Transmission lines carry electric power over great distances, at very high voltage for greater efficiency. To reduce high voltage to usable voltage, transformers are used, whose operation depends on electromagnetic induction. Induction is also the basis for electric generators, which produce the electric power in the first place.

CHAPTER

21 ELECTROMAGNETIC INDUCTION AND FARADAY'S LAW; AC CIRCUITS

In Chapter 20, we discussed two ways in which electricity and [illegible] ism are related: (1) an electric current produces a magnetic [illegible] (2) a magnetic field exerts a force on an electric current or [illegible] electric charge. These discoveries were made in 1820–1821. Scient[illegible] began to wonder: if electric currents produce a magnetic field, is it [illegible] ble that a magnetic field can produce an electric current? Ten years [illegible] the American Joseph Henry (1797–1878) and the Englishman [illegible] Faraday (1791–1867) independently found that it was possible. [illegible] tually made the discovery first. But Faraday published his result[illegible] and investigated the subject in more detail. We now discuss this phe[illegible] non and some of its world-changing applications.

21–1 Induced EMF

In his attempt to produce an electric current from a magnetic field, [illegible] used an apparatus like that shown in Fig. 21–1. A coil of wire, X, [illegible] nected to a battery. The current that flowed through X produced a [illegible] ic field that was intensified by the iron core. Faraday hoped that by [illegible]

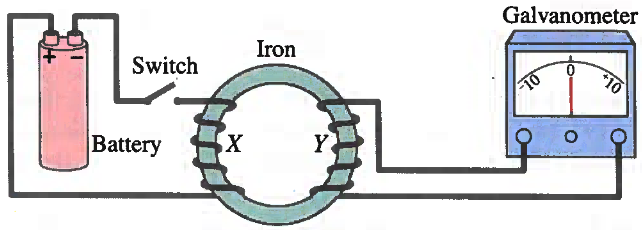


FIGURE 21–1 Faraday's experiment to induce an emf.

strong enough battery, a steady current in X would produce a great enough magnetic field to produce a current in a second coil Y. This second circuit, Y, contained a galvanometer to detect any current but contained no battery. He met no success with steady currents. But the long-sought effect was finally observed when Faraday saw the galvanometer in circuit Y deflect strongly at the moment he closed the switch in circuit X. And the galvanometer deflected strongly in the opposite direction when he opened the switch. A *steady* current in X had produced *no* current in Y. Only when the current in X was starting or stopping was a current produced in Y.

Constant **B** *induces no emf*

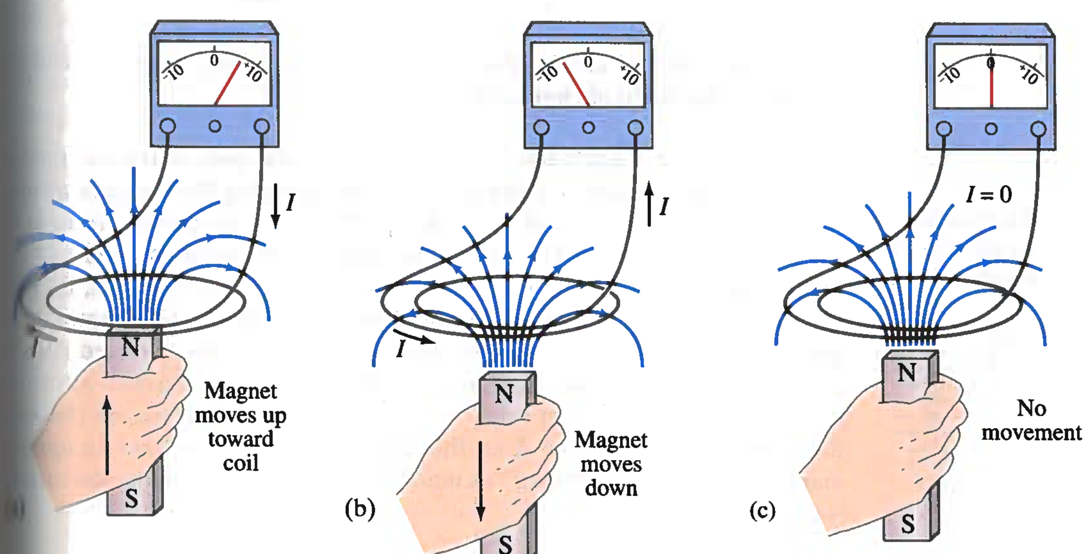
Faraday concluded that although a steady magnetic field produces no current, a *changing* magnetic field can produce an electric current! Such a current is called an **induced current.** When the magnetic field through coil Y changes, a current flows as if there were a source of emf in the circuit. We therefore say that an

induced emf is produced by a changing magnetic field.

Changing **B** *induces an emf*

Faraday did further experiments on **electromagnetic induction**, as this phenomenon is called. For example, Fig. 21–2 shows that if a magnet is moved quickly into a coil of wire, a current is induced in the wire. If the magnet is quickly removed, a current is induced in the opposite direction. Furthermore, if the magnet is held steady and the coil of wire is moved toward or away from the magnet, again an emf is induced and a current flows. Motion or change is required to induce an emf. It doesn't matter whether the magnet or the coil moves.

FIGURE 21–2 (a) A current is induced when a magnet is moved toward a coil. (b) The induced current is opposite when the magnet is moved away from the coil. Note that the galvanometer zero is at the center of the scale and the needle deflects left or right, depending on the direction of the current. In (c) no current is induced if the magnet does not move relative to the coil.



21–2 Faraday's Law of Induction; Lenz's Law

Faraday investigated quantitatively what factors influence the magnitude of the emf induced. He found first of all that it depends on time: the more rapidly the magnetic field changes, the greater the induced emf. But the emf is not simply proportional to the rate of change of the magnetic field **B**. Rather it is proportional to the rate of change of the **magnetic flux**, Φ_B, passing through the loop of area A, which is defined as

Magnetic flux defined

$$\Phi_B = B_\perp A = BA\cos\theta. \quad (21\text{–}1)$$

Here $B_\perp$ is the component of the magnetic field **B** perpendicular to the face of the coil, and θ is the angle between **B** and a line drawn perpendicular to the face of the coil. These quantities are shown in Fig. 21–3 for a square coil of side l whose area $A = l^2$. When the face of the coil is parallel to **B**, $\theta = 90°$ and $\Phi_B = 0$. When **B** is perpendicular to the coil, $\theta = 0°$ and

$$\Phi_B = BA. \qquad [\mathbf{B} \perp \text{coil face}]$$

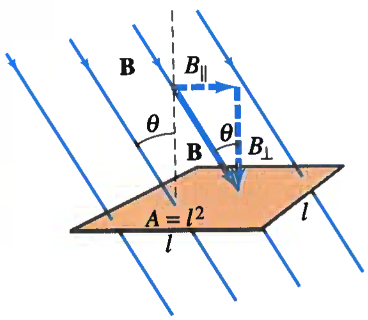


FIGURE 21–3 Determining the flux through a flat loop of wire. This loop is square, of side l and area $A = l^2$.

As we saw earlier, the lines of **B** (like lines of **E**) can be drawn such that the number of lines per unit area is proportional to the field strength. Then the flux Φ_B can be thought of as being proportional to the *total number of lines passing through the coil*. This is illustrated in Fig. 21–4, where the coil is viewed from the side (on edge). For $\theta = 90°$, no lines pass through the coil and $\Phi_B = 0$, whereas Φ_B is a maximum when $\theta = 0°$. The unit of magnetic flux is the tesla-meter^2; this is called a **weber**: 1 Wb = 1 T·m^2.

With this definition of the flux, we can now write down the results of Faraday's investigations. If the flux through N loops of wire changes by an amount $\Delta\Phi_B$ during a time Δt, the average induced emf during this time is

FARADAY'S LAW OF INDUCTION

$$\mathscr{E} = -N\frac{\Delta\Phi_B}{\Delta t}. \quad (21\text{–}2)$$

This fundamental result is known as **Faraday's law of induction**, and is one of the basic laws of electromagnetism.

