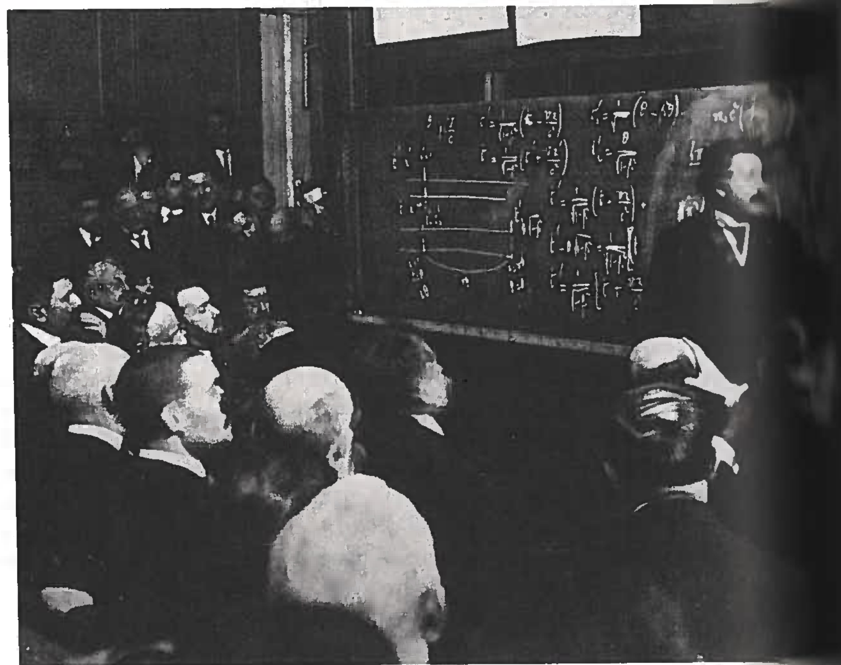
Albert Einstein (1879–1955), one of the great minds of the twentieth century, creator of the special and general theories of relativity, here shown lecturing.



C H A P T E R

26 SPECIAL THEORY OF RELATIVITY

FIGURE 26–1 Albert Einstein and his second wife.

Physics at the end of the nineteenth century looked back on a period of great progress. The theories developed over the preceding three centuries had been very successful in explaining a wide range of natural phenomena. Newtonian mechanics beautifully explained the motion of objects on Earth and in the heavens. Furthermore, it formed the basis for successful treatments of fluids, wave motion, and sound. Kinetic theory explained the behavior of gases and other materials. Maxwell's theory of electromagnetism not only brought together and explained electric and magnetic phenomena, but it predicted the existence of electromagnetic (EM) waves that would behave in every way just like light—so light came to be thought of as an electromagnetic wave. Indeed, it seemed that the natural world, as seen through the eyes of physicists, was very well explained. A few puzzles remained, but it was felt that these would soon be explained using already known principles.

But it did not turn out so simply. Instead, these few puzzles were to be solved only by the introduction, in the early part of the twentieth century, of two revolutionary new theories that changed our whole conception of nature: the *theory of relativity* and *quantum theory*.

Physics as it was known at the end of the nineteenth century (what we've covered up to now in this book) is referred to as **classical physics**. The new physics that grew out of the great revolution at the turn of the twentieth century is now called **modern physics**. In this chapter, we present the special theory of relativity, which was first proposed by Albert Einstein (1879–1955; Fig. 26–1) in 1905. In the following chapter, we introduce the equally momentous quantum theory.

Classical vs. modern physics

26–1 Galilean–Newtonian Relativity

Einstein's special theory of relativity deals with how we observe events, particularly how objects and events are observed from different frames of reference.[†] This subject had, of course, already been explored by Galileo and Newton. We first briefly discuss these earlier ideas, before seeing (starting in Section 26–3) how the theory of relativity changed them.

The special theory of relativity deals with events that are observed and measured from so-called **inertial reference frames**, which (as mentioned in Chapter 4) are reference frames in which Newton's first law, the law of inertia, is valid. (Newton's first law states that, if an object experiences no net force, the object either remains at rest or continues in motion with constant velocity in a straight line.) It is easiest to analyze events when they are observed and measured from inertial frames, and the Earth, though not quite an inertial frame (it rotates), is close enough that for most purposes we can consider it an inertial frame. Rotating or otherwise accelerating frames of reference are noninertial frames,[‡] and Einstein dealt with such complicated frames of reference in his general theory of relativity (Chapter 33).

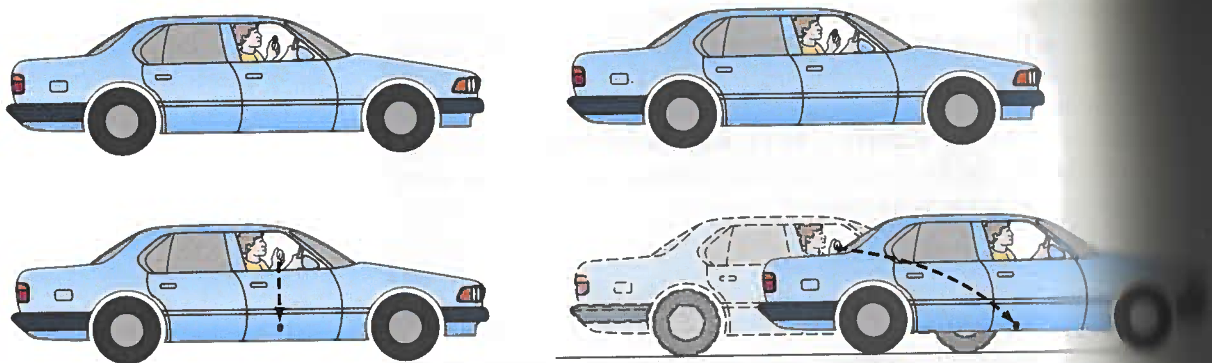
A reference frame that moves with constant velocity with respect to an inertial frame is itself also an inertial frame, since Newton's laws hold in it as well. When we say that we observe or make measurements from a certain reference frame, it means that we are at rest in that reference frame.

Both Galileo and Newton were aware of what we now call the **relativity principle** applied to mechanics: that *the basic laws of physics are the same in all inertial reference frames*. You may have recognized its validity in everyday life. For example, objects move in the same way in a smoothly moving (constant-velocity) train or airplane as they do on Earth. (This assumes no vibrations or rocking—for they would make the reference frame noninertial.) When you walk, drink a cup of soup, play Ping-Pong, or drop a pencil on the floor while traveling in a train, airplane, or ship moving at constant velocity, the bodies move just as they do when you are at rest on Earth. Suppose you are in a car traveling rapidly along at constant velocity. If you release a coin from above your head inside the car, how will it fall? It falls straight downward with respect to the car, and hits the floor

Relativity principle: the laws of physics are the same in all inertial reference frames

[†] A reference frame is a set of coordinate axes fixed to some body such as the Earth, a train, the Moon, and so on. See Section 2–1.

[‡] On a rotating platform (say a merry-go-round), for example, an object at rest starts moving outward even though no body exerts a force on it. This is therefore not an inertial frame. See Appendix C, Fig. C–1.



(a)
Reference frame = car

(b)
Reference frame = Earth

FIGURE 26–2 A coin is dropped by a person in a moving car. (a) In the reference frame of the car, the coin falls straight down. (b) In a reference frame fixed on the Earth, the coin follows a curved (parabolic) path. The upper views show the moment of the coin's release, and the lower views are a short time later.

directly below the point of release, Fig. 26–2a. (If you drop the [illegible] the car's window, this won't happen because the moving air drags [illegible] backward relative to the car.) This is just how objects fall on the [illegible] straight down—and thus our experiment in the moving car is in [illegible] with the relativity principle.

Note in this example, however, that to an observer on the [illegible] coin follows a curved path, Fig. 26–2b. The actual path followed [illegible] coin is different as viewed from different frames of reference. This [illegible] not violate the relativity principle because this principle states that [illegible] *laws* of physics are the same in all inertial frames. The same law of [illegible] ty, and the same laws of motion, apply in both reference frames. [illegible] acceleration of the coin is the same in both reference frames. The [illegible] ence in Figs. 26–2a and b is that in the Earth's frame of reference, the [illegible] has an initial velocity (equal to that of the car). The laws of physics [illegible] fore predict it will follow a parabolic path like any projectile. In the [illegible] reference frame, there is no initial velocity, and the laws of physics [illegible] that the coin will fall straight down. The laws are the same in both [illegible] ence frames, although the specific paths are different.†

Galilean–Newtonian relativity involves certain unprovable assump[illegible] that make sense from everyday experience. It is assumed that the length [illegible] objects are the same in one reference frame as in another, and that [illegible] passes at the same rate in different reference frames. In classical me[illegible] then, space and time are considered to be **absolute**: their measure[illegible] doesn't change from one reference frame to another. The mass of an [illegible] as well as all forces, are assumed to be unchanged by a change in [illegible] reference frame.

The position of an object is, of course, different when specified in [illegible] ent reference frames, and so is velocity. For example, a person may [illegible] side a bus toward the front with a speed of 5 km/h. But if the bus [illegible] 40 km/h with respect to the Earth, the person is then moving with a [illegible] 45 km/h with respect to the Earth. The acceleration of a body, how[illegible] the same in any inertial reference frame according to classical mecha[illegible] This is because the change in velocity, and the time interval, will be the [illegible]

†Galileo, in his great book *Dialogues on the Two Chief Systems of the World*, desc[illegible] ilar experiment and predicted the same results. Galileo's example involved a sailor [illegible] a knife from the top of the mast of a sailing vessel. If the vessel moves at constant [illegible] where will the knife hit the deck (ignoring Earth's rotation and air resistance)?

For example, the person in the bus may accelerate from 0 to 5 km/h in 1.0 seconds, so $a = 5\,\text{km/h/s}$ in the reference frame of the bus. With respect to the Earth, the acceleration is $(45\,\text{km/h} - 40\,\text{km/h})/(1.0\,\text{s}) = 5\,\text{km/h/s}$, which is the same.

Since neither F, m, nor a changes from one inertial frame to another, then Newton's second law, $F = ma$, does not change. Thus Newton's second law satisfies the relativity principle. It is easily shown that the other laws of mechanics also satisfy the relativity principle.

That the laws of mechanics are the same in all inertial reference frames implies that no one inertial frame is special in any sense. We express this important conclusion by saying that **all inertial reference frames are equivalent** for the description of mechanical phenomena. No one inertial reference frame is any better than another. A reference frame fixed to a car or an aircraft traveling at constant velocity is as good as one fixed on the Earth. When you travel smoothly at constant velocity in a car or airplane, it is just as valid to say you are at rest and the Earth is moving as it is to say the reverse. There is no experiment you can do to tell which frame is "really" at rest and which is moving. Thus, there is no way to single out one particular reference frame as being at absolute rest.

All inertial reference frames are equally valid

A complication arose, however, in the last half of the nineteenth century. When Maxwell presented his comprehensive and very successful theory of electromagnetism (Chapter 22), he showed that light can be considered an electromagnetic wave. Maxwell's equations predicted that the velocity of light c would be $3.00 \times 10^8\,\text{m/s}$; and this is just what is measured, within experimental error. The question then arose: in what reference frame does light have precisely the value predicted by Maxwell's theory? For it was assumed that light would have a different speed in different frames of reference. For example, if observers were traveling on a rocket ship at a speed of $1.0 \times 10^8\,\text{m/s}$ away from a source of light, we might expect them to measure the speed of the light reaching them to be $3.0 \times 10^8\,\text{m/s} - 1.0 \times 10^8\,\text{m/s} = 2.0 \times 10^8\,\text{m/s}$. But Maxwell's equations have no provision for relative velocity. They predicted the speed of light to be $c = 3.0 \times 10^8\,\text{m/s}$. This seemed to imply there must be some special reference frame where c would have this value.

We discussed in Chapters 11 and 12 that waves travel on water and along ropes or strings, and sound waves travel in air and other materials. Nineteenth-century physicists viewed the material world in terms of the laws of mechanics, so it was natural for them to assume that light too must travel in some *medium*. They called this transparent medium the **ether** and assumed it permeated all space.† It was therefore assumed that the velocity of light given by Maxwell's equations must be with respect to the ether.

