, you worry about parking your car in your driveway, which has an incline of 12°. Your neighbor Ralph's driveway has an incline of 9.0°, and Bonnie's driveway across the street has one of 6.0°. The coefficient of static friction between tire rubber and ice is 0.15. Which driveway(s) will be safe to park in?

49. (II) A child slides down a slide with a 28° incline, and at the bottom her speed is precisely half what it would have been if the slide had been frictionless. Calculate the coefficient of kinetic friction between the slide and the child.

50. (II) A coffee cup on the dashboard of a car slides forward on the dash when the driver decelerates from 40 km/h to rest in 3.5 s or less, but not if he decelerates in a longer time. What is the coefficient of static friction between the cup and the dash?

51. (II) A wet bar of soap (mass = 150 grams) slides without friction down a ramp 2.0 m long inclined at 7.3°. How long does it take to reach the bottom? Neglect friction. How would this change if the soap's mass were 250 grams?

52. (II) The block shown in Fig. 4–48 lies on a smooth plane tilted at an angle $\theta = 22.0°$ to the horizontal. (*a*) Determine the acceleration of the block as it slides down the plane. (*b*) If the block starts from rest 9.10 m up the plane from its base, what will be the block's speed when it reaches the bottom of the incline? Ignore friction.

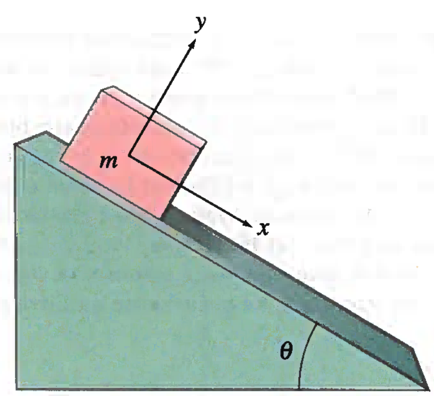


FIGURE 4–48 Block on inclined plane. Problems 52, 53, and 54.

53. (II) A block is given an initial speed of 3.0 m/s up the 22.0° plane shown in Fig. 4–48. (*a*) How far up the plane will it go? (*b*) How much time elapses before it returns to its starting point? Ignore friction.

54. (II) Repeat (*a*) Problem 52 and (*b*) Problem 53, assuming $\mu_k = 0.20$ between the block and plane.

55. (II) A roller coaster reaches the top of the steepest hill with a speed of 6.0 km/h. It then descends the hill, which is at an average angle of 45° and is 45.0 m long. What will its speed be when it reaches the bottom? Assume $\mu_k = 0.12$.

56. (II) An 18.0-kg box is released on a 37.0° incline and accelerates down the incline at $0.270\ \mathrm{m/s^2}$. Find the friction force impeding its motion. How large is the coefficient of friction?

57. (II) In the design of a supermarket, there are to be several ramps connecting different parts of the store. Customers will have to push grocery carts up the ramps, Fig. 4–49, and it is obviously desired that this not be too difficult. An engineer has done a survey and found that almost no one complains if the force required is no more than 50 N. Will a slope $\theta = 5°$ be too steep, assuming a 30-kg grocery cart (full of groceries)? Assume friction (wheels against ground, wheel on the axles, and so on), can be accounted for by a coefficient $\mu_k = 0.10$.

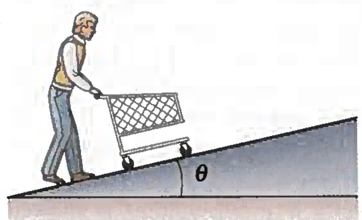


FIGURE 4–49 Problem 57.

58. (II) Figaro the cat (5.0 kg) is hanging on the tablecloth, pulling Cleo's fishbowl (11 kg) toward the edge of the table (Fig. 4–50). The coefficient of kinetic friction between the tablecloth (ignore its mass) under the fishbowl and the table is 0.44. (*a*) What is the acceleration of Figaro and the fishbowl? (*b*) If the fishbowl is 0.90 m from the edge of the table, how much time does it take for Figaro to pull Cleo off the table?

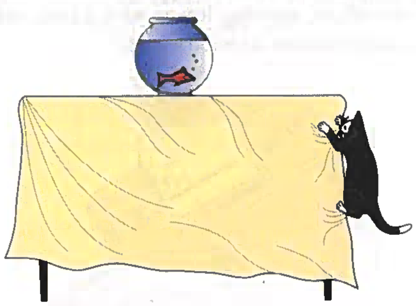


FIGURE 4–50 Problem 58.