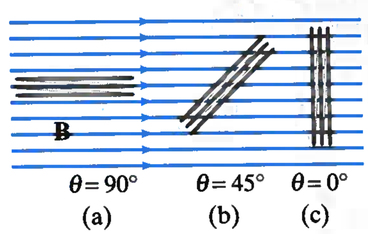
The minus sign in Eq. 21–2 is placed there to remind us in which direction the induced emf acts. Experiments show that

Lenz's law

> **an induced emf always gives rise to a current whose magnetic field opposes the original change in flux.**

This is known as **Lenz's law**. Let us apply it to the case of relative motion between a magnet and a coil, Fig. 21–2. The changing flux induces an emf, which produces a current in the coil. And this induced current produces its own magnetic field. In Fig. 21–2a the distance between the coil and the magnet decreases. So the magnetic field, and therefore the flux, through the coil increases. The magnetic field of the magnet points upward. To oppose this upward increase, the field produced by the induced current points *downward*. Thus, Lenz's law tells us that the current moves as shown (use the right-hand rule). In Fig. 21–2b, the flux *decreases* (because the magnet is moved away), so the induced current produces an upward magnetic field that is "trying" to maintain the status quo. Thus the current is as shown.

FIGURE 21–4 Magnetic flux Φ_B is proportional to the number of lines of **B** that pass through the loop.



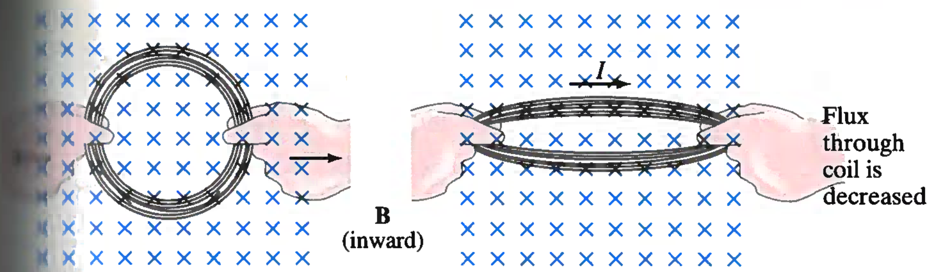


FIGURE 21–5 A current can be induced by changing the area of the coil. In both this case and that of Fig. 21–6, the flux through the coil is reduced. Here the brief induced current acts in the direction shown so as to try to maintain the original flux ($\Phi = BA$) by producing its own magnetic field into the page. That is, as the area A decreases, the current acts to increase B in the original (inward) direction.

Let us consider what would happen if Lenz's law were not true, but were just the reverse. The induced current in this imaginary situation would produce a flux in the same direction as the original change. This greater change in flux would produce an even larger current followed by a still greater change in flux, and so on. The current would continue to grow indefinitely, producing power ($=I^2R$) even after the original stimulus ended. This would violate the conservation of energy. Such "perpetual motion" devices do not exist. Thus, Lenz's law as stated above (and not its opposite) is consistent with the law of conservation of energy.

It is important to note that an emf is induced whenever there is a change in flux. Since magnetic flux $\Phi_B = BA\cos\theta$, we see that an emf can be induced in three ways: (1) by a changing magnetic field B; (2) by changing the area of the loop in the field; or (3) by changing the loop's orientation θ with respect to the field. Figures 21–1 and 21–2 illustrated case 1. Examples of cases 2 and 3 are illustrated in Figs. 21–5 and 21–6, respectively.

CONCEPTUAL EXAMPLE 21–1 **Induction stove.** Some modern stove burners are based on induction. That is, an ac current passes around a coil that is the "burner" (a burner that never gets hot). Why will it heat a metal pan but not a glass container?

PHYSICS APPLIED
Induction stove

RESPONSE The ac current sets up a changing magnetic field that passes through the pan bottom. This changing field induces a current through the pan bottom, and since the pan offers resistance, electric energy is transformed to heat, heating the pot and its contents. A glass container offers very high resistance so very little current is induced and very little energy transferred. Recall Eq. 18–6c, $P = V^2/R$.

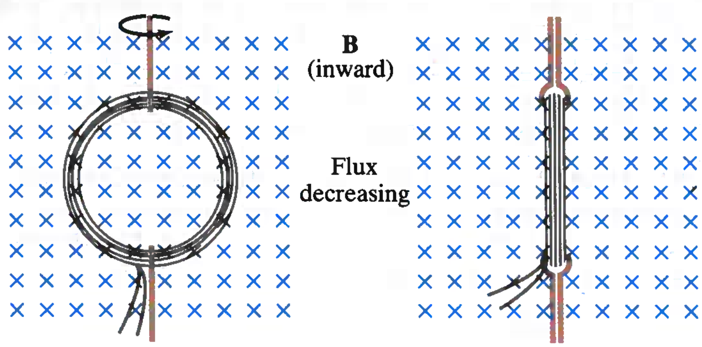
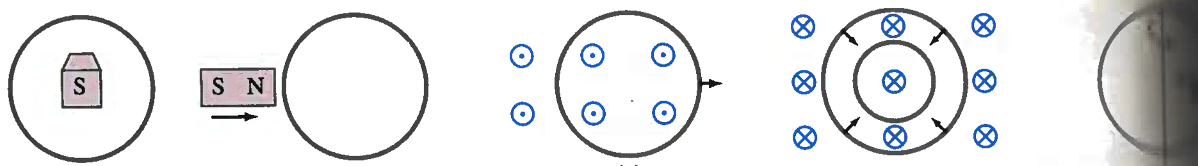


FIGURE 21–6 A current can be induced by rotating a coil in a magnetic field.



(a) N magnetic pole moving toward coil from above the page

(b) N magnetic pole moving toward the coil in the plane of the page

(c) Pulling the coil to the right out of a magnetic field that points out of the page

(d) Shrinking a coil in a magnetic field pointing into the page

(e) Rotating th[illegible] vertical dia[illegible] the left side [illegible] and pushing [illegible] away from the magnetic field from right to left [illegible] of the [illegible]

FIGURE 21–7 Example 21–2.

CONCEPTUAL EXAMPLE 21–2 **Practice with Lenz's law.** In [illegible] direction is the current induced in the coil for each situation in Fig. [illegible]

RESPONSE (*a*) Magnetic field lines point out from the N pol[illegible] magnet, so as the magnet moves down toward the coil, the field [illegible] into the page and is getting stronger. The current will be induc[illegible] counterclockwise direction to produce a field **B** *out* of the page [illegible] its own flux counteracts the externally imposed change.

(*b*) The field is in the plane of the page, so the flux through the [illegible] zero throughout the process; hence there is no change in magn[illegible] with time, and there will be no induced emf or current in the coil.

(*c*) Initially, the magnetic flux pointing out of the page passes [illegible] the coil. If you remove the coil, the induced current will be in a di[illegible] to make up the deficiency: the current flow will be countercloc[illegible] produce an outward (toward the reader) magnetic field.

(*d*) The flux is into the page and the coil area shrinks so the flux will [illegible] crease; hence the induced current will be clockwise to try to prod[illegible] own flux into the page to make up for the flux decrease.

(*e*) Initially there is no flux through the coil (why?). When you [illegible] rotate the coil, the flux begins passing through the coil increasing [illegible] left. To counteract this, the coil will have current induced in a [illegible] clockwise direction so as to produce its own flux to the right.