The "ether"

However, it appeared that Maxwell's equations did *not* satisfy the relativity principle. They were not the same in all inertial reference frames. They were simplest in the frame where $c = 3.00 \times 10^8\,\text{m/s}$; that is, in a reference frame at rest in the ether. In any other reference frame, extra terms would have to be added to take into account the relative velocity. Thus, although most of the laws of physics obeyed the relativity principle,

†The medium for light waves could not be air, since light travels from the Sun to Earth through nearly empty space. Therefore, another medium was postulated, the ether. The ether was not only transparent, but, because of difficulty in detecting it, was assumed to have zero density.

the laws of electricity and magnetism apparently did not. Inst... seemed to single out one reference frame that was better th... other—a reference frame that could be considered absolutely at r...

Scientists soon set out to determine the speed of the Earth rel... this absolute frame, whatever it might be. A number of clever exp... were designed. The most direct were performed by A. A. Michelso... E. W. Morley in the 1880s. The details of their experiment are d... the next Section. Briefly, what they did was measure the differen... speed of light in different directions. They expected to find a diffe... pending on the orientation of their apparatus with respect to the eth... just as a boat has different speeds relative to the land when it mo... stream, downstream, or across the stream, so too light would be exp... have different speeds depending on the velocity of the ether past th...

Strange as it may seem, they detected no difference at all. Th... great puzzle. A number of explanations were put forth over a period... but they led to contradictions or were otherwise not generally accep...

Then in 1905, Albert Einstein proposed a radical new theory th... onciled these many problems in a simple way. But at the same time,... shall soon see, it completely changed our ideas of space and time.

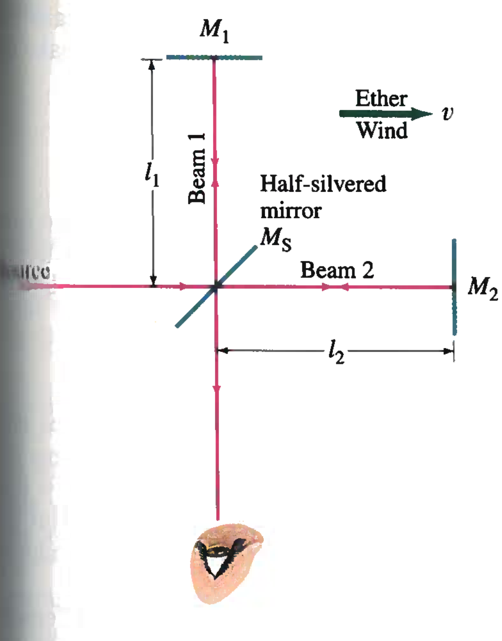
* 26–2 The Michelson–Morley Experiment

The Michelson–Morley experiment was designed to measure the sp... the *ether*—the medium in which light was assumed to travel—with re... to the Earth. The experimenters thus hoped to find an absolute ref... frame, one that could be considered to be at rest.

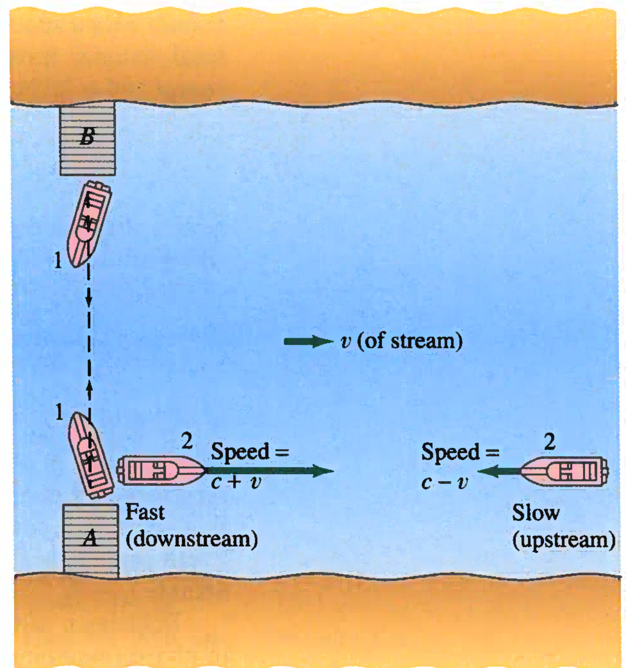
One of the possibilities nineteenth-century scientists consider... that the ether is fixed relative to the Sun, for even Newton had tak... Sun as the center of the universe. If this were the case (there was no g... antee, of course), the Earth's speed of about 3×10^4 m/s in its orbit ... the Sun would produce a change of 1 part in 10^4 in the speed of ... (3.0×10^8 m/s). Direct measurement of the speed of light to this ac... was not possible. But A. A. Michelson, later with the help of E. W. M... was able to use his interferometer (Section 24–9) to measure the diff... in the speed of light in different directions to this accuracy. This fam... periment is based on the principle shown in Fig. 26–3. Part (a) is a di... of the Michelson interferometer, and it is assumed that the "ether w... moving with speed v to the right. (Alternatively, the Earth is assum... move to the left with respect to the ether at speed v.) The light fr... source is split into two beams by the half-silvered mirror M_S. One b... travels to mirror M_1 and the other to mirror M_2. The beams are refl... M_1 and M_2 and are joined again after passing through M_S. The no... posed beams interfere with each other and the resultant is viewed b... observer's eye as an interference pattern (discussed in Section 24–9).

Whether constructive or destructive interference occurs at the ... of the interference pattern depends on the relative phases of the ... beams after they have traveled their separate paths. To examine this ... consider an analogy of a boat traveling up and down, and across, a ri... whose current moves with speed v, as shown in Fig. 26–3b. In still w... the boat can travel with speed c (not the speed of light in this case).

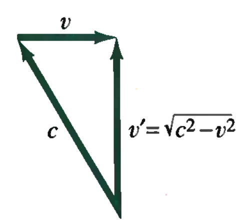
First we consider beam 2 in Fig. 26–3a, which travels parallel to ... "ether wind." In its journey from M_S to M_2, we expect the light to ...



(a)



(b)



(c)

FIGURE 26–3 The Michelson–Morley experiment. (a) Michelson interferometer. (b) Boat analogy: boat 1 goes across the stream and back; boat 2 goes downstream and back upstream. (c) Calculation of the velocity of boat (or light beam) traveling perpendicular to the current (or ether wind).

with speed $c + v$, just as a boat traveling downstream (see Fig. 26–3b) acquires the speed of the river current. Since the beam travels a distance l_2, the time it takes to go from M_S to M_2 is $t = l_2/(c + v)$. To make the return trip from M_2 to M_S, the light must move against the ether wind (like the boat going upstream), so its relative speed is expected to be $c - v$. The time for the return trip is $l_2/(c - v)$. The total time required for beam 2 to travel from M_S to M_2 and back to M_S is

$$t_2 = \frac{l_2}{c + v} + \frac{l_2}{c - v}$$

$$= \frac{2l_2}{c(1 - v^2/c^2)}.$$

The second line was obtained from the first by finding the common denominator and factoring out c^2 in the denominator.

Now let us consider beam 1, which travels crosswise to the ether wind. Here the boat analogy (part b) is especially helpful. The boat is to go from wharf A to wharf B directly across the stream. If it heads directly across, the stream's current will drag it downstream. To reach wharf B, the boat must head at an angle upstream. The precise angle depends on the magnitudes of c and v, but is of no interest to us in itself. Part (c) of Fig. 26–3 shows how to calculate the velocity v' of the boat relative to Earth as it crosses the stream. Since c, v, and v' form a right triangle, we have that $v' = \sqrt{c^2 - v^2}$. The boat has the same velocity when it returns. If we now apply these principles to light beam 1 in Fig. 26–3a, we see that the beam travels with a speed $\sqrt{c^2 - v^2}$ in going from M_S to M_1 and back again. The total distance traveled is $2l_1$, so the time required for beam 1 to make the round trip is $2l_1/\sqrt{c^2 - v^2}$, or

$$t_1 = \frac{2l_1}{c\sqrt{1 - v^2/c^2}}.$$

Notice that the denominator in this equation for t_1 involves a square root, whereas that for t_2 does not.

If $l_1 = l_2 = l$, we see that beam 2 will lag behind beam 1 by an amount

$$\Delta t = t_2 - t_1 = \frac{2l}{c}\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right).$$

If $v = 0$, then $\Delta t = 0$, and the two beams will return in phase since they were initially in phase. But if $v \neq 0$, then $\Delta t \neq 0$, and the two beams will return out of phase. If this change of phase from the condition $v = 0$ to that for $v = v$ could be measured, then v could be determined. But the Earth cannot be stopped. Furthermore, it is not possible to independently assume $l_1 = l_2$.

Michelson and Morley realized that they could detect the difference in phase (assuming that $v \neq 0$) if they rotated their apparatus by 90°, for then the interference pattern between the two beams should change. In the rotated position, beam 1 would now move parallel to the ether and beam 2 perpendicular to it. Thus the roles could be reversed, and in the rotated position the times (designated by primes) would be

$$t_1' = \frac{2l_1}{c(1 - v^2/c^2)} \quad \text{and} \quad t_2' = \frac{2l_2}{c\sqrt{1 - v^2/c^2}}.$$

The time lag between the two beams in the nonrotated position (unprimed) would be

$$\Delta t = t_2 - t_1 = \frac{2l_2}{c(1 - v^2/c^2)} - \frac{2l_1}{c\sqrt{1 - v^2/c^2}}.$$

In the rotated position, the time difference would be

$$\Delta t' = t_2' - t_1' = \frac{2l_2}{c\sqrt{1 - v^2/c^2}} - \frac{2l_1}{c(1 - v^2/c^2)}.$$

When the rotation is made, the fringes of the interference pattern (Section 24–9) will shift an amount determined by the difference:

$$\Delta t - \Delta t' = \frac{2}{c}(l_1 + l_2)\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right).$$

This expression can be considerably simplified if we assume that $v/c \ll 1$. For in this case we can use the binomial expansion,† so

$$\frac{1}{1 - v^2/c^2} \approx 1 + \frac{v^2}{c^2} \quad \text{and} \quad \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

†The binomial expansion (see Appendix A) states that $(1 \pm x)^n = 1 \pm nx + [n(n-1)/2!]x^2 + \cdots$. In our case we have, therefore, $(1 - x)^{-1} \approx 1 + x$, and $(1 - x)^{-1/2} \approx 1 + \frac{1}{2}x$, where only the first term is kept, since $x = v^2/c^2$ is assumed to be small.

Then

$$\Delta t - \Delta t' \approx \frac{2}{c}(l_1 + l_2)\left(1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\approx (l_1 + l_2)\frac{v^2}{c^3}.$$

Now we take $v = 3.0 \times 10^4\,\text{m/s}$, the speed of the Earth in its orbit around the Sun. In Michelson and Morley's experiments, the arms l_1 and l_2 were about 11 m long. The time difference would then be about

$$(22\,\text{m})(3.0 \times 10^4\,\text{m/s})^2/(3.0 \times 10^8\,\text{m/s})^3 \approx 7.0 \times 10^{-16}\,\text{s}.$$

For visible light of wavelength $\lambda = 5.5 \times 10^{-7}\,\text{m}$, say, the frequency would be $f = c/\lambda = (3.0 \times 10^8\,\text{m/s})/(5.5 \times 10^{-7}\,\text{m}) = 5.5 \times 10^{14}\,\text{Hz}$, which means that wave crests pass by a point every $1/(5.5 \times 10^{14}\,\text{Hz}) = 1.8 \times 10^{-15}\,\text{s}$. Thus, with a time difference of $7.0 \times 10^{-16}\,\text{s}$, Michelson and Morley should have noted a movement in the interference pattern of $(7.0 \times 10^{-16}\,\text{s})/(1.8 \times 10^{-15}\,\text{s}) = 0.4$ fringe. They could easily have detected this, since their apparatus was capable of observing a fringe shift as small as 0.01 fringe.

The null result

But they found ***no significant fringe shift whatever***! They set their apparatus at various orientations. They made observations day and night so that they would be at various orientations with respect to the Sun (due to the Earth's rotation). They tried at different seasons of the year (the Earth at different locations due to its orbit around the Sun). Never did they observe a significant fringe shift.