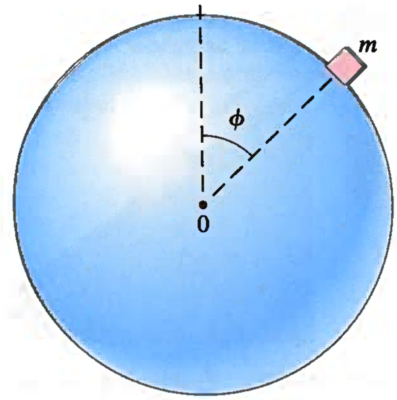


FIGURE 4–51 Problem 59.

59. (III) A small mass m is set on the surface of a sphere, Fig. 4–51. If the coefficient of static friction is $\mu_s = 0.60$, at what angle ϕ would the mass start sliding? [Hint: compare to Fig. 4–48; how are θ and ϕ related?]

60. (III) The 70-kg climber in Fig. 4–52 is supported in the "chimney" by the friction forces exerted on his shoes and back. The static coefficients of friction between his shoes and the wall, and between his back and the wall, are 0.80 and 0.60, respectively. What is the minimum normal force he must exert? Assume the walls are vertical and that friction forces are both at a maximum.

FIGURE 4–52 Problem 60.

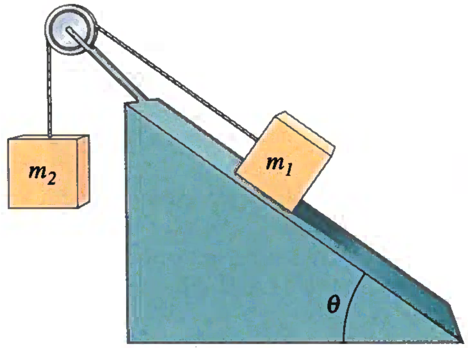


FIGURE 4–53 Problems 61, 62, and 63.

61. (III) A block (mass m_1) lying on a frictionless inclined plane is connected to a mass m_2 by a massless cord passing over a pulley, as shown in Fig. 4–53. (*a*) Determine a formula for the acceleration of the system in terms of m_1, m_2, θ, and g. (*b*) What conditions apply to masses m_1 and m_2 for the acceleration to be in one direction (say, m_1 down the plane), or in the opposite direction?

62. (III) Suppose the coefficient of kinetic friction between m_1 and the plane in Fig. 4–53 is $\mu_k = 0.15$, and that $m_1 = m_2 = 2.7$ kg. As m_2 moves down, determine the magnitude and direction of the acceleration of m_1 and m_2, given $\theta = 25°$.

63. (III) What smallest value of μ_k will keep the system of Problem 62 from accelerating?

64. (III) If a bicyclist of mass 65 kg (including the bicycle) can coast down a 6.0° hill at a steady speed of 6.0 km/h because of air resistance, how much force must be applied to climb the hill at the same speed (and the same air resistance)?

GENERAL PROBLEMS

65. According to a simplified model of a mammalian heart, at each pulse, approximately 20 g of blood is accelerated from 0.25 m/s to 0.35 m/s during a period of 0.10 s. What is the magnitude of the force exerted by the heart muscle?

66. A person has a reasonable chance of surviving an automobile crash if the deceleration is no more than 30 "g's." Calculate the force on a 70-kg person accelerating at this rate. What distance is traveled if brought to rest at this rate from 90 km/h?

67. (*a*) If the horizontal acceleration produced by an earthquake is a,and if an object is going to "hold its place" on the ground, show that the coefficient of static friction with the ground must be at least $\mu_s = a/g$. (*b*) The famous Loma Prieta earthquake that stopped the 1989 World Series produced ground accelerations of up to 4.0 m/s^2 in the San Francisco Bay Area. Would a chair have started to slide on a linoleum floor if the coefficient of static friction were 0.25?

68. A 1000-kg car pulls a 450-kg trailer. The car exerts a horizontal force of 3.5×10^3 N against the ground in order to accelerate. What force does the car exert on the trailer? Assume an effective friction coefficient of 0.15 for the trailer.

69. Police lieutenants, examining the scene of an accident involving two cars, measure the skid marks of one of the cars, which nearly came to a stop before colliding, to be 80 m long. The coefficient of kinetic friction between rubber and the pavement is about 0.80. Estimate the initial speed of that car assuming a level road.

70. A car starts rolling down a 1-in-4 hill (1-in-4 means that for each 4 m traveled along the road, the elevation change is 1 m). How fast is it going when it reaches the bottom after traveling 50 m? (*a*) Ignore friction. (*b*) Assume an effective coefficient of friction equal to 0.10.

71. A fisherman in a boat is using a "10-lb test" fishing line. This means that the line can exert a force of 45 N without breaking (1 lb = 4.45 N). (*a*) How heavy a fish can the fisherman land if he pulls the fish up vertically at constant speed? (*b*) If he accelerates the fish upwards at 2.0 m/s^2, what maximum weight fish can he land?

72. An elevator in a tall building is allowed to reach a maximum speed of 3.5 m/s going down. What must the tension be in the cable to stop this elevator over a distance of 3.0 m if the elevator has a mass of 1300 kg including occupants?