EXAMPLE 21–3 **Pulling a coil from a magnetic field.** A square [illegible] side 5.0 cm contains 100 loops and is positioned perpendicular to a [illegible] form 0.60-T magnetic field, as shown in Fig. 21–8. It is quickly [illegible] formly pulled from the field (moving perpendicular to **B**) to a [illegible] where B drops abruptly to zero. It takes 0.10 s for the whole coil to [illegible] the field-free region. Find (*a*) the change in flux through the coil, (*b*) [illegible] emf and current induced, and (*c*) how much energy is dissipated [illegible] coil if its resistance is 100 Ω. (*d*) What was the average force requ[illegible]

SOLUTION (*a*) First we find how the magnetic flux, $\Phi_B = BA$, ch[illegible] during the time interval $\Delta t = 0.10$ s. The area of the coil is $A = (0.050$ [illegible] $2.5 \times 10^{-3}\,\mathrm{m}^2$. The flux is initially $\Phi_B = BA = (0.60\,\mathrm{T})(2.5 \times 10^{-3}$ [illegible] 1.5×10^{-3} Wb. After 0.10 s, the flux is zero. Hence the change in flu[illegible]

$$\Delta\Phi_B = 0 - 1.5 \times 10^{-3}\,\mathrm{Wb} = -1.5 \times 10^{-3}\,\mathrm{Wb}.$$

(*b*) The rate of change of flux is constant during the 0.10 s, so the [illegible] field (Eq. 21–2) during this period is

$$\mathscr{E} = -(100)\frac{(0 - 1.5 \times 10^{-3}\,\text{Wb})}{(0.10\,\text{s})} = 1.5\,\text{V}.$$

The current is

$$I = \frac{\mathscr{E}}{R} = \frac{1.5\,\text{V}}{100\,\Omega} = 15\,\text{mA}.$$

(c) The total energy dissipated is

$$E = Pt = I^2Rt = (1.5 \times 10^{-2}\,\text{A})^2(100\,\Omega)(0.10\,\text{s}) = 2.3 \times 10^{-3}\,\text{J}.$$

(d) From the conservation of energy principle, the result in (c) is equal to the work W needed to pull the coil out of the field. Since $W = \bar{F}d$, the average force is

$$\bar{F} = \frac{W}{d} = \frac{2.3 \times 10^{-3}\,\text{J}}{5.0 \times 10^{-2}\,\text{m}} = 0.046\,\text{N},$$

where $d = 5.0\,\text{cm}$ because there is no flux change (hence no force) until one edge of the coil leaves the field.

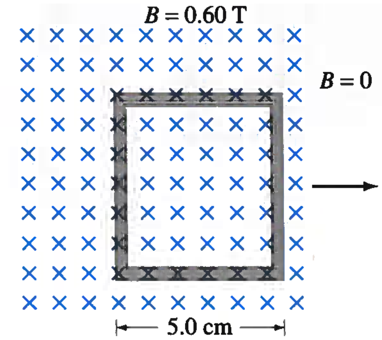


FIGURE 21–8 Example 21–3. The square coil in a magnetic field $B = 0.60\,\text{T}$ is pulled abruptly to the right to a region where $B = 0$.

21–3 EMF Induced in a Moving Conductor

Another way to induce an emf is shown in Fig. 21–9, and this situation helps illuminate the nature of the induced emf. Assume that a uniform magnetic field **B** is perpendicular to the area bounded by the U-shaped conductor and the movable rod resting on it. If the rod is made to move at a speed v, it travels a distance $\Delta x = v\,\Delta t$ in a time Δt. Therefore, the area of the loop increases by an amount $\Delta A = l\,\Delta x = lv\,\Delta t$ in a time Δt. By Faraday's law, there is an induced emf $\mathscr{E}$ whose magnitude is given by

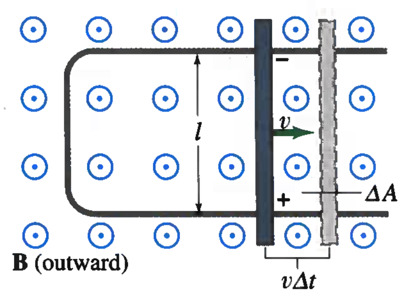
$$\mathscr{E} = \frac{\Delta\Phi_B}{\Delta t} = \frac{B\,\Delta A}{\Delta t} = \frac{Blv\,\Delta t}{\Delta t} = Blv. \qquad \textbf{(21–3)}$$

Motional emf

This equation is valid as long as B, l, and v are mutually perpendicular. (If they are not, we use only the components of each that are mutually perpendicular.) An emf induced in this way is sometimes called *motional emf.*

We can also obtain Eq. 21–3 without using Faraday's law. We saw in Chapter 20 that a charged particle moving perpendicular to a magnetic field B with speed v experiences a force $F = qvB$. When the rod of Fig. 21–9 moves to the right with speed v, the electrons in the rod move with this same speed. Therefore, each feels a force $F = qvB$, which acts upward in the figure. If the rod were not in contact with the U-shaped conductor, electrons would collect at the upper end of the rod, leaving the lower end positive. There must thus be an induced emf. If the rod does slide on the U-shaped conductor, the electrons will flow into it. There will then be a clockwise (conventional) current flowing in the loop. To calculate the emf, we determine the work W needed to move a charge q from one end of the rod to the other against this potential difference: W = force × distance = $(qvB)(l)$. The emf equals the work done per unit charge, so $\mathscr{E} = W/q = qvBl/q = Blv$, just as above.†

FIGURE 21–9 A conducting rod is moved to the right on a U-shaped conductor in a uniform magnetic field **B** that points out of the paper.



† This argument, which is basically the same as for the Hall effect, explains this one way of inducing an emf. It does not explain the general case of electromagnetic induction, however.

FIGURE 21–10 Example 21–4.

EXAMPLE 21–4 **Does a moving airplane develop a dangerous emf?** [illegible] plane travels 1000 km/h in a region where the Earth's field is 5.0 [illegible] and is nearly vertical (Fig. 21–10). What is the potential difference [illegible] between the wing tips that are 70 m apart?

SOLUTION Since $v = 1000\ \text{km/h} = 280\ \text{m/s}$, and $\mathbf{v} \perp \mathbf{B}$, we [illegible]

$$\mathscr{E} = Blv = (5.0 \times 10^{-5}\ \text{T})(70\ \text{m})(280\ \text{m/s}) = 1.0\ \text{V}$$

Not much to worry about.

21–4 Changing Magnetic Flux Produces an Electric Field

Electric field is produced by a changing magnetic flux

As we just discussed, the electrons in the moving conductor of [illegible] feel a force. This implies that there is an electric field in the [illegible] Since electric field is defined as the force per unit charge, $E = F/q$, [illegible] fective field E in the rod must be (since $F = qvB$)

$$E = \frac{F}{q} = \frac{qvB}{q} = vB.$$

In the situation where a changing magnetic field (rather than [illegible] ing conductor) induces an emf (as, for example, in Fig. 21–2), there [illegible] an induced current. And again this implies that there is an electric [illegible] the wire. Thus we come to the important conclusion that

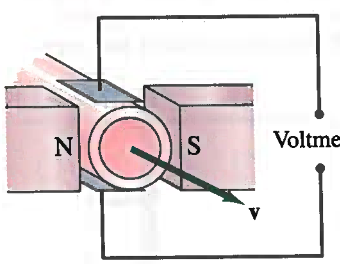
a changing magnetic flux produces an electric field.

This applies not only to wires and other conductors, but is a general [illegible] that applies to any region in space: an electric field will be produced [illegible] point in space where there is a changing magnetic field.

PHYSICS APPLIED

Blood flow measurement

FIGURE 21–11 Measurement of blood velocity from the induced emf.



EXAMPLE 21–5 **Electromagnetic blood-flow measurement.** The [illegible] blood flow can be measured using the apparatus shown in Fig. [illegible] since blood contains charged ions. Suppose that the blood [illegible] 2.0 mm in diameter, the magnetic field is 0.080 T, and the measured [illegible] is 0.10 mV. What is the flow velocity of the blood?

SOLUTION We solve for v in Eq. 21–3, and we find that

$$v = \frac{\mathscr{E}}{Bl} = \frac{(1.0 \times 10^{-4}\ \text{V})}{(0.080\ \text{T})(2.0 \times 10^{-3}\ \text{m})} = 0.63\ \text{m/s}.$$

(In actual practice, an alternating current is used to produce an [illegible] ing magnetic field. The induced emf is then alternating.)