This "null" result was one of the great puzzles of physics at the end of the nineteenth century. To explain it was a difficult challenge. One possibility to explain the null result was to apply an idea put forth independently by G. F. Fitzgerald and H. A. Lorentz (in the 1890s) in which they proposed that any length (including the arm of an interferometer) contracts by a factor $\sqrt{1 - v^2/c^2}$ in the direction of motion through the ether. According to Lorentz, this could be due to the ether affecting the forces between the molecules of a substance, which were assumed to be electrical in nature. This theory was eventually replaced by the far more comprehensive theory proposed by Albert Einstein in 1905—the special theory of relativity.

26–3 Postulates of the Special Theory of Relativity

The problems that existed at the turn of the century with regard to electromagnetic theory and Newtonian mechanics were beautifully resolved by Einstein's introduction of the theory of relativity in 1905. Einstein, however, was apparently not influenced directly by the null result of the Michelson–Morley experiment. What motivated Einstein were certain questions regarding electromagnetic theory and light waves. For example, he asked himself: "What would I see if I rode a light beam?" The answer was that instead of a traveling electromagnetic wave, he would see alternating electric and magnetic fields at rest whose magnitude changed in space, but did not change in time. Such fields, he realized, had never been detected and indeed were not consistent with Maxwell's electromagnetic theory. He argued, therefore, that it was unreasonable to think that the speed of light relative to any … could be reduced to zero, or in fact reduced at all. This idea became … ond postulate of his theory of relativity.

Einstein concluded that the inconsistencies he found in … netic theory were due to the assumption that an absolute space … his famous 1905 paper, he proposed doing away completely with … of the ether and the accompanying assumption of an absolute … frame at rest. This proposal was embodied in two postulates. The … tulate was an extension of the Newtonian relativity principle … not only the laws of mechanics but also those of the rest of … cluding electricity and magnetism:

The two postulates of special relativity

***First postulate* (*the relativity principle*): The laws of physics … same form in all inertial reference frames.**

The second postulate is consistent with the first:

***Second postulate* (*constancy of the speed of light*): Light … through empty space with a definite speed *c* independent of … of the source or observer.**

These two postulates form the foundation of Einstein's **special … relativity**. It is called "special" to distinguish it from his later "… ory of relativity," which deals with noninertial (accelerating) … frames (discussed in Chapter 33). The special theory, which is what … cuss here, deals only with inertial frames.

The second postulate may seem hard to accept, for it violates … monsense notions. First of all, we have to think of light traveling … empty space. Giving up the ether is not too hard, however, for … had never been detected. But the second postulate also tells us … speed of light in vacuum is always the same, 3.00×10^8 m/s, … what the speed of the observer or the source. Thus, a person … ward or away from a source of light will measure the same speed … light as someone at rest with respect to the source. This conflicts … everyday notions, for we would expect to have to add in the velocity … observer. Part of the problem is that in our everyday experience, … not measure velocities anywhere near as large as the speed of light. … we can't expect our everyday experience to be helpful when dealing … such a high velocity. On the other hand, the Michelson–Morley … ment is fully consistent with the second postulate.†

Einstein's proposal has a certain beauty. For by doing away with … of an absolute reference frame, it was possible to reconcile classical … ics with Maxwell's electromagnetic theory. The speed of light predicted … Maxwell's equations *is* the speed of light in vacuum in *any* reference …

Einstein's theory required giving up commonsense notions … and time, and in the following sections we will examine some … interesting consequences of Einstein's theory. Our arguments for … part will be simple ones. We will use a technique that Einstein …

†The Michelson–Morley experiment can also be considered as evidence for the … late, for it was intended to measure the motion of the Earth relative to an absolute … frame. Its failure to do so implies the absence of any such preferred frame.

will imagine very simple experimental situations in which little mathematics is needed. In this way, we can see many of the consequences of relativity theory without getting involved in detailed calculations. Einstein called these "gedanken" experiments, which is German for "thought" experiments. Some of the more mathematical details of special relativity are treated in Appendix E.

26–4 Simultaneity

One of the important consequences of the theory of relativity is that we can no longer regard time as an absolute quantity. No one doubts that time flows onward and never turns back. But, as we shall see in this section and the next, the time interval between two events, and even whether two events are simultaneous, depends on the observer's reference frame.

Two events are said to occur simultaneously if they occur at exactly the same time. But how do we know if two events occur precisely at the same time? If they occur at the same point in space—such as two apples falling on your head at the same time—it is easy. But if the two events occur at widely separated places, it is more difficult to know whether the events are simultaneous since we have to take into account the time it takes for the light from them to reach us. Because light travels at finite speed, a person who sees two events must calculate back to find out when they actually occurred. For example, if two events are *observed* to occur at the same time, but one actually took place farther from the observer than the other, then the former must have occurred earlier, and the two events are not simultaneous.

A "thought" experiment

We will now make use of a simple thought experiment. We assume an observer, called O, is located exactly halfway between points A and B where two events occur, Fig. 26–4. The two events may be lightning that strikes the points A and B, as shown, or any other type of events. For brief events like lightning, only short pulses of light will travel outward from A and B and reach O. O "sees" the events when the pulses of light reach point O. If the two pulses reach O at the same time, then the two events are said to be simultaneous. This is because the two light pulses travel at the same speed (postulate 2), and since the distance OA equals OB, the time for the light to travel from A to O and B to O must be the same. Observer O can then definitely state that the two events occurred simultaneously. On the other hand, if O sees the light from one event before that from the other, then it is certain the former event occurred first.

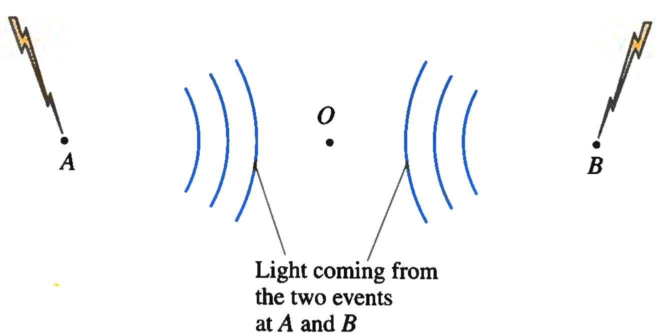


FIGURE 26–4 A moment after lightning strikes points A and B, the pulses of light are traveling toward the observer O, but O "sees" the lightning only when the light reaches O.

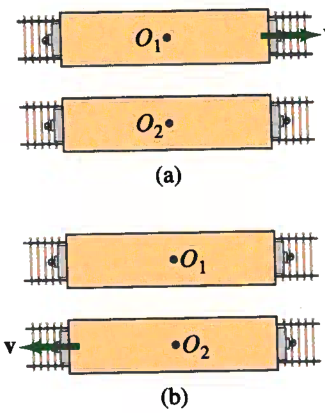


FIGURE 26–5 Observers O_1 and O_2, on two different trains (two different reference frames), are moving with relative velocity v. O_2 says that O_1 is moving to the right (a); O_1 says that O_2 is moving to the left (b). Both viewpoints are legitimate—it all depends on your reference frame.

The question we really want to examine is this: if two events are simultaneous to an observer in one reference frame, are they also simultaneous to another observer moving with respect to the first? Let us call the observers O_1 and O_2 and assume they are fixed in reference frames 1 and 2 that move with speed v relative to one another. These two reference frames can be thought of as trains (Fig. 26–5). O_2 says that O_1 is moving to the right with speed v, as in (a); and O_1 says O_2 is moving to the left with speed v, as in (b). Both viewpoints are legitimate according to the relativity principle. (There is, of course, no third point of view which will tell us which one is "really" moving.)

Now suppose two events occur that are observed and measured by both observers. Let us assume again that the two events are the striking of lightning and that the lightning marks both trains where it struck: at A_1 and B_1 on O_1's train, and at A_2 and B_2 on O_2's train. For simplicity, we assume that O_1 happens to be exactly halfway between A_1 and B_1, and that O_2 is halfway between A_2 and B_2. We now put ourselves in one reference frame or the other, from which we make our observations and measurements. Let us put ourselves in O_2's reference frame, so we observe O_1 moving to the right with speed v. Let us assume that the two events occur *simultaneously* in O_2's frame, and at the instant when O_1 and O_2 are opposite each other, Fig. 26–6a. A short time later, Fig. 26–6b, the light from A_2 and B_2 reaches O_2 at the same time (we assumed this). Since O_2 knows (or measures) the distances O_2A_2 and O_2B_2 as equal, O_2 knows the two events are simultaneous in the O_2 reference frame.

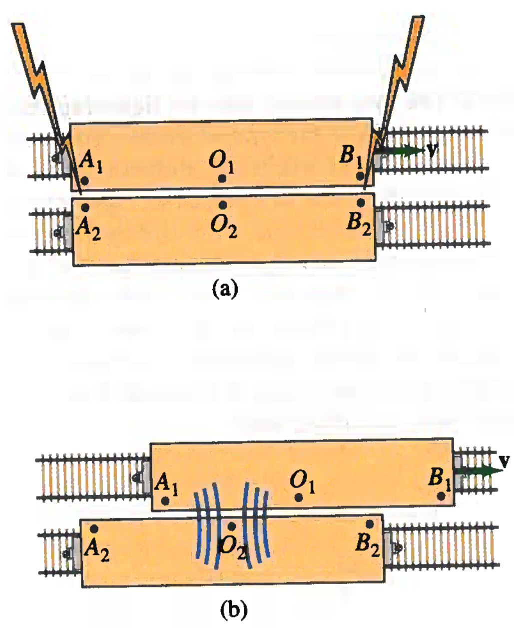


FIGURE 26–6 Thought experiment on simultaneity. To observer O_2, the reference frame of O_1 is moving to the right. In (a), one lightning bolt strikes the two reference frames at A_1 and A_2, and a second lightning bolt strikes at B_1 and B_2. (b) A moment later, the light from the two events reaches O_2 at the same time, so according to observer O_2, the two bolts of lightning strike simultaneously. But in O_1's reference frame, the light from B_1 has already reached O_1, whereas the light from A_1 has not yet reached O_1. So in O_1's reference frame, the event at B_1 must have preceded the event at A_1. Time is not absolute.

But what does observer O_1 observe and measure? From our (O_2) reference frame, we can predict what O_1 will observe. We see that O_1 moves to the right during the time the light is traveling to O_1 from A_1 and B_1. As shown in Fig. 26–6b, we can see from our O_2 reference frame that the light from B_1 has already passed O_1, whereas the light from A_1 has not yet reached O_1. Therefore, it is clear that O_1 will observe the light coming from B_1 before he observes the light coming from A_1. Now O_1's frame is as good as O_2's. Light travels at the same speed c for O_1 as for O_2 (the second postulate)†; and in the O_1 reference frame, this speed c is of course the same for light traveling from A_1 to O_1 as it is for light traveling from B_1 to O_1. Furthermore the distance O_1A_1 equals O_1B_1. Hence, since O_1 observes the light from B_1 before he observes the light from A_1 (we established this above, looking from the O_2 reference frame, Fig. 26–6b), then observer O_1 can only conclude that the event at B_1 occurred before the event at A_1. The two events are not simultaneous for O_1, even though they are for O_2.

Simultaneity is relative

We thus find that *two events* which are simultaneous to one observer are not necessarily simultaneous to a second observer.

It may be tempting to ask: "Which observer is right, O_1 or O_2?" The answer, according to relativity, is that they are *both* right. There is no "best" reference frame we can choose to determine which observer is right. Both frames are equally good. We can only conclude that simultaneity is not an absolute concept, but is relative. We are not aware of it in everyday life, however, because the effect is noticeable only when the relative speed of the two reference frames is very large (near c), or the distances involved are very large.

Because of the principle of relativity, the argument we gave for the thought experiment of Fig. 26–6 can be done from O_1's reference frame as well. In this case, O_1 will be at rest and will see event B_1 occur before A_1. But O_1 will recognize (by drawing a diagram equivalent to Fig. 26–6—try and see!) that O_2, who is moving with speed v to the left, will see the two events as simultaneous.