73. Two boxes, $m_1 = 1.0$ kg with a coefficient of kinetic friction of 0.10, and $m_2 = 2.0$ kg with a coefficient of 0.20, are placed on a plane inclined at $\theta = 30°$. (*a*) What acceleration does each block experience? (*b*) If a taut string is connected to the blocks (Fig. 4–54), with m_2 initially farther down the slope, what is the acceleration of each block? (*c*) If the initial configuration is reversed with m_1 starting lower with a taut string, what is the acceleration of each block?

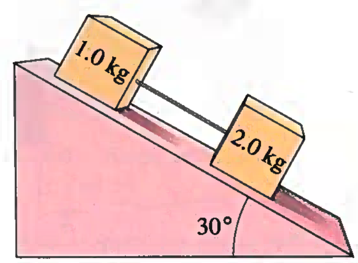


FIGURE 4–54 Problem 73.

74. A 75.0-kg person stands on a scale in an elevator. What does the scale read (in kg) when (*a*) the elevator is at rest, (*b*) the elevator is climbing at a constant speed of 3.0 m/s, (*c*) the elevator is falling at 3.0 m/s, (*d*) the elevator is accelerating upward at 3.0 m/s^2, (*e*) the elevator is accelerating downward at 3.0 m/s^2?

75. A small block of mass m is given an initial speed v_0 up a ramp inclined at angle θ to the horizontal. It travels a distance d up the ramp and comes to rest. (*a*) Determine a formula for the coefficient of kinetic friction between the block and ramp. (*b*) What can you say about the value of the coefficient of static friction?

76. A motorcyclist is coasting with the engine off at a steady speed of 17 m/s but enters a sandy stretch where the coefficient of friction is 0.80. Will the cyclist emerge from the sandy stretch without having to start the engine if the sand lasts for 15 m? If so, what will be the speed upon emerging?

77. A city planner is working on the redesign of a hilly portion of a city. An important consideration is how steep the roads can be so that even low-powered cars can get up the hills without slowing down. It is given that a particular small car, with a mass of 1100 kg, can accelerate on a level road from rest to 21 m/s (75 km/h) in 14.0 s. Using this data, calculate the maximum steepness of a hill.

78. A bicyclist can coast down a 5.0° hill at a constant 6.0 km/h. If the force of friction (air resistance) is proportional to the speed v so that $F_{fr} = cv$, calculate (*a*) the value of the constant c, and (*b*) the average force that must be applied in order to descend the hill at 20 km/h. The mass of the cyclist plus bicycle is 80 kg.

79. Jean, who likes physics experiments, dangles her watch from a thin piece of string while the jetliner she is in takes off from Dulles Airport (Fig. 4–55). She notices that the string makes an angle of 25° with respect to the vertical while the aircraft accelerates for takeoff, which takes about 18 seconds. Estimate the takeoff speed of the aircraft.

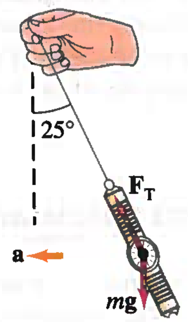


FIGURE 4–55 Problem 79.

80. (*a*) What minimum force F is needed to lift the piano (mass M) using the pulley apparatus shown in Fig. 4–56? (*b*) Determine the tension in each section of rope: F_{T1}, F_{T2}, F_{T3}, and F_{T4}.

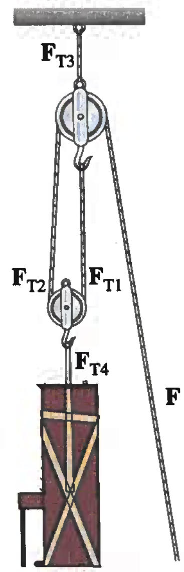


FIGURE 4–56 Problem 80.

81. A 28.0-kg block is connected to an empty 1.00-kg bucket by a cord running over a frictionless pulley (Fig. 4–57). The coefficient of static friction between the table and the block is 0.450 and the coefficient of kinetic friction between the table and the block is 0.320. Sand is gradually added to the bucket until the system just begins to move. (*a*) Calculate the mass of sand added to the bucket. (*b*) Calculate the acceleration of the system.

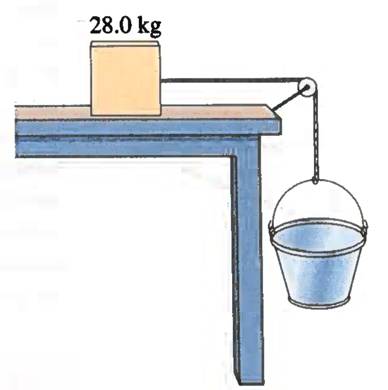


FIGURE 4–57 Problem 81.