21–5 Electric Generators

Probably the most important practical result of Faraday's great discovery was the development of the **electric generator** or **dynamo**. A generator transforms mechanical energy into electric energy. This is just the opposite of what a motor does. Indeed, a generator is basically the inverse of a motor.† A simplified diagram of an **ac generator** is shown in Fig. 21–12. A generator consists of many coils of wire (only one is shown) wound on an armature that can rotate in a magnetic field. The axle is turned by some mechanical means (falling water, car motor belt), and an emf is induced in the rotating coil. An electric current is thus the *output* of a generator. In Fig. 21–12 the right-hand rule tells us that, with the armature rotating counterclockwise, the (conventional) current in the wire labeled *a* on the armature is outward; therefore it is outward at brush *a*. (Each brush presses against a continuous slip ring.) After one-half revolution, wire *a* will be where wire *b* is now in the drawing, and the current then at brush *a* will be inward. Thus the current produced is alternating. Let us look at this in more detail.

➡ **PHYSICS APPLIED**

AC generator

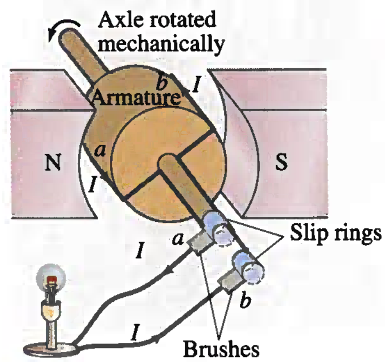


FIGURE 21–12 An ac generator.

In Fig. 21–13, the loop is being made to rotate clockwise in a uniform magnetic field **B**. The velocity of the two lengths *ab* and *cd* at this instant are shown. Although the sections of wire *bc* and *da* are moving, the force on electrons in these sections is toward the side of the wire, not along its length. The emf generated is thus due only to the force on charges in the sections *ab* and *cd*. From the right-hand rule, we see that the direction of the induced current in *ab* is from *a* toward *b*. And in the lower section, it is from *c* to *d*; so the flow is continuous in the loop. The magnitude of the emf generated in *ab* is given by Eq. 21–3, except that we must take the component of the velocity perpendicular to *B*:

$$\mathscr{E} = Blv_{\perp},$$

where *l* is the length of *ab*. From the diagram we can see that $v_{\perp} = v\sin\theta$, where θ is the angle the face of the loop makes with the vertical. The emf induced in *cd* has the same magnitude and is in the same direction. Therefore they add, and the total emf is

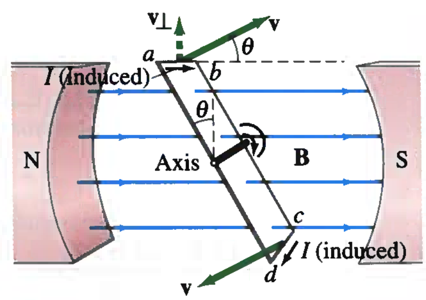
$$\mathscr{E} = 2NBlv\sin\theta,$$

where we have multiplied by *N*, the number of loops in the coil (if there is more than one). If the coil is rotating with constant angular velocity ω, then the angle $\theta = \omega t$. We also have from the angular equations (Chapter 8) that $v = \omega r = \omega(h/2)$, where *h* is the length of *bc* or *ad*. Thus $\mathscr{E} = 2NBl\omega(h/2)\sin\omega t$, or

$$\mathscr{E} = NBA\omega\sin\omega t, \qquad \textbf{(21–5)}$$

where $A = lh$ is the area of the loop. This equation holds for any shape coil, not just for a rectangle as derived. Thus, the output emf of the generator is sinusoidally alternating (Fig. 21–14 and Section 18–8). Since ω is expressed in radians per second, we can write $\omega = 2\pi f$, where *f* is the frequency.

FIGURE 21–13 The emf is induced in the segments *ab* and *cd*, whose velocity components perpendicular to the field **B** are $v\sin\theta$.



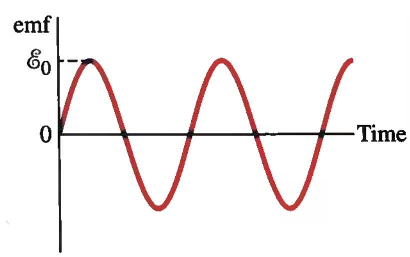


FIGURE 21–14 An ac generator produces an alternating current. The output emf $\mathscr{E} = \mathscr{E}_0\sin\omega t$, where $\mathscr{E}_0 = NAB\omega$ (Eq. 21–5).

† You can, for example, actually run a car generator backward as a motor by connecting its output terminals to a battery.

➥ **PHYSICS APPLIED**

Power plants

FIGURE 21–15 Water-driven generators at the base of Boulder Dam, Nevada.

Over 99 percent of the electricity used in the United States is produced from generators (Fig. 21–15). The frequency f is 60 Hz for generators in the United States and Canada, although 50 Hz is used in many countries. In electric power generating plants, the armature is mounted on a heavy shaft connected to a turbine, which is the modern equivalent of a waterwheel. Water falling over a dam can turn the turbine at a hydroelectric plant. Most of the power generated at present in the United States, however, is done at steam plants, where the burning of fossil fuels (coal, oil, natural gas) boils water to produce high-pressure steam that turns the turbines. Likewise, in nuclear power plants, the nuclear energy released is used to produce steam to turn turbines. Thus, a heat engine (Chapter 15) connected to a generator is the principal means of generating electric power.

The frequency of 60 Hz is maintained very precisely by power companies, and in doing problems, we will assume it is at least as precise as other numbers given.

EXAMPLE 21–6 **An ac generator.** The armature of a 60-Hz ac generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0 \times 10^{-2}\,\mathrm{m}^2$, how many loops must the coil contain if the peak output is to be $\mathscr{E}_0 = 170\,\mathrm{V}$?

SOLUTION From Eq. 21–5, we see that the maximum emf is $\mathscr{E}_0 = NBA\omega$. Since $\omega = 2\pi f = (6.28)(60\,\mathrm{s}^{-1}) = 377\,\mathrm{s}^{-1}$, we have

$$N = \frac{\mathscr{E}_0}{BA\omega} = \frac{170\,\mathrm{V}}{(0.15\,\mathrm{T})(2.0 \times 10^{-2}\,\mathrm{m}^2)(377\,\mathrm{s}^{-1})} = 150\ \text{turns}$$

➥ **PHYSICS APPLIED**

DC generator

A **dc generator** is much like an ac generator, except the slip rings are replaced by split-ring commutators, Fig. 21–16a, just as in a dc motor. The output of such a generator is as shown and can be smoothed out by placing a capacitor in parallel with the output (Section 19–7). More common is the use of many armature windings, as in Fig. 21–16b, which produces a smoother output.

➥ **PHYSICS APPLIED**

Alternators

In the past, automobiles used dc generators. More common now, however, are ac generators or **alternators**, which avoid the problems of wear and electrical arcing (sparks) across the split-ring commutators of dc generators. Alternators differ from the generators discussed above in the following way. In an alternator, current from the battery produces a magnetic field in an electromagnet, called the *rotor*, which is made to rotate by a belt from the engine. Surrounding the rotating rotor are a set of stationary

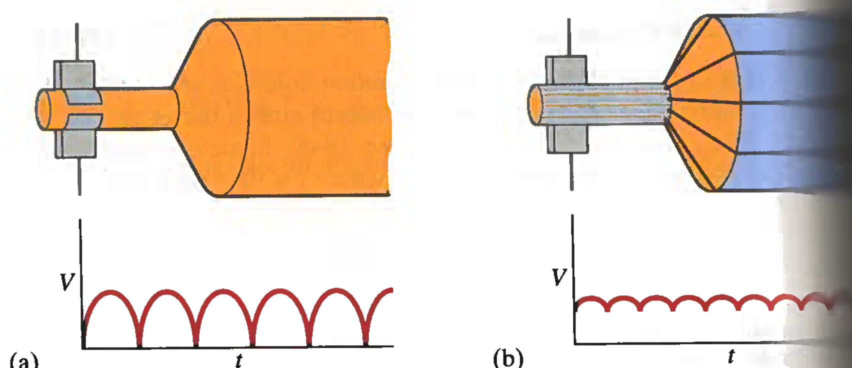


FIGURE 21–16 (a) A dc generator with one set of commutators, and (b) a dc generator with many sets of commutators and windings.