26–5 Time Dilation and the Twin Paradox

The fact that two events simultaneous to one observer may not be simultaneous to a second observer suggests that time itself is not absolute. Could it be that time passes differently in one reference frame than in another? This is, indeed, just what Einstein's theory of relativity predicts, as the following thought experiment shows.

† Note that O_1 does not see himself catching up with one light beam and running away from the other (that is O_2's viewpoint of what happens for O_1). O_1 sees both light beams traveling at the same speed, c.

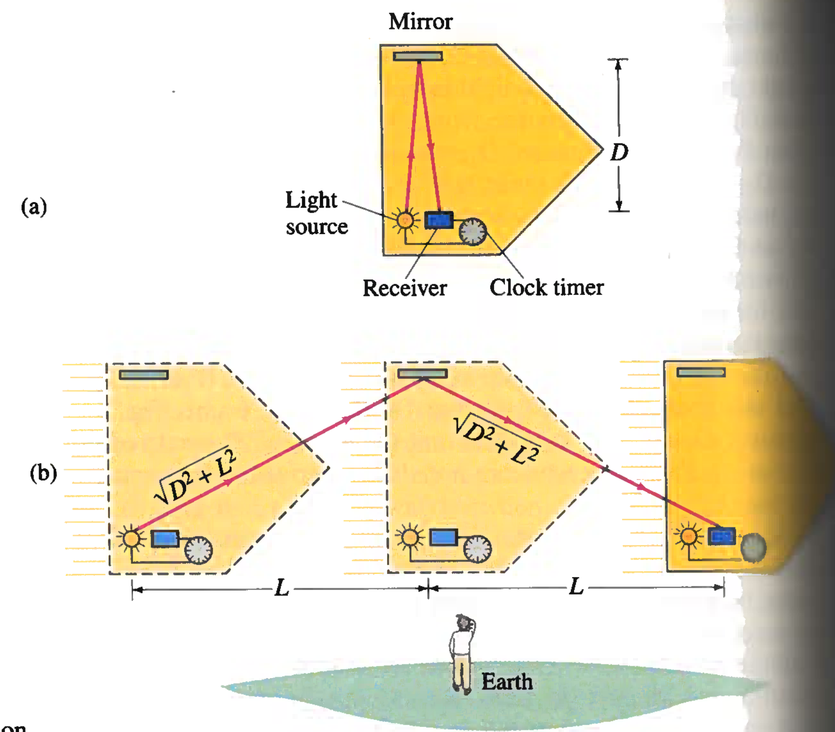


FIGURE 26–7 Time dilation can be shown by a thought experiment: the time it takes for light to travel over and back on a spaceship is longer for the observer on Earth (b) than for the observer on the spaceship (a).

Figure 26–7 shows a spaceship traveling past Earth at high
point of view of an observer on the spaceship is shown in part (a),
of an observer on Earth in part (b). Both observers have accurate
The person on the spaceship (a) flashes a light and measures the
takes the light to travel across the spaceship and return after
from a mirror. The light travels a distance $2D$ at speed c, so the
quired, which we call Δt_0, is

$$\Delta t_0 = \frac{2D}{c}.$$

This is the time as measured by the observer on the spaceship.

The observer on Earth, Fig. 26–7b, observes the same process.
this observer, the spaceship is moving. So the light travels the
path shown in going across the spaceship, reflecting off the mirror, and
turning to the sender. Although the light travels at the same speed
observer (the second postulate), it travels a greater distance.
time required, as measured by the observer on Earth, will be *greater*
that measured by the observer on the spaceship. The time interval
observed by the observer on Earth can be calculated as follows.
time Δt, the spaceship travels a distance $2L = v\,\Delta t$ where v is the
the spaceship (Fig. 26–7b). Thus, the light travels a total distance
diagonal path of $2\sqrt{D^2 + L^2}$, and therefore

$$c = \frac{2\sqrt{D^2 + L^2}}{\Delta t} = \frac{2\sqrt{D^2 + v^2(\Delta t)^2/4}}{\Delta t}.$$

We square both sides, and then solve for Δt, to find

$$c^2 = \frac{4D^2}{(\Delta t)^2} + v^2,$$

$$\Delta t = \frac{2D}{c\sqrt{1 - v^2/c^2}}.$$

We combine this with the formula above for Δt_0 ($\Delta t_0 = 2D/c$) and find:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}. \qquad \textbf{(26–1)}$$

Time-dilation formula

Since $\sqrt{1 - v^2/c^2}$ is always less than 1, we see that $\Delta t > \Delta t_0$. That is, the time interval between the two events (the sending of the light, and its reception on the spaceship) is *greater* for the observer on Earth than for the observer on the spaceship. This is a general result of the theory of relativity, and is known as **time dilation**. Stated simply, the time-dilation effect says that

clocks moving relative to an observer are measured by that observer to run more slowly (as compared to clocks at rest).

Time dilation: moving clocks run slowly

However, we should not think that the clocks are somehow at fault. Time is actually measured to pass more slowly in any moving reference frame as compared to your own. This remarkable result is an inevitable outcome of the two postulates of the theory of relativity.

The concept of time dilation may be hard to accept, for it violates our commonsense understanding. We can see from Eq. 26–1 that the time dilation effect is negligible unless v is reasonably close to c. If v is much less than c, then the term v^2/c^2 is much smaller than the 1 in the denominator of Eq. 26–1, and then $\Delta t \approx \Delta t_0$ (see Example 26–2). The speeds we experience in everyday life are much smaller than c, so it is little wonder we don't ordinarily notice time dilation. Experiments have tested the time-dilation effect, and have confirmed Einstein's predictions. In 1971, for example, extremely precise atomic clocks were flown around the world in jet planes. The speed of the planes (10^3 km/h) was much less than c, so the clocks had to be accurate to nanoseconds (10^{-9} s) in order to detect any time dilation. They were this accurate, and they confirmed Eq. 26–1 to within experimental error. Time dilation had been confirmed decades earlier, however, by observation on "elementary particles" (see Chapter 32) which have very small masses (typically 10^{-30} to 10^{-27} kg) and so require little energy to be accelerated to speeds close to c. Many of these elementary particles are not stable and decay after a time into smaller particles. One example is the muon, whose mean lifetime is 2.2 μs when at rest. Careful experiments showed that when a muon is traveling at high speeds, its lifetime is measured to be longer than when it is at rest, just as predicted by the time-dilation formula.

Why we don't usually notice time dilation

EXAMPLE 26–1 **Lifetime of a moving muon.** (*a*) What will be the mean lifetime of a muon as measured in the laboratory if it is traveling at $v = 0.60c = 1.8 \times 10^8$ m/s with respect to the laboratory? Its mean lifetime at rest is 2.2×10^{-6} s. (*b*) How far does a muon travel in the laboratory, on average, before decaying?

SOLUTION (*a*) If an observer were to move along with the muon (the muon would be at rest to this observer), the muon would have a mean life of 2.2×10^{-6} s. To an observer in the lab, the muon lives longer because of time dilation. From Eq. 26–1 with $v = 0.60c$, we have

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{2.2 \times 10^{-6}\,\text{s}}{\sqrt{1 - \frac{0.36c^2}{c^2}}} = \frac{2.2 \times 10^{-6}\,\text{s}}{\sqrt{0.64}} = 2.8 \times 10^{-6}\,\text{s}$$

(*b*) At a speed of 1.8×10^8 m/s, classical physics would tell us that with a mean life of 2.2 μs, an average muon would travel $(1.8 \times 10^8\,\text{m/s})(2.2 \times 10^{-6}\,\text{s}) = 400$ m. But relativity predicts an average distance of $(1.8 \times 10^8\,\text{m/s})(2.8 \times 10^{-6}\,\text{s}) = 500$ m, and it is this longer distance that is measured experimentally.

We need to make a comment about the use of Eq. 26–1 and the meaning of Δt and Δt_0. The equation is true only when Δt_0 represents the time interval between the two events in a reference frame where the two events occur at *the same point in space* (as in Fig. 26–7a where the two events are the light flash being sent and being received). This time interval Δt_0 is called the **proper time**. Then Δt in Eq. 26–1 represents the time interval between the two events as measured in a reference frame moving with speed v with respect to the first. In Example 26–1 above, Δt_0 (and not Δt) was set equal to 2.2×10^{-6} s because it is only in the rest frame of the muon that the two events ("birth" and "decay") occur at the same point in space.

Proper time

EXAMPLE 26–2 **Time dilation at 100 km/h.** Let's check time dilation for everyday speeds. A car traveling 100 km/h covers a certain distance in 10.00 s according to the driver's watch. What does an observer on Earth measure for the time interval?

SOLUTION The car's speed relative to Earth is 100 km/h = $(1.00 \times 10^5\,\text{m})/(3600\,\text{s}) = 27.8$ m/s. We set $\Delta t_0 = 10.00$ s in the time dilation formula (the driver is at rest in the reference frame of the car), and then Δt is

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10.00\,\text{s}}{\sqrt{1 - \left(\frac{27.8\,\text{m/s}}{3.00 \times 10^8\,\text{m/s}}\right)^2}} = \frac{10.00\,\text{s}}{\sqrt{1 - 8.59 \times 10^{\text{[illegible]}}}}$$

If you put these numbers into a calculator, you will obtain Δt = [illegible] since the denominator differs from 1 by such a tiny amount. Indeed, the time measured by an observer on Earth would be no different from that measured by the driver, even with the best of today's instruments. A computer that could calculate to a large number of decimal places could reveal a difference between Δt and Δt_0. But we can estimate the difference [illegible] easily using the binomial expansion (Appendix A), which says that in a formula of the form $(1 \pm x)^n$, if $x \ll 1$, then to a good approximation,

PROBLEM SOLVING

Use of the binomial expansion

$$(1 \pm x)^n \approx 1 \pm nx.$$

In our time-dilation formula, we have the factor $1/\sqrt{1 - v^2/c^2} = (1 - v^2/c^2)^{-1/2}$. Thus (setting $x = v^2/c^2$ and $n = -\frac{1}{2}$ in the binomial expansion):

$$\Delta t = \Delta t_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2} \approx \Delta t_0\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\approx 10.00\ \text{s}\left[1 + \frac{1}{2}\left(\frac{27.8\ \text{m/s}}{3.00 \times 10^8\ \text{m/s}}\right)^2\right] \approx 10.00\ \text{s} + 4 \times 10^{-15}\ \text{s}.$$

So the difference between Δt and Δt_0 is predicted to be 4×10^{-15} s, an immeasurably small amount.

Time dilation has aroused interesting speculation about space travel. According to classical (Newtonian) physics, to reach a star 100 light-years away would not be possible for ordinary mortals (1 light-year is the distance light can travel in 1 year $= 3.0 \times 10^8\ \text{m/s} \times 3.15 \times 10^7\ \text{s} = 9.5 \times 10^{15}\ \text{m}$). Even if a spaceship could travel at close to the speed of light, it would take over 100 years to reach such a star. But time dilation tells us that the time involved would be less for an astronaut. In a spaceship traveling at $v = 0.999c$, the time for such a trip would be only about $\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2} = (100\ \text{yr})\sqrt{1 - (0.999)^2} = 4.5\ \text{yr}$. Thus time dilation allows such a trip, but the enormous practical problems of achieving such speeds will not be overcome in the near future.

Notice, in this example, that whereas 100 years would pass on Earth, only 4.5 years would pass for the astronaut on the trip. Is it just the clocks that would slow down for the astronaut? The answer is no. All processes, including life processes, run more slowly for the astronaut according to the Earth observer. But to the astronaut, time would pass in a normal way. The astronaut would experience 4.5 years of normal sleeping, eating, reading, and so on. And people on Earth would experience 100 years of ordinary activity.

Not long after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. According to this **twin paradox**, suppose one of a pair of 20-year-old twins takes off in a spaceship traveling at very high speed to a distant star and back again, while the other twin remains on Earth. According to the Earth twin, the traveling twin will age less. Whereas 20 years might pass for the Earth twin, perhaps only 1 year (depending on the spacecraft's speed) would pass for the traveler. Thus, when the traveler returns, the earthbound twin could expect to be 40 years old whereas the traveling twin would be only 21.