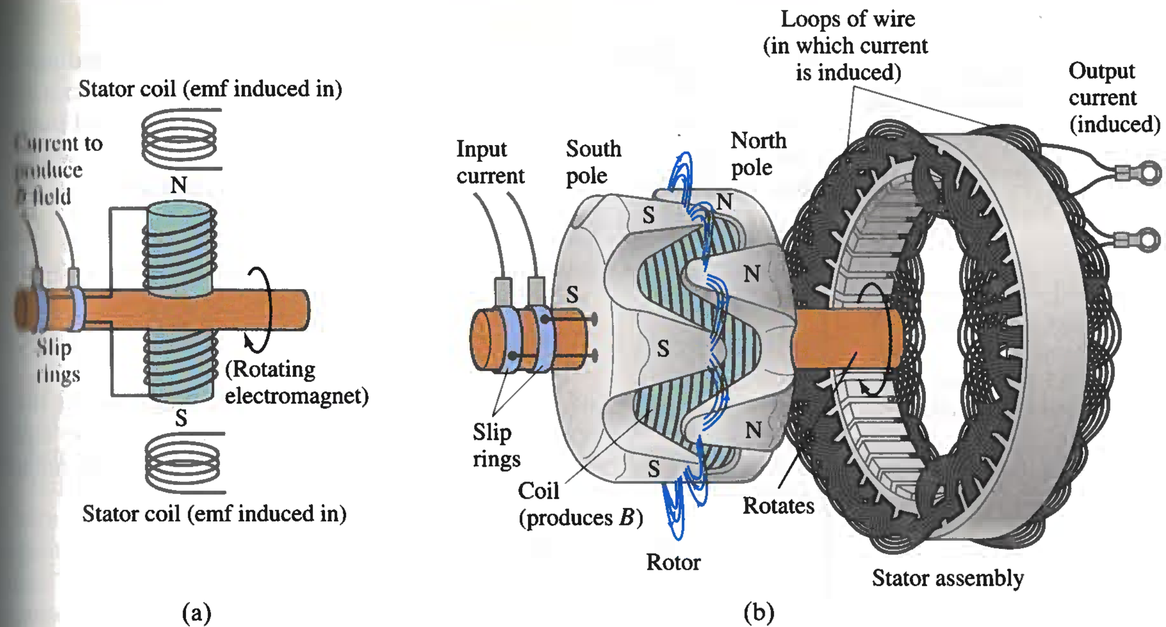


FIGURE 21–17 (a) Schematic (simplified) diagram of an alternator. The input electromagnet current to the rotor is connected through continuous slip rings. Sometimes the rotor electromagnet is replaced by a permanent magnet. (b) Actual shape of an alternator. The rotor is made to turn by a belt from the engine. The current in the wire coil of the rotor produces a magnetic field inside it on its axis that points horizontally from left to right, thus making north and south poles of the plates attached at either end. These end plates are made with triangular fingers that are bent over the coil—hence there are alternating N and S poles quite close to one another, with magnetic field lines between them as shown by the blue lines. As the rotor turns, these field lines pass through the fixed stator coils (shown on the right for clarity, but in operation the rotor rotates within the stator), inducing a current in them, which is the output.

coils called the *stator*, Fig. 21–17. The magnetic field of the rotor passes through the stator coils and, since the rotor is rotating, the field through the fixed stator coils is changing. Hence an alternating current is induced in the stator coils, which is the output. This ac output is changed to dc for charging the battery by the use of semiconductor diodes, which allow current to flow in one direction only (see Section 29–8).

Output emf of alternator

21–6 Counter EMF and Torque; Eddy Currents

A motor turns and produces mechanical energy when a current is made to flow in it. From our description in Section 20–10 of a simple dc motor, you might expect that the armature would accelerate indefinitely due to the torque on it. However, as the armature of the motor turns, the magnetic flux through the coil changes and an emf is generated. This induced emf acts to oppose the motion (Lenz's law) and is called the **back emf** or **counter emf**. The greater the speed of the motor, the greater the counter emf. A motor normally turns and does work on something, but if there were no load, the motor's speed would increase until the counter emf equaled the input voltage. In the normal situation, when there is a mechanical load, the speed of the motor is limited also by the load. The counter emf will then be less than the external voltage. The greater the mechanical load, the slower the motor rotates and the lower is the counter emf ($\mathscr{E} \propto \omega$, Eq. 21–5).

Back emf

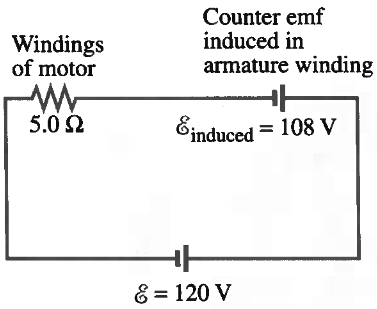


FIGURE 21–18 Circuit of a motor showing induced counter emf.

EXAMPLE 21–7 **Counter emf in a motor.** The armature w
dc motor have a resistance of 5.0 Ω. The motor is connect
line, and when the motor reaches full speed against its nor
counter emf is 108 V. Calculate (*a*) the current into the mot
just starting up, and (*b*) the current when it reaches full spe

SOLUTION (*a*) Initially, the motor is not turning (or turnin
so there is no induced counter emf. Hence, from Ohm's law, the

$$I = \frac{V}{R} = \frac{120\ \text{V}}{5.0\ \Omega} = 24\ \text{A}.$$

Effect of back emf on current

(*b*) At full speed, the counter emf is a source of emf that oppo
terior source. We represent this counter emf as a battery in th
lent circuit shown in Fig. 21–18. In this case, Ohm's law (or K
rule) gives

$$120\ \text{V} - 108\ \text{V} = I\,(5.0\ \Omega).$$

Therefore

$$I = 12\ \text{V}/5.0\ \Omega = 2.4\ \text{A}.$$

This Example illustrates the fact that the current is very hig
motor first starts up. This is why the lights in your house may dim
motor of the refrigerator (or other large motor) starts up. The l
current causes the voltage at the outlets to drop (the house wiring h
ance, so there is some voltage drop across it when large currents

PHYSICS APPLIED

Burning out a motor

CONCEPTUAL EXAMPLE 21–8 **Motor overload.** When using
pliance such as a blender, electric drill, or sewing machine, if th
ance is overloaded or jammed so that the motor slows app
stops while the power is still connected, the device can burn ou
ruined. Explain why this happens.

RESPONSE The motors are designed to run at a certain speed fo
applied voltage and the designer must take the expected counte
account. If the rotation speed is reduced, the counter emf will not b
as expected ($\mathscr{E} \propto \omega$, Eq. 21–5), and the current will increase, and
come large enough that the windings of the motor heat up to the poi
ining the motor.

In a generator, the situation is the reverse of that for a motor.
saw, the mechanical turning of the armature induces an emf in th
which is the output. If the generator is not connected to an exte
cuit, the emf exists at the terminals but no current flows. In this
takes little effort to turn the armature. But if the generator *is* conne
a device that draws current, then a current flows in the coils of th
ture. Because this current-carrying coil is in a magnetic field, there
a torque exerted on it (as in a motor), and this torque opposes the
(use the right-hand rule for the force on a wire, in Fig. 21–12).
called a **counter torque**. The greater the electrical load—that is, th
current that is drawn—the greater will be the counter torque. Hen
external applied torque will have to be greater to keep the generator

Counter torque

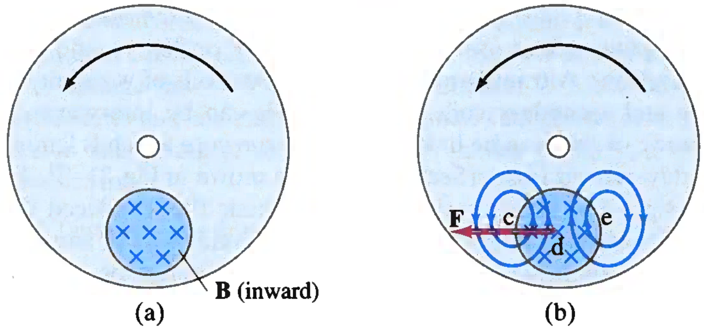


FIGURE 21–19 Production of eddy currents in a rotating wheel.

This of course makes sense from the conservation-of-energy principle. More mechanical-energy input is needed to produce more electrical-energy output.

Induced currents are not always confined to well-defined paths such as in wires. Consider, for example, the rotating metal wheel in Fig. 21–19a. A magnetic field is applied to a limited area as shown and points into the paper. The section of wheel in the magnetic field has an emf induced in it because the conductor is moving (carrying electrons with it). The flow of (conventional) current is upward in the region of the magnetic field (Fig. 21–19b), and the current follows a downward return path outside that region. Why? According to Lenz's law, the induced currents oppose the change that causes them. Consider the part of the wheel labeled c in Fig. 21–19b, where the magnetic field is zero but is just about to enter a region where **B** points into the page. To oppose this change, the induced current is counterclockwise to produce a field pointing out of the page (right-hand rule). Similarly region d is about to move to e, where **B** is zero; hence the current is clockwise to produce an inward field opposed to this change. These currents are referred to as **eddy currents** and can be present in any conductor that is moving across a magnetic field or through which the magnetic flux is changing. In Fig. 21–19, the magnetic field exerts a force **F** on the induced currents that opposes (use the right-hand rule) the rotational motion. Eddy currents can be used in this way as a smooth braking device on, say, a rapid-transit car. In order to stop the car, an electromagnet can be turned on that applies its field either to the wheels or to the moving steel rail below. Eddy currents can also be used to dampen (reduce) the oscillation of a vibrating system. A common example is in a galvanometer, where induced eddy currents keep the needle from overshooting or oscillating violently. Eddy currents, however, can be a problem. For example, eddy currents induced in the armature of a motor or generator produce heat ($P = I\mathscr{E}$) and waste energy. To reduce the eddy currents, the armatures are *laminated*; that is, they are made of very thin sheets of iron that are well insulated from one another. (See Fig. 21–21 in the next Section.) Thus the total path length of the eddy currents is confined to each slab, which increases the total resistance; hence the current is less and there is less wasted energy.

Eddy currents

➡ **PHYSICS APPLIED**

Brakes and damping

21–7 Transformers; Transmission of Power

A transformer is a device for increasing or decreasing an ac voltage. Transformers are found everywhere: in TV sets to give the high voltage needed for the picture tube, in converters for plugging in a portable "Walkman,"

➡ **PHYSICS APPLIED**

Transformers

FIGURE 21–20 Repairing a transformer on a utility pole.

on utility poles (Fig. 21–20) to reduce the high voltage from the company to that usable in houses (110 V or 220 V), and in many other applications. A **transformer** consists of two coils of wire known as the **primary** and **secondary** coils. The two coils can be interwoven (with insulated wire); or they can be linked by a soft iron core which is laminated to minimize eddy-current losses (Section 21–6), as shown in Fig. 21–21. Transformers are designed so that (nearly) all the magnetic flux produced by the current in the primary also passes through the secondary coil, and we assume this is true in what follows. We also assume that energy losses in the resistance of the coils and hysteresis in the iron can be ignored—a good approximation for real transformers, which are often better than 99 percent efficient.

When an ac voltage is applied to the primary, the changing magnetic field it produces will induce an ac voltage of the same frequency in the secondary. However, the voltage will be different according to the number of loops in each coil. From Faraday's law, the voltage or emf induced in the secondary is

$$V_S = N_S \frac{\Delta\Phi_B}{\Delta t},$$

where N_S is the number of turns in the secondary coil, and $\Delta\Phi_B/\Delta t$ is the rate at which the magnetic flux changes. The input primary voltage, V_P, is also related to the rate at which the flux changes

$$V_P = N_P \frac{\Delta\Phi_B}{\Delta t},$$

where N_P is the number of turns in the primary coil.[†] We divide these two equations, assuming little or no flux is lost, to find

Transformer equation

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}.$$

This *transformer equation* tells how the secondary (output) voltage is related to the primary (input) voltage; V_S and V_P in Eq. 21–6 can be the rms values for both, or peak values for both.

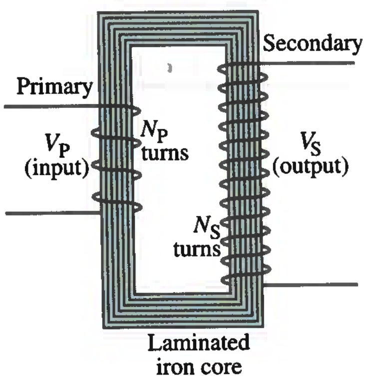


FIGURE 21–21 Step-up transformer ($N_P = 4$, $N_S = 12$).

If N_S is greater than N_P, we have a **step-up transformer**. The secondary voltage is greater than the primary voltage. For example, if the secondary has twice as many turns as the primary, then the secondary voltage will be twice that of the primary. If N_S is less than N_P, we have a **step-down transformer**.

Although ac voltage can be increased (or decreased) with a transformer, we don't get something for nothing. Energy conservation tells us that the power output can be no greater than the power input. A well-designed transformer can be greater than 99 percent efficient, so little energy is lost to heat. The power input thus essentially equals the power output. Since power $P = VI$ (Eq. 18–5), we have

$$V_P I_P = V_S I_S,$$

or

Transformer equation II

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}.$$

[†]This follows because the changing flux produces a counter emf, $N_P\,\Delta\Phi_B/\Delta t$ in the primary that exactly balances the applied voltage V_P if the resistance of the primary can be ignored (Kirchhoff's rules).

EXAMPLE 21–9 Portable radio transformer. A transformer for home use of a portable radio reduces 120-V ac to 9.0-V ac. (Such a device also contains diodes to change the 9.0-V ac to dc. See Chapter 29.) The secondary contains 30 turns and the radio draws 400 mA. Calculate: (*a*) the number of turns in the primary; (*b*) the current in the primary; and (*c*) the power transformed.

SOLUTION (*a*) This is a step-down transformer, and from Eq. 21–6 we have

$$N_P = N_S \frac{V_P}{V_S} = \frac{(30)(120\ \text{V})}{(9.0\ \text{V})} = 400\ \text{turns}.$$

(*b*) From Eq. 21–7:

$$I_P = I_S \frac{N_S}{N_P} = (0.40\ \text{A})\left(\frac{30}{400}\right) = 0.030\ \text{A}.$$

(*c*) The power transformed is

$$P = I_S V_S = (9.0\ \text{V})(0.40\ \text{A}) = 3.6\ \text{W},$$

which is, assuming 100 percent efficiency, the same as the power in the primary, $P = (120\ \text{V})(0.030\ \text{A}) = 3.6\ \text{W}$.