Twin paradox

This is the viewpoint of the twin on the Earth. But what about the traveling twin? If all inertial reference frames are equally good, won't the traveling twin make all the claims the Earth twin does, only in reverse? Can't the astronaut twin claim that since the Earth is moving away at high speed, time passes more slowly on Earth and the twin on Earth will age less? This is the opposite of what the Earth twin predicts. They cannot both be right, for after all the spacecraft returns to Earth and a direct comparison of ages and clocks can be made.

There is, however, not a paradox at all. The consequences of the cial theory of relativity—in this case, time dilation—can be ap by observers in inertial reference frames. The Earth is such a nearly so), whereas the spacecraft is not. The spacecraft accel start and end of its trip and, more importantly, when it turns ar far point of its journey. During these acceleration periods, the predictions based on special relativity are not valid. The twin in an inertial frame and can make valid predictions. Thus, there dox. The traveling twin's point of view expressed above is not predictions of the Earth twin *are* valid, and the prediction that ing twin returns having aged less is the proper one.†

26–6 Length Contraction

Not only time intervals are different in different reference fram intervals—lengths and distances—are different as well, accordin special theory of relativity, and we illustrate this with a thought ex

Observers on Earth watch a spacecraft traveling at speed v from to, say, Neptune, Fig. 26–8a. The distance between the planets, by the Earth observers, is L_0. The time required for the trip, mea Earth, is $\Delta t = L_0/v$. In Fig. 26–8b we see the point of view of the spacecraft. In this frame of reference, the spaceship is at rest Neptune move with speed v. (We assume v is much greater than the speed of Neptune and Earth, so the latter can be ignored.) The tween the departure of Earth and arrival of Neptune (as observed spacecraft) is the "proper time" (since the two events occur at the point in space—i.e., on the spacecraft). Therefore the time interval the spacecraft observers than for the Earth observers, because of tion. From Eq. 26–1, the time for the trip as viewed by the $\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2}$. Since the spacecraft observers measure speed but less time between these two events, they must also

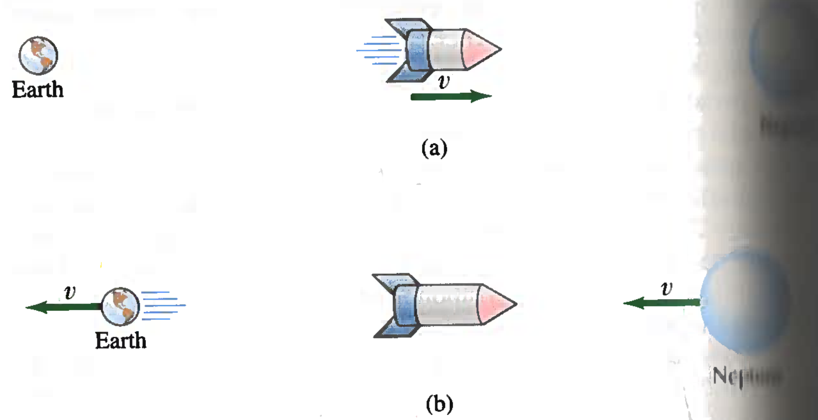


FIGURE 26–8 (a) A spaceship traveling at very high speed from Earth to Neptune, as seen from Earth's frame of reference. (b) As viewed by an observer on the spaceship, Earth and Neptune are moving at the very high velocity v: Earth leaves the spaceship, and a time Δt_0 later planet Neptune arrives at the spaceship. [Note in (b) that each planet does not look shortened because at high speeds we see the trailing edge (as in Fig. 26–10), and the net effect is to leave its appearance as a circle.]

†Einstein's general theory of relativity, which deals with accelerating reference firms this result.

distance as less. If we let L be the distance between the planets as viewed by the spacecraft observers, then $L = v\,\Delta t_0$. We have already seen that $\Delta t_0 = \Delta t\sqrt{1 - v^2/c^2}$ and $\Delta t = L_0/v$, so we have $L = v\,\Delta t_0 = v\,\Delta t\sqrt{1 - v^2/c^2} = L_0\sqrt{1 - v^2/c^2}$. That is,

$$L = L_0\sqrt{1 - v^2/c^2}. \tag{26–2}$$

Length-contraction formula

This is a general result of the special theory of relativity and applies to lengths of objects as well as to distance. The result can be stated most simply in words as:

the length of an object is measured to be shorter when it is moving relative to the observer than when it is at rest.

Length contraction: moving objects are shorter (in the direction of motion)

This is called **length contraction**. The length L_0 in Eq. 26–2 is called the **proper length**. It is the length of the object—or distance between two points whose *positions are measured at the same time*—as measured by observers at rest with respect to it. Equation 26–2 gives the length L that will be measured by observers when the object travels past them at speed v. It is important to note, however, that length contraction occurs *only along the direction of motion*. For example, the moving spaceship in Fig. 26–8a is shortened in length, but its height is the same as when it is at rest.

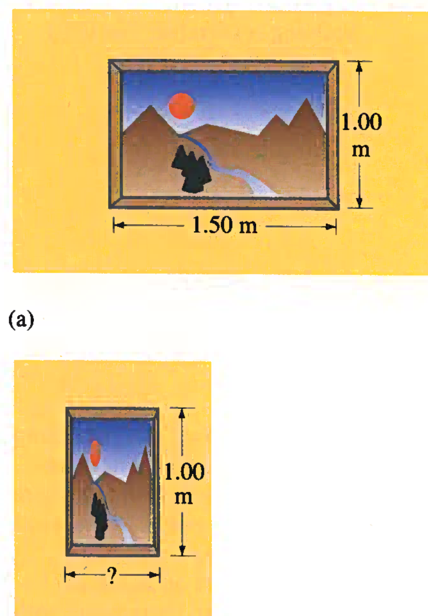
Length contraction, like time dilation, is not noticeable in everyday life because the factor $\sqrt{1 - v^2/c^2}$ in Eq. 26–2 differs from 1.00 significantly only when v is very large.

EXAMPLE 26–3 **Painting's contraction.** A rectangular painting measures 1.00 m tall and 1.50 m wide. It is hung on the side wall of a spaceship which is moving past the Earth at a speed of $0.90c$. See Fig. 26–9a. (*a*) What are the dimensions of the picture according to the captain of the spaceship? (*b*) What are the dimensions as seen by an observer on the Earth?

SOLUTION (*a*) The painting (as well as everything else in the spaceship) looks perfectly normal to everyone on the spaceship, so the captain sees a 1.00 m by 1.50 m painting.
(*b*) Only the dimension in the direction of motion is shortened, so the height is unchanged at 1.00 m, Fig. 26–9b. The length, however, is contracted to

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$
$$= (1.50\text{ m})\sqrt{1 - (0.90)^2} = 0.65\text{ m}.$$

So the picture has dimensions 1.00 m × 0.65 m.



(a)
(b)

FIGURE 26–9 Example 26–3.

Equation 26–2 tells us what the length of an object will be *measured* to be when traveling at speed v. The *appearance* of the object is another matter. Suppose, for example, you are traveling to the left past a small building at speed $v = 0.85c$. This is equivalent to the building moving past

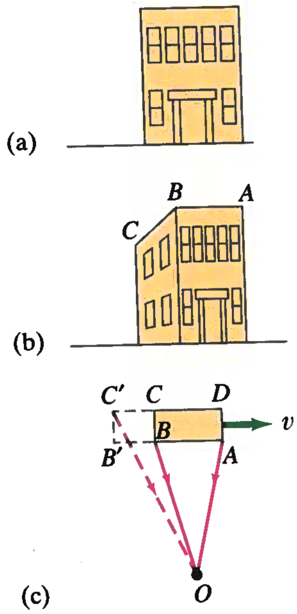


FIGURE 26–10 Building seen (a) at rest, and (b) moving at high speed. (c) Diagram explains why the side of the building is seen (see the text).

you to the right at speed v. The building will look narrower (and the height), but you will also be able to see the side of the building even are directly in front of it. This is shown in Fig. 26–10b—part (a) shows building at rest. The fact that you see the side is not really a rel fect, but is due to the finite speed of light. To see how this occurs, at Fig. 26–10c which is a top view of the building, looking down. At stant shown, the observer O is directly in front of the building. Light points A and B reach O at the same time. If the building were at from point C could never reach O. But the building is moving at speed and does "get out of the way" so that light from C can reach deed, at the instant shown, light from point C when it was at an cation (C' on the diagram) can reach O because the building has In order to reach the observer at the same time as light from light from C had to leave at an earlier time since it must travel distance. Thus it is light from C' that reaches the observer at the time as light from A and B. This, then, is how an observer might the front and side of an object at the same time even when front of it.† It can be shown, by the same reasoning, that spherical will actually still have a circular outline even at high speeds. That the planets in Fig. 26–8b are drawn round rather than contracted.

26–7 Four-Dimensional Space–Time

Let us imagine a person is on a train moving at a very high 0.65c, Fig. 26–11. This person begins a meal at 7:00 and finishes at cording to a clock on the train. The two events, beginning and meal, take place at the same point on the train. So the proper tween these two events is 15 min. To observers on Earth, the take longer—20 min according to Eq. 26–1. Let us assume that was served on a 20-cm-diameter plate. To observers on the Earth,

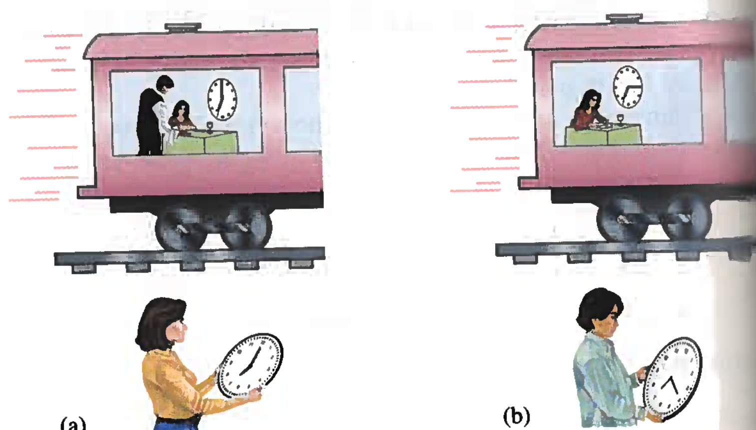


FIGURE 26–11 According to an accurate clock on a fast-moving train, a person (a) begins dinner at 7:00 and (b) finishes at 7:15. At the beginning of the meal, observers on Earth set their watches to correspond with the clock on the train. These observers measure the eating time as 20 minutes.

†It would be an error to think that the building in Fig. 26–10b would look rotated. correct since in that case side A would look shorter than side B. In fact, if the rectly in front, these sides appear equal in height. Thus the building looks front face, but we also see the side, as described above. Also, though Fig. 26–10b, the walls of the building would appear curved, because of differing from the observer's eye of the various points from top to bottom along a vertical

only 15 cm wide (length contraction). Thus, to observers on the Earth, the meal looks smaller but lasts longer.

In a sense these two effects, time dilation and length contraction, balance each other. When viewed from the Earth, what the meal seems to lose in size it gains in length of time it lasts. Space, or length, is exchanged for time.

Considerations like this led to the idea of **four-dimensional space–time**: space takes up three dimensions and time is a fourth dimension. Space and time are intimately connected. Just as when we squeeze a balloon we make one dimension larger and another smaller, so when we examine objects and events from different reference frames, a certain amount of space is exchanged for time, or vice versa.

Although the idea of four dimensions may seem strange, it refers to the idea that any object or event is specified by four quantities—three to describe where in space, and one to describe when in time. The really unusual aspect of four-dimensional space–time is that space and time can intermix: a little of one can be exchanged for a little of the other when the reference frame is changed.

It is difficult for most of us to understand the idea of four-dimensional space–time. Somehow we feel, just as physicists did before the advent of relativity, that space and time are completely separate entities. Yet we have found in our thought experiments that they are not completely separate. Our difficulty in accepting this is reminiscent of the situation in the seventeenth century at the time of Galileo and Newton. Before Galileo, the vertical direction, that in which objects fall, was considered to be distinctly different from the two horizontal dimensions. Galileo showed that the vertical dimension differs only in that it happens to be the direction in which gravity acts. Otherwise, all three dimensions are equivalent, a viewpoint we all accept today. Now we are asked to accept one more dimension, time, which we had previously thought of as being somehow different. This is not to say that there is no distinction between space and time. What relativity has shown is that space and time determinations are not independent of one another.

26–8 Momentum and Mass

The three basic mechanical quantities are length, time intervals, and mass. The first two have been shown to be relative—their value depends on the reference frame from which they are measured. We might ask if mass, too, is a relative quantity.

Analysis of collision processes between two particles shows that if we want to preserve conservation of momentum as a principle also in relativity, we must redefine momentum as

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}. \qquad (26\text{–}3)$$

Relativistic momentum

At speeds much less than the speed of light, Eq. 26–3 gives the classical momentum, $p = m_0 v$. We have written m_0 rather than m because Eq. 26–3 suggests a relativistic interpretation of mass. Namely, that *the* *mass of an object is measured to increase as its speed increases* according to the formula

Mass increase formula

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$

In this **mass-increase** formula, m_0 is the **rest mass** of the object—the mass it has as measured in a reference frame in which it is at rest; and m is the mass it will be measured to have in a reference frame in which it moves at speed v.

Relativistic momentum and mass increase have been tested countless times on tiny elementary particles (such as muons), and they have been found to increase in accord with Eqs. 26–3 and 26–4.

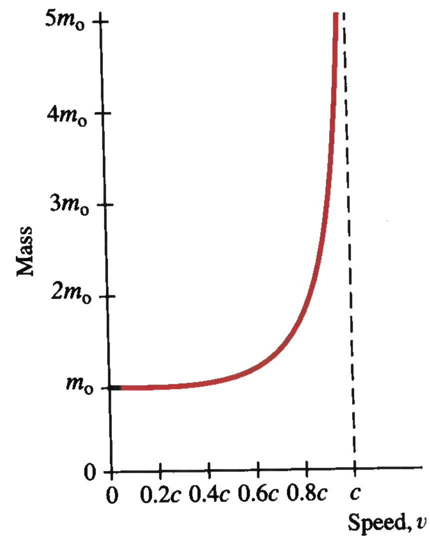


FIGURE 26–12 Mass of a particle (rest mass m_0) as a function of speed v (given as a fraction of c, the speed of light).

EXAMPLE 26–4 **Mass of moving electron.** Calculate the mass of an electron when it has a speed of (*a*) 4.00×10^7 m/s in the CRT of a TV set, and (*b*) $0.98c$ in an accelerator used for cancer therapy.

SOLUTION The rest mass of an electron is $m_0 = 9.11 \times 10^{-31}$ kg. (*a*) At $v = 4.00 \times 10^7$ m/s, the electron's mass will be

$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{9.11 \times 10^{-31}\ \text{kg}}{\sqrt{1 - \dfrac{(4.00 \times 10^7\ \text{m/s})^2}{(3.00 \times 10^8\ \text{m/s})^2}}} = 9.19 \times 10^{-31}\ \text{kg}.$$

Even at such a high speed ($v \approx 0.1c$), the electron's mass is only about 1 percent higher than its rest mass. But in (*b*), we have

$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \dfrac{(0.98c)^2}{c^2}}} = \frac{m_0}{\sqrt{1 - (0.98)^2}} = 5.0m_0.$$

An electron traveling at 98 percent the speed of light has a mass 5 times its rest mass!

Figure 26–12 is a graph of mass versus speed for any particle.

26–9 The Ultimate Speed

A basic result of the special theory of relativity is that the speed of an object cannot equal or exceed the speed of light. That the speed of light is a natural upper speed limit in the universe can be seen from any of Eqs. 26–1 through 26–4. It is perhaps easiest to see it from Eq. 26–4, the mass-increase formula, $m = m_0/\sqrt{1 - v^2/c^2}$. As an object is accelerated to greater and greater speeds, its mass becomes larger and larger. Indeed, if v were to equal c, the denominator in this equation would be zero and the mass m would become infinite. To accelerate an object up to $v = c$ would thus require infinite energy, and so is not possible. Similarly, Eqs. 26–1 and 26–2 tell us that length would disappear and time become infinite as v approaches c.

26–10 $E = mc^2$; Mass and Energy

When a steady net force is applied to an object of rest mass m_0, the object increases in speed. Since the force is acting over a distance, work is done on the object and its kinetic energy increases. As the speed of the object approaches c, the speed cannot increase indefinitely because it cannot exceed c. On the other hand, the mass of the object increases with increasing speed. That is, the work done on an object not only increases its speed but also contributes to increasing its *mass*. Since the work done on an object increases its energy, this new twist from the theory of relativity leads to the idea that mass is a form of energy, a crucial part of Einstein's theory.

To find the mathematical relationship between mass and energy, Einstein assumed that the work–energy theorem (Chapter 6) is still valid in relativity. That is, the net work done on a particle is equal to its change in kinetic energy (KE). Using this theorem, Einstein showed that at high speeds the formula $\text{KE} = \frac{1}{2}mv^2$ is not correct. You might think that using Eq. 26–4 for m would give $\text{KE} = \frac{1}{2}m_0v^2/\sqrt{1 - v^2/c^2}$, but this formula, too, is wrong. Instead, Einstein showed that the kinetic energy of a particle is given by

$$\text{KE} = mc^2 - m_0c^2, \qquad \textbf{(26–5)}$$

Relativistic kinetic energy

where m is the mass of the particle traveling at speed v and m_0 is its rest mass.

But what does the second term in Eq. 26–5—the m_0c^2—mean? Consistent with the idea that mass is a form of energy, Einstein called m_0c^2 the **rest energy** of the object. We can rearrange Eq. 26–5 to get $mc^2 = m_0c^2 + \text{KE}$. We call mc^2 the *total energy E* of the particle (assuming no potential energy), and we see that the total energy equals the rest energy plus the kinetic energy:

$$E = mc^2, \qquad \textbf{(26–6a)}$$

$E = mc^2$, mass related to energy

$$E = m_0c^2 + \text{KE}. \qquad \textbf{(26–6b)}$$

Here we have Einstein's famous formula $E = mc^2$.

For a particle at rest in a given reference frame, its total energy is $E = m_0c^2$, which we have called its rest energy. This formula mathematically relates the concepts of energy and mass. But if this idea is to have any meaning from a practical point of view, then mass ought to be convertible to energy and vice versa. That is, if mass is just one form of energy, then it should be convertible to other forms of energy just as other types of energy are interconvertible. Einstein suggested that this might be possible, and indeed changes of mass to other forms of energy, and vice versa, have been experimentally confirmed countless times. The interconversion of mass and energy is most easily detected in nuclear and elementary particle physics. For example, the neutral pion (π^0) of rest mass 2.4×10^{-28} kg is observed to decay into pure electromagnetic radiation (photons). The π^0 completely disappears in the process. The amount of electromagnetic energy produced is found to be exactly equal to that predicted by Einstein's formula, $E = m_0c^2$. The reverse process is also commonly observed in the laboratory: electromagnetic radiation under certain conditions can be converted into material particles such as electrons. On a larger scale, the energy produced in nuclear power plants is a result of the loss in mass of the uranium fuel as it undergoes the process called fission

Mass and energy interchangeable

(Chapter 31). Even the radiant energy we receive from the Sun i... ample of $E = mc^2$; the Sun's mass is continually decreasing as it ... electromagnetic energy outward.

The relation $E = mc^2$ is now believed to apply to all pro... though the changes are often too small to measure. That is, when the ... ergy of a system changes by an amount ΔE, the mass of the ... changes by an amount Δm given by

$$\Delta E = (\Delta m)(c^2).$$

In a chemical reaction where heat is gained or lost, the masses of the ... tants and the products will be different. Even when water is heated ... stove, the mass of the water increases very slightly. This example is ... to understand from the point of view of kinetic theory (Chapter ... cause as heat is added, the temperature and therefore the average ... the molecules increases; and Eq. 26–4 tells us that the mass also in...

EXAMPLE 26–5 **Pion's KE.** A π^0 meson ($m_0 = 2.4 \times 10^{-28}$ kg) ... at a speed $v = 0.80c = 2.4 \times 10^8$ m/s. What is its kinetic energy? ... pare to a classical calculation.

SOLUTION The mass of the π^0 moving with a speed of $v = 0.80c$ ...

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{2.4 \times 10^{-28}\text{ kg}}{\sqrt{1 - (0.80)^2}} = 4.0 \times 10^{-28}\text{ kg}$$

Thus its KE is

PROBLEM SOLVING

Relativistic KE

$$\text{KE} = (m - m_0)c^2 = (4.0 \times 10^{-28}\text{ kg} - 2.4 \times 10^{-28}\text{ kg})(3.0 \times 10^8 \ldots$$
$$= 1.4 \times 10^{-11}\text{ J}.$$

Notice that the units of mc^2 are kg·m²/s², which is the joule. A classical cal... culation would give $\text{KE} = \frac{1}{2}m_0v^2 = \frac{1}{2}(2.4 \times 10^{-28}\text{ kg})(2.4 \times 10^8 \text{ m/s})^2 = $ 6.9×10^{-12} J, about half as much, but this is not a correct result.

EXAMPLE 26–6 **Energy from pion mass.** How much energy would ... released if the π^0 meson in the last example is transformed comple... into electromagnetic radiation?

Rest energy

SOLUTION The rest energy of the π^0 is

$$E_0 = m_0c^2 = (2.40 \times 10^{-28}\text{ kg})(3.00 \times 10^8\text{ m/s})^2 = 2.16 \times 10^{\ldots}$$

This is how much energy would be released if the π^0 decayed at re... saw in Chapter 17, Section 17–4, that the energies of atomic particles ... often expressed in terms of the electron volt (eV) unit:

$$1\text{ eV} = 1.60 \times 10^{-19}\text{ J}, \qquad \text{and} \qquad 1\text{ MeV} = 10^6\text{ eV} = 1.60 \times 10^{\ldots}$$

Thus the rest mass of the π^0 is equivalent to

$$\frac{2.16 \times 10^{-11}\text{ J}}{1.60 \times 10^{-13}\text{ J/MeV}} = 135\text{ MeV}$$

of energy. If the π^0 had $\text{KE} = 1.4 \times 10^{-11}$ J, the total energy ... would be $(2.16 + 1.4) \times 10^{-11}\text{ J} = 3.6 \times 10^{-11}$ J, or 230 MeV.

EXAMPLE 26–7 **Energy from nuclear decay.** The energy required or released in nuclear reactions and decays comes from a change in mass between the initial and final particles. In one type of radioactive decay (Chapter 30), an atom of uranium ($m = 232.03714\,\text{u}$) decays to an atom of thorium ($m = 228.02873\,\text{u}$) plus an atom of helium ($m = 4.00260\,\text{u}$) where the masses given are in atomic mass units ($1\,\text{u} = 1.6605 \times 10^{-27}\,\text{kg}$). Calculate the energy released in this decay.

SOLUTION The initial mass is 232.03714 u, and after the decay it is $228.02873\,\text{u} + 4.00260\,\text{u} = 232.03133\,\text{u}$, so there is a decrease in mass of 0.00581 u. This mass, which equals $(0.00581\,\text{u})(1.66 \times 10^{-27}\,\text{kg}) = 9.64 \times 10^{-30}\,\text{kg}$, is changed into energy. By $E = mc^2$, we have

Energy released in nuclear process

$$E = (9.64 \times 10^{-30}\,\text{kg})(3.0 \times 10^8\,\text{m/s})^2 = 8.68 \times 10^{-13}\,\text{J}.$$

Since $1\,\text{MeV} = 1.60 \times 10^{-13}\,\text{J}$, the energy released is 5.4 MeV.

Equation 26–5 for the kinetic energy can be written in terms of the speed v of the object with the help of Eq. 26–4:

$$\text{KE} = m_0c^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right). \qquad \textbf{(26–7)}$$

At low speeds, $v \ll c$, we can expand the square root in Eq. 26–7 using the binomial expansion (see Appendix A or Example 26–2). Then we get

$$\text{KE} \approx m_0c^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \cdots - 1\right)$$
$$\approx \tfrac{1}{2}m_0v^2,$$

where the dots in the first expression represent very small terms in the expansion which we have neglected since we assumed that $v \ll c$. Thus at low speeds, the relativistic form for kinetic energy reduces to the classical form, $\text{KE} = \frac{1}{2}m_0v^2$. This is, of course, what we would like. It makes relativity a more valuable theory in that it can predict accurate results at low speed as well as at high. Indeed, the other equations of special relativity also reduce to their classical equivalents at ordinary speeds: length contraction, time dilation, and mass increase all disappear for $v \ll c$ since $\sqrt{1 - v^2/c^2} \approx 1$.