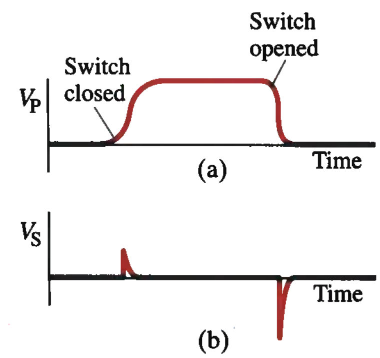


FIGURE 21–22 A dc voltage turned on and off as shown in (a) produces voltage pulses in the secondary (b). Voltage scales in (a) and (b) are not necessarily the same.

It is important to recognize that a transformer operates only on ac. A dc current in the primary does not produce a changing flux and therefore induces no emf in the secondary. However, if a dc voltage is applied to the primary through a switch, at the instant the switch is opened or closed there will be an induced current in the secondary. For example, if the dc is turned on and off as shown in Fig. 21–22a, the voltage induced in the secondary is as shown in Fig. 21–22b. Notice that the secondary voltage drops to zero when the dc voltage is steady.

Transformers play an important role in the transmission of electricity. Power plants are often situated some distance from metropolitan areas. Hydroelectric plants are located at a dam site and nuclear plants need much cooling water. Fossil-fuel plants too are often situated far from a city because of lack of availability of land or to avoid contributing to air pollution. In any case, electricity must often be transmitted over long distances (Fig. 21–23). There is always some power loss in the transmission lines, and this loss can be minimized if the power is transmitted at high voltage, using transformers, as the following Example shows.

PHYSICS APPLIED

Transformers help power transmission

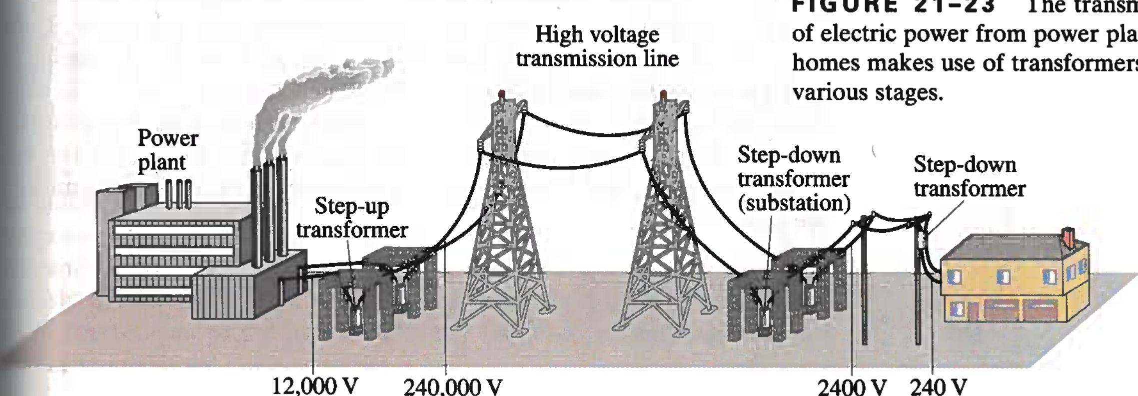


FIGURE 21–23 The transmission of electric power from power plants to homes makes use of transformers at various stages.

EXAMPLE 21–10 **Transmission lines.** An average of 120 kW of ele-
tric power is sent to a small town from a power plant 10 km aw
transmission lines have a total resistance of 0.40 Ω. Calculate the
loss if the power is transmitted at (*a*) 240 V and (*b*) 24,000 V

SOLUTION For each case we determine the current I in the li
then find the power loss from $P = I^2R$. (*a*) If 120 kW is sent at 2
total current will be

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\ \mathrm{W}}{2.4 \times 10^2\ \mathrm{V}} = 500\ \mathrm{A}.$$

The power loss in the lines, P_L, is then

$$P_L = I^2R = (500\ \mathrm{A})^2(0.40\ \Omega) = 100\ \mathrm{kW}.$$

Thus, over 80 percent of all the power would be wasted as heat [illegible]
power lines!
(*b*) When $V = 24{,}000$ V,

$$I = \frac{P}{V} = \frac{1.2 \times 10^5\ \mathrm{W}}{2.4 \times 10^4\ \mathrm{V}} = 5.0\ \mathrm{A}.$$

The power loss is then

$$P_L = I^2R = (5.0\ \mathrm{A})^2(0.40\ \Omega) = 10\ \mathrm{W},$$

which is less than $\frac{1}{100}$ of 1 percent.

It should be clear that the greater the voltage, the less the current and
the less power is wasted in the transmission lines. It is for this reason
power is usually transmitted at very high voltages, as high as 700 kV.

Power is generated at somewhat lower voltages than this, and the
voltage in homes and factories is also much lower. The great advantage
of ac, and a major reason it is in nearly universal use, is that the vo
can easily be stepped up and down by a transformer. The output vo
of an electric generating plant is stepped up prior to transmission. U
arrival in a city, it is stepped down in stages at electric substations pr
to distribution. The voltage in lines along city streets is typically [illegible]
and is stepped down to 240 V or 120 V for home use by transfo
(Fig. 21–20).

Direct current transmission has gained in popularity recently. Al
though changing voltage with dc is more difficult and expensive, it of
some advantages over ac. A few of these are as follows. Alternating cu
rent produces alternating magnetic fields which induce current in ne
wires and so reduce transmitted power; this is absent in dc. It is possi
transmit dc at a higher average voltage than ac since for dc, the rms
equals the peak; and breakdown of insulation or of air is determined
the peak voltage.

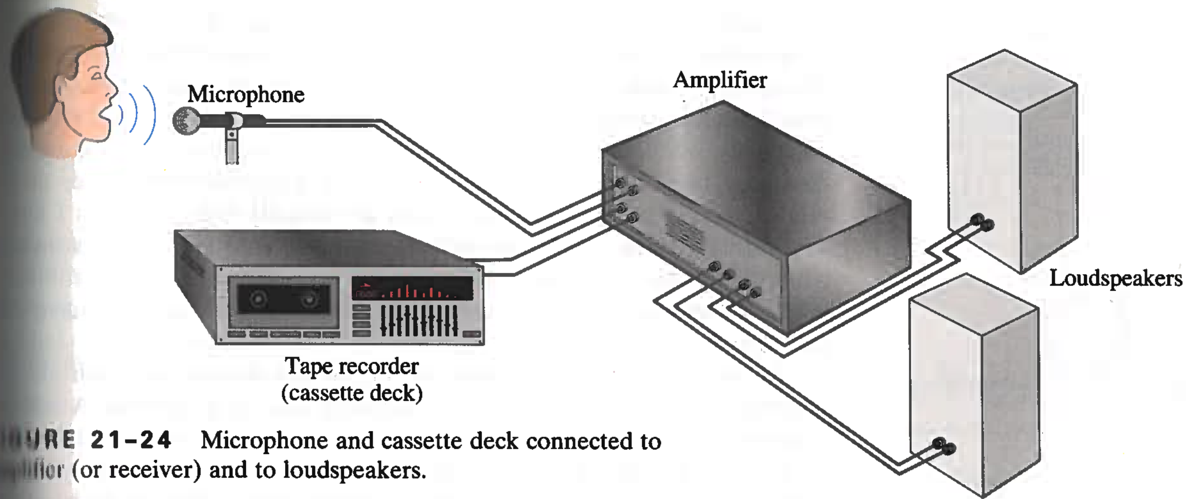


FIGURE 21–24 Microphone and cassette deck connected to amplifier (or receiver) and to loudspeakers.

21–8 Applications of Induction: Sound Systems, Computer Memory, and the Seismograph

Figure 21–24 shows several components of a sound system. Two of these components, the microphone and the tape recorder, make use of the principle of electromagnetic induction. [The output of each goes to an amplifier that amplifies (increases) the signal and sends it to a loudspeaker to be heard; alternately, the microphone signal could go directly to the tape recorder for recording.]