A useful relation between the total energy E of a particle and its momentum p can also be derived. The relativistic momentum of a particle of mass m and speed v is given by Eq. 26–3:

Relativistic momentum

$$p = mv = \frac{m_0v}{\sqrt{1 - v^2/c^2}}.$$

Then, since $E = mc^2$, we can write (in the first line we insert "$v^2 - v^2$" which is zero, but will help us):

$$E^2 = m^2c^4 = m^2c^2(c^2 + v^2 - v^2)$$
$$= m^2c^2v^2 + m^2c^2(c^2 - v^2)$$
$$= p^2c^2 + \frac{m_0^2c^4(1 - v^2/c^2)}{1 - v^2/c^2},$$

Energy–momentum relation

$$E^2 = p^2c^2 + m_0^2c^4, \qquad \textbf{(26–8)}$$

where we have assumed there is no potential energy. Thus, the total energy can be written in terms of the momentum p, or in terms of the kinetic energy (Eq. 26–6).

Units:
eV/c *for p*
eV/c^2 *for m*

In the tiny world of atoms and nuclei, it is common to quote ... in eV (electron volts) or multiples such as MeV (10^6 eV). Mom... Eq. 26–8) can be quoted in units of eV/c (or MeV/c). And ... quoted (from $E = mc^2$) in units of eV/c^2 (or MeV/c^2).

26–11 Relativistic Addition of Velocities

Consider a rocket ship that travels away from the Earth with ... assume that this rocket has fired off a second rocket that trav... u' with respect to the first (Fig. 26–13). We might expect that th... of rocket 2 with respect to Earth is $u = v + u'$, which in the case ... the figure is $u = 0.60c + 0.60c = 1.20c$. But, as discussed in Se... no object can travel faster than the speed of light in any refere... Indeed, Einstein showed that since length and time are different ... ent reference frames, the old addition-of-velocities formula is no ... valid. Instead, the correct formula is

*Relativistic addition of velocities formula (**u** and **v** along same line)*

$$u = \frac{v + u'}{1 + vu'/c^2}$$

for motion along a straight line. We derive this formula in App... u' is in the opposite direction from v, then u' must have a minus ... $u = (v - u')/(1 - vu'/c^2)$.

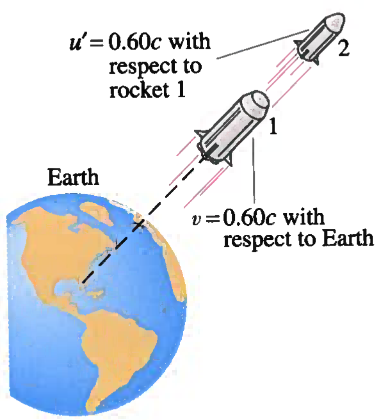


FIGURE 26–13 Rocket 2 is fired from rocket 1 with speed $u' = 0.60c$. What is the speed of rocket 2 with respect to the Earth?

EXAMPLE 26–8 **Relative velocity, relativistically.** Calculate the ... of rocket 2 in Fig. 26–13 with respect to Earth.

SOLUTION Rocket 2 moves with speed $u' = 0.60c$ with ... rocket 1. Rocket 1 has speed $v = 0.60c$ with respect to Earth. The ... of rocket 2 with respect to Earth is therefore

$$u = \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c.$$

Notice that Eq. 26–9 reduces to the classical form for veloc... compared to the speed of light since $1 + vu'/c^2 \approx 1$ for v and ... Thus, $u \approx v + u'$.

Let us test our formula in one more case, that of the speed ... Suppose that rocket 1 in Fig. 26–13 sends out a beam of light ...

$v' = c$. Equation 26–9 tells us that the speed of this light with respect to Earth is

$$u = \frac{0.60c + c}{1 + \frac{(0.60c)(c)}{c^2}} = \frac{1.60c}{1.60} = c,$$

which is fully consistent with the second postulate of relativity.

26–12 The Impact of Special Relativity

A great many experiments have been performed to test the predictions of the special theory of relativity. Within experimental error, no contradictions have been found. Scientists have therefore accepted relativity as an accurate description of nature.

At speeds much less than the speed of light, the relativistic formulas reduce to the old classical ones, as we have discussed. We would, of course, hope—or rather, insist—that this be true since Newtonian mechanics works so well for objects moving with speeds $v \ll c$. This insistence that a more general theory (such as relativity) give the same results as a more restricted theory (such as classical mechanics which works for $v \ll c$) is called the **correspondence principle**. The two theories must correspond where their realms of validity overlap. Relativity thus does not contradict classical mechanics. Rather, it is a more general theory, of which classical mechanics is now considered to be a limiting case.

Correspondence principle

The importance of relativity is not simply that it gives more accurate results, especially at very high speeds. Much more than that, it has changed the way we view the world. The concepts of space and time are now seen to be relative, and intertwined with one another, whereas before they were considered absolute and separate. Even our concepts of matter and energy have changed: either can be converted to the other. The impact of relativity extends far beyond physics. It has influenced the other sciences, and even the world of art and literature; it has, indeed, entered the general culture.

From a practical point of view, we do not have much opportunity in our daily lives to use the mathematics of relativity. For example, the factor $\sqrt{1 - v^2/c^2}$, which appears in many relativistic formulas, has a value of 0.995 when $v = 0.10c$. Thus, for speeds even as high as $0.10c = 3.0 \times 10^7$ m/s, the factor $\sqrt{1 - v^2/c^2}$ in relativistic formulas gives a numerical correction of less than 1 percent. For speeds less than $0.10c$, or unless mass and energy are interchanged, we thus don't usually need to use the more complicated relativistic formulas, and can use the simpler classical formulas.

The special theory of relativity we have studied in this chapter deals with inertial (nonaccelerating) reference frames. In Chapter 33 we will discuss briefly the more complicated "general theory of relativity" which can deal with noninertial reference frames.

SUMMARY

An **inertial reference frame** is one in which Newton's law of inertia holds. Inertial reference frames can move at constant velocity relative to one another; accelerating reference frames are noninertial.

The **special theory of relativity** is based on two principles: the **relativity principle**, which states that the laws of physics are the same in all inertial reference frames, and the principle of the **constancy of the speed of light**, which states that the speed of light in empty space has the same value in all inertial reference frames.

One consequence of relativity theory is that two events that are simultaneous in one reference frame may not be simultaneous in another. Other effects are **time dilation**: moving clocks are measured to run slowly; **length contraction**: the length of a moving object is measured to be shorter (in its direction of motion) than when it is at rest; *mass increase*: the mass of a body is measured to increase with speed. Quantitatively,

$$L = L_0\sqrt{1 - v^2/c^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where L, Δt, and m are the length, time interval, and mass of objects (or events) that are observed as they move by at speed v; L_0, Δt_0, and [illegible] **proper length, proper time**, and **rest mass** [illegible] the same quantities as measured in the [illegible] of the objects or events. Velocity addition [illegible] must be done in a special way. All these [illegible] significant only at high speeds, close to the [illegible] of light, which itself is the ultimate speed [illegible] universe.

The theory of relativity has changed [illegible] tions of space and time, and of mass and [illegible] Space and time are seen to be intimately [illegible] ed, with time being the fourth dimension [illegible] tion to the three dimensions of space. [illegible] energy are interconvertible. The equation

$$E = mc^2$$

tells how much energy E is needed to [illegible] mass m, or vice versa. Said another way [illegible] is the amount of energy an object has [illegible] its mass m. The law of conservation of [illegible] include mass as a form of energy. The kinetic [illegible] gy of an object moving at speed v is given by

$$\text{KE} = mc^2 - m_0c^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)[illegible]$$

where m_0 is the rest mass of the object. [illegible] mentum p of an object is related to its total [illegible] E (assuming no potential energy) by

$$E^2 = p^2c^2 + m_0^2c^4.$$

QUESTIONS

1. You are in a windowless car in an exceptionally smooth train. Is there any physical experiment you can do in the train car to determine whether you are moving?
2. You might have had the experience of being at a red light when, out of the corner of your eye, you see the car beside you creep forward. Instinctively you stomp on the brake pedal, thinking that you are rolling backward. What does this say about absolute and relative motion?
3. A worker stands on top of a moving railroad car, and throws a heavy ball straight up (from his point of view). Ignoring air resistance, will the ball land on the car or behind it?
4. Does the Earth really go around the [illegible] also valid to say that the Sun goes around [illegible] Discuss in view of the first principle of relativity [illegible] there is no best reference frame).
5. If you were on a spaceship traveling at 0.5 [illegible] a star, at what speed would the starlight [illegible]
6. Will two events that occur at the same place [illegible] time for one observer be simultaneous [illegible] observer moving with respect to the first?
7. Analyze the thought experiment of Section [illegible] O_1's point of view. (Make a diagram [illegible] Fig. 26–6.)

8. The time-dilation effect is sometimes expressed as "moving clocks run slowly." Actually, this effect has nothing to do with motion affecting the functioning of clocks. What then does it deal with?

9. Does time dilation mean that time actually passes more slowly in moving reference frames or that it only *seems* to pass more slowly?

10. A young-looking woman astronaut has just arrived home from a long trip. She rushes up to an old gray-haired man and in the ensuing conversation refers to him as her son. How might this be possible?

11. If you were traveling away from Earth at speed $0.5c$, would you notice a change in your heartbeat? Would your mass, height, or waistline change? What would observers on Earth using telescopes say about you?

12. Discuss how our everyday lives would be different if the speed of light were only 25 m/s.

13. Do mass increase, time dilation, and length contraction occur at ordinary speeds, say 90 km/h?

14. Suppose the speed of light were infinite. What would happen to the relativistic predictions of length contraction, time dilation, and mass increase?

15. Explain how the length-contraction and time-dilation formulas might be used to indicate that c is the limiting speed in the universe.

16. Consider an object of mass m to which is applied a constant force for an indefinite period of time. Discuss how its velocity and mass change with time.

17. A white-hot iron bar is cooled to room temperature. Does its mass change?

18. Does the equation $E = mc^2$ conflict with the conservation of energy principle? Explain.

19. Does $E = mc^2$ apply to particles that travel at the speed of light? Does it apply only to them?

20. An electron is limited to travel at speeds less than c. Does this put an upper limit on the momentum of an electron? If so, what is this upper limit?

21. If mass is a form of energy, does this mean that a spring has more mass when compressed than when relaxed?

22. It is not correct to say that "matter can neither be created nor destroyed." What must we say instead?

23. Is our intuitive notion that velocities simply add, as we did in Section 3–8, completely wrong?

PROBLEMS

SECTIONS 26–5 AND 26–6

1. (I) Lengths and time intervals (as well as mass) depend on the factor

$$\sqrt{1 - v^2/c^2}$$

according to the theory of relativity (Eqs. 26–1, 26–2, 26–4). Evaluate this correction factor for speeds of: (*a*) $v = 20{,}000$ m/s (typical speed of a satellite); (*b*) $v = 0.0100c$; (*c*) $v = 0.100c$; (*d*) $v = 0.900c$; (*e*) $v = 0.990c$; (*f*) $v = 0.999c$.

2. (I) A spaceship passes you at a speed of $0.850c$. You measure its length to be 48.2 m. How long would it be when at rest?

3. (I) A beam of a certain type of elementary particle travels at a speed of 2.70×10^8 m/s. At this speed, the average lifetime is measured to be 4.76×10^{-6} s. What is the particle's lifetime at rest?

4. (I) If you were to travel to a star 100 light-years from Earth at a speed of 2.60×10^8 m/s, what would you measure this distance to be?

5. (II) You are sitting in your car when a very fast sports car passes you at a speed of $0.37c$. A person in that car says his car is 6.00 m long and yours is 6.21 m long. What do you measure for these two lengths?

6. (II) What is the speed of a beam of pions if their average lifetime is measured to be 4.10×10^{-8} s? At rest, their lifetime is 2.60×10^{-8} s.

7. (II) Suppose you decide to travel to a star 90 light-years away. How fast would you have to travel so the distance would be only 25 light-years?

8. (II) At what speed do the relativistic formulas for length and time intervals differ from classical values by 1.00 percent? (This is a reasonable way to estimate when to do relativistic calculations rather than classical.)

9. (II) Suppose a news report stated that starship *Enterprise* had just returned from a 5-year voyage while traveling at $0.89c$. (*a*) If the report meant 5.0 years of *Earth time*, how much time elapsed on the ship? (*b*) If the report meant 5.0 years of *ship time*, how much time passed on Earth?

10. (II) A certain star is 75.0 light-years away. How long would it take a spacecraft traveling $0.950c$ to reach that star from Earth, as measured by observers: (*a*) on Earth, (*b*) on the spacecraft? (*c*) What is the distance traveled according to observers on the spacecraft? (*d*) What will the spacecraft occupants compute their speed to be from the results of (*b*) and (*c*)?

11. (II) A friend of yours travels by you in her fast sports vehicle at a speed of $0.580c$. You measure it to be 5.80 m long and 1.20 m high. (*a*) What will be its length and height at rest? (*b*) How many seconds would you say elapsed on your friend's watch when 20.0 s passed on yours? (*c*) How fast did you appear to be traveling to your friend? (*d*) How many seconds would she say elapsed on your watch when she saw 20.0 s pass on hers?

12. (III) How fast must a pion be moving, on average, to travel 10.0 m before it decays? The average lifetime, at rest, is 2.60×10^{-8} s.

SECTION 26–8

13. (I) What is the mass of a proton traveling at $v = 0.90c$?

14. (I) At what speed will an object's mass be twice its rest mass?

15. (II) At what speed v will the mass of an object be 10 percent greater than its rest mass?

16. (II) Escape velocity from the Earth is 40,000 km/h. What would be the percent increase in mass of a 7.2×10^5-kg spacecraft traveling at that speed?

17. (II) (*a*) What is the speed of an electron whose mass is 10,000 times its rest mass? Such speeds are reached in the Stanford Linear Accelerator, SLAC. (*b*) If the electrons travel in the lab through a tube 3.0 km long (as at SLAC), how long is this tube in the electron's reference frame?

SECTION 26–10

18. (I) What is the kinetic energy of an electron whose mass is 3.0 times its rest mass?

19. (I) A certain chemical reaction requires 4.82×10^4 J of energy input for it to go. What is the increase in rest mass of the products over the reactants?

20. (I) When a uranium nucleus at rest breaks apart in the process known as fission in a nuclear reactor, the resulting fragments have a total kinetic energy of about 200 MeV. How much mass was lost in the process?

21. (I) Calculate the rest energy of an electron in joules and in MeV (1 MeV = 1.60×10^{-13} J).

22. (I) Calculate the rest mass of a proton in MeV/c^2.

23. (I) The total annual energy consumption in the United States is about 8×10^{19} J. How much mass would have to be converted to energy to fuel this need?

24. (II) How much energy can be obtained from conversion of 1.0 gram of mass? How much mass could this energy raise to a height of 100 m?

25. (II) Show that when the kinetic energy of a particle equals its rest energy, the speed of the particle is about $0.866c$.

26. (II) (*a*) How much work is required to [illegible] proton from rest up to a speed of 0.99[illegible] would be the momentum of this proton?

27. (II) (*a*) By how much does the mass of the [illegible] crease each year as a result solely of the [illegible] reaching it? (*b*) How much mass does the [illegible] per year? (Radiation from the Sun reaches the [illegible] at a rate of about 1400 W/m^2 of area [illegible] to the energy flow.)

28. (II) Calculate the kinetic energy and mom[illegible] a proton traveling 2.50×10^8 m/s.

29. (II) What is the momentum of a 750-MeV [illegible] (that is, one with KE = 750 MeV)?

30. (II) What is the speed of a proton accelerated [illegible] potential difference of 75 MV?

31. (II) What is the speed of an electron wh[illegible] 1.00 MeV?

32. (II) What is the speed and apparent rest [illegible] electron when it hits a television screen [illegible] accelerated by the 25,000 V of the picture [illegible]

33. (II) Two identical particles of rest mass m_0 [illegible] each other at equal and opposite speeds, [illegible] sion is completely inelastic and results in [illegible] particle at rest due to momentum con[illegible] What is the rest mass of the new partic[illegible] much energy was lost in the collision? How [illegible] netic energy is lost in this collision?

34. (II) Calculate the mass of a proton (m_0 [illegible] 10^{-27} kg) whose kinetic energy is half its total [illegible] gy. How fast is it traveling?

35. (II) What is the speed and momentum of an [illegible] ($m_0 = 9.11 \times 10^{-31}$ kg) whose kinetic energ[illegible] its rest energy?

36. (II) Suppose a spacecraft of rest mass [illegible] accelerated to $0.21c$. (*a*) How much kinetic [illegible] would it have? (*b*) If you used the classical [illegible] for KE, by what percentage would you be in [illegible]

37. (II) Calculate the kinetic energy and mom[illegible] proton ($m_0 = 1.67 \times 10^{-27}$ kg) traveling 9.8 [illegible] By what percentages would your calculat[illegible] been in error if you had used classical formul[illegible]

38. (II) The americium nucleus, $^{241}_{95}$Am, decays [illegible] tunium nucleus, $^{237}_{93}$Np, by emitting an alpha [illegible] of mass 4.00260 u and kinetic energy 5.5 MeV [illegible] mate the mass of the neptunium nucleus, [illegible] recoil, given that the americium mass is 241[illegible]

39. (II) An electron ($m_0 = 9.11 \times 10^{-31}$ kg) is [illegible] from rest to speed v by a conservative for[illegible] process, its potential energy decreases by 7.00 [illegible] Determine the electron's speed, v.

40. (II) Make a graph of the kinetic energy ve[illegible] mentum for (*a*) a particle of nonzero rest [illegible] (*b*) a particle with zero rest mass.

(II) What magnetic field intensity is needed to keep 900-GeV protons revolving in a circle of radius 1.0 km (at, say, the Fermilab synchrotron)? Use the relativistic mass. The proton's rest mass is $0.938\ \text{GeV}/c^2$. ($1\ \text{GeV} = 10^9\ \text{eV}$.)

(II) A negative muon traveling at 33 percent the speed of light collides head on with a positive muon traveling at 50 percent the speed of light. The two muons (each of mass $105.7\ \text{MeV}/c^2$) annihilate, and produce electromagnetic energy of what total amount?

(II) Show that the energy of a particle of charge e revolving in a circle of radius r in a magnetic field B is given by $E\,(\text{in eV}) = Brc$ in the relativistic limit ($v \approx c$).

(III) Show that the kinetic energy (KE) of a particle of rest mass m_0 is related to its momentum p by the equation $p = \sqrt{(\text{KE})^2 + 2(\text{KE})(m_0c^2)}/c$.

SECTION 26–11

(I) A person on a rocket traveling at 0.50c (with respect to the Earth) observes a meteor come from behind and pass her at a speed she measures as 0.50c. How fast is the meteor moving with respect to the Earth?

46. (II) Two spaceships leave the Earth in opposite directions, each with a speed of 0.50c with respect to the Earth. (*a*) What is the velocity of spaceship 1 relative to spaceship 2? (*b*) What is the velocity of spaceship 2 relative to spaceship 1?

47. (II) An observer on Earth sees an alien vessel approach at a speed of 0.60c. The *Enterprise* comes to the rescue (Fig. 26–14), overtaking the aliens while moving directly toward Earth at a speed of 0.90c relative to Earth. What is the relative speed of one vessel as seen by the other?

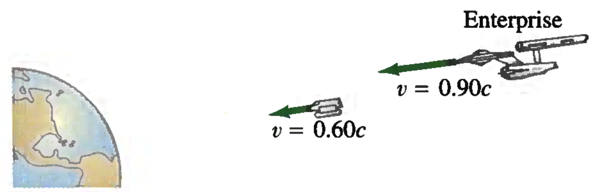


FIGURE 26–14 Problem 47.

48. (II) A spaceship leaves Earth traveling 0.65c. A second spaceship leaves the first at a speed of 0.91c with respect to the first. Calculate the speed of the second ship with respect to Earth if it is fired (*a*) in the same direction the first spaceship is already moving, (*b*) directly backward toward Earth.

GENERAL PROBLEMS

As a rule of thumb, anything traveling faster than about 0.1c is called *relativistic*—i.e., for which the correction using special relativity is a significant effect. Is the electron in a hydrogen atom (radius 0.5×10^{-10} m) relativistic? (Treat the electron as though it were in a circular orbit around the proton.)

An atomic clock is taken to the North Pole, while another stays at the Equator. How far will they be out of synchronization after a year has elapsed?

The nearest star to Earth is Proxima Centauri, 4.3 light-years away. (*a*) At what constant velocity must a spacecraft travel from Earth if it is to reach the star in 4.0 years, as measured by travelers on the spacecraft? (*b*) How long does the trip take according to Earth observers?

Derive a formula showing how the density of an object changes with speed v relative to an observer.

An airplane travels 1500 km/h around the world, returning to the same place, in a circle of radius essentially equal to that of the Earth. Estimate the difference in time to make the trip as seen by Earth and airplane observers. [*Hint*: Use the binomial expansion, Appendix A.]

54. How many grams of matter would have to be totally destroyed to run a 100-W lightbulb for 1 year?

55. What minimum amount of electromagnetic energy is needed to produce an electron and a positron together? A positron is a particle with the same rest mass as an electron, but has the opposite charge. (Note that electric charge is conserved in this process. See Section 27–4.)

56. A 1.68-kg mass oscillates on the end of a spring whose spring constant is $k = 48.7\ \text{N/m}$. If this system is in a spaceship moving past Earth at 0.900c, what is its period of oscillation according to (*a*) observers on the ship, and (*b*) observers on Earth?

57. An electron ($m_0 = 9.11 \times 10^{-31}$ kg) enters a uniform magnetic field $B = 1.8$ T, and moves perpendicular to the field lines with a speed $v = 0.92c$. What is the radius of curvature of its path?

58. A free neutron can decay into a proton, an electron, and a neutrino. The neutrino's rest mass is zero, and the other masses can be found in the table inside the front cover. Determine the total kinetic energy shared among the three particles when a neutron decays at rest.

59. The Sun radiates energy at a rate of about 4×10^{26} W. (*a*) At what rate is the Sun's mass decreasing? (*b*) How long does it take for the Sun to lose a mass equal to that of Earth? (*c*) Estimate how long the Sun could last if it radiated constantly at this rate.

60. An unknown particle is measured to have a negative charge and a speed of 2.24×10^8 m/s. Its momentum is determined to be 3.07×10^{-22} kg·m/s. Identify the particle by finding its rest mass.

61. How much energy would be required to break a helium nucleus into its constituents, two protons and two neutrons? The rest masses of a proton (including an electron), a neutron, and helium are, respectively, 1.00783 u, 1.00867 u, and 4.00260 u. (This is called the *total binding energy* of the $^4_2$He nucleus.)

62. What is the percentage increase in the mass of a car traveling 110 km/h as compared to at rest?

63. Two protons, each having a speed of 0.[illegible] laboratory, are moving toward each other [illegible] mine (*a*) the momentum of each proton in the [illegible] ratory, (*b*) the total momentum of the two [illegible] the laboratory, and (*c*) the momentum of [illegible] as seen by the other proton.

64. A pi meson of rest mass m_π decays at [illegible] muon (rest mass m_μ) and a neutrino of [illegible] mass. Show that the kinetic energy of the [illegible] $\text{KE}_\mu = (m_\pi - m_\mu)^2c^2/2m_\pi$.

65. A farm boy studying physics believes that he [illegible] 15.0-m-long pole into a 12.0-m-long barn if he [illegible] fast enough (carrying the pole). Can he do it? [illegible] in detail. How does this fit with the idea that [illegible] is running the barn looks even shorter than [illegible]

66. Show analytically that a particle with mom[illegible] and energy E has a speed given by

$$v = \frac{pc^2}{E} = \frac{pc}{\sqrt{m_0^2c^2 + p^2}}.$$