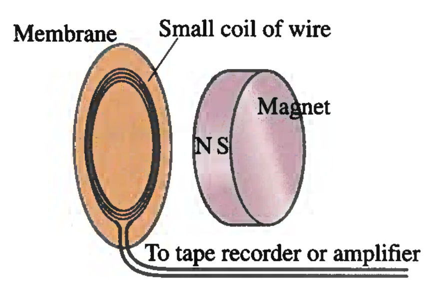
➥ PHYSICS APPLIED
Microphones

There are various types of *microphones*, and many operate on the principle of induction. In one form, a microphone is just the inverse of a loudspeaker (Section 20–10). A small coil connected to a membrane is suspended close to a small permanent magnet, as shown in Fig. 21–25. The coil moves in the magnetic field when sound waves strike the membrane. The frequency of the induced emf will be just that of the impinging sound waves, and this emf is the "signal" that can be amplified and sent to loudspeakers, or sent to a tape recorder to be recorded on tape. In a "ribbon" microphone, a thin metal ribbon is suspended between the poles of a permanent magnet. The ribbon vibrates in response to sound waves, and the emf induced in the ribbon is proportional to its velocity.

➥ PHYSICS APPLIED
Tape recording

Tape recording and tape playback is done by *tape heads* inside a tape recorder (or cassette deck). Recording tape for use in audio and video tape recorders contains a thin layer of magnetic oxide on a thin plastic tape. During recording, the audio or video signal voltage is sent to the recording head, which acts as a tiny electromagnet (Fig. 21–26) that magnetizes the tiny section of tape passing over the narrow gap in the head at each instant. In playback, the changing magnetism of the moving tape at the gap causes corresponding changes in the magnetic field within the soft iron head, which in turn induces an emf in the coil (Faraday's law). This induced emf is the output signal that can be amplified and sent to a loudspeaker (or, in the case of a video signal, to the picture tube). In audio and video recorders, the signals are usually *analog*—they vary continuously in amplitude over time. The variation in degree of magnetization of the tape at any point reflects the variation in amplitude of the audio or video signal.

FIGURE 21–25 Diagram of microphone that works by induction.



PHYSICS APPLIED

Computers and digital information

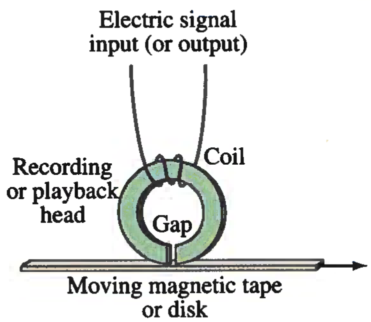


FIGURE 21–26 Recording and/or playback head for tape or disk. In recording (or "writing"), the electric input signal to the head, which acts as an electromagnet, magnetizes the passing tape or disk. In playback (or "reading"), the changing magnetic field of the passing tape or disk induces a changing magnetic field in the head, which in turn induces in the coil an emf that is the output signal.

Digital information, such as used on computer disks (flopp... hard disks) or on magnetic computer tape and some types of di... recorders, is read and written using heads that are basically th... just described (Fig. 21–26). The essential difference is in the ... which are not analog, but are digital, and in particular binary, ... that only two values are possible for each predetermined spa... tape or disk. The two possible values are usually referred to as 1 ... The signal voltage does not vary continuously but rather takes ... two values, say +5 volts and 0 volts, corresponding to the 1 or 0 ... information is carried as a series of "bits," each of which can ha... one of two values, 1 or 0.

In another field, geophysics, an important device based on ... magnetic induction is one type of *seismograph* or *geophone*. A ... graph is placed in direct contact with the Earth and converts the ... the Earth—whether due to an earthquake or to an explosion (su... mineral prospecting or for detecting a bomb test)—into an elec... nal. A seismograph contains a magnet and a coil of wire, one of ... fixed rigidly to the case, which moves as the Earth does where it ... ed. The other element is inertial and is suspended from the ... spring. In the type shown in Fig. 21–27, the coil moves with the Ea... the relative motion of the magnet and coil produces an induced emf ... coil, which is the output of the device. In many geophones, the coil ... tial and the magnet moves with the Earth.

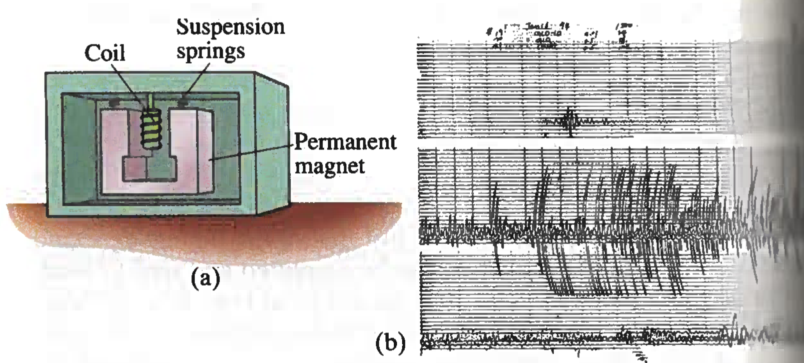


FIGURE 21–27 (a) A seismograph or geophone. (b) A seismograph reading (Northridge, California earthquake, January 17, 1994).

* 21–9 Inductance

Mutual inductance. If two coils of wire are placed near one ... as in Fig. 21–28, a changing current in one will induce an emf in the ... According to Faraday's law, the emf $\mathscr{E}_2$ induced in coil 2 is proporti... the rate of change of flux passing through it. Since the flux is propor... to the current flowing in coil 1, $\mathscr{E}_2$ must be proportional to the ra... change of the current in coil 1, $\Delta I_1/\Delta t$. Thus we can write

Mutual inductance

$$\mathscr{E}_2 = -M\frac{\Delta I_1}{\Delta t},$$

where the constant of proportionality, M, is called the **mutual induct...** (The minus sign is because of Lenz's law.) Mutual inductance has ... $\mathrm{V{\cdot}s/A} = \Omega{\cdot}\mathrm{s}$, which is called the **henry** (H), after Joseph Henry: 1 H ... The value of M depends on whether iron is present or not, and on the ...

of the coil configuration: that is, on the size of the coils, on the number of turns, and on their separation. If we look at the inverse situation—a changing current in coil 2 inducing an emf in coil 1—the proportionality constant, M, turns out to have the same value,

$$\mathscr{E}_1 = -M\frac{\Delta I_2}{\Delta t}. \qquad \textbf{(21–8b)}$$

A transformer is an example of mutual inductance in which the coupling is maximized so that nearly all flux lines pass through both coils. However, mutual inductance has other uses as well. For example, some pacemakers, which are used to maintain blood flow in heart patients (Section 19–8), are powered externally. Power in an external coil is transmitted via mutual inductance to a second coil in the pacemaker at the heart. This has the advantage over battery-powered pacemakers in that surgery is not needed to replace a battery when it wears out.

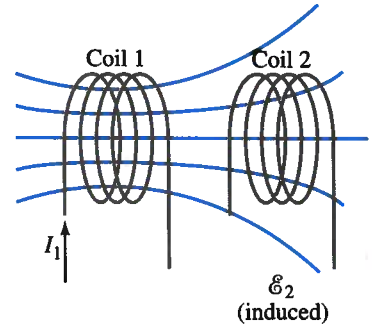


FIGURE 21–28 A changing current in one coil will induce a current in the second coil.

Self-inductance. The concept of inductance applies also to an isolated single coil. When a changing current passes through a coil or solenoid, a changing magnetic flux is produced inside the coil, and this in turn induces an emf. This induced emf opposes the change in flux (Lenz's law); it is much like the back emf generated in a motor. For example, if the current through the coil is increasing, the increasing magnetic flux induces an emf that opposes the original current and tends to retard its increase. If the current is decreasing in the coil, the decreasing flux induces an emf in the same direction as the current, tending to maintain the original current. In either case, the induced emf $\mathscr{E}$ is proportional to the rate of change in current (and is in the direction opposed to the change):

Self-inductance (induced emf for an inductor)

$$\mathscr{E} = -L\frac{\Delta I}{\Delta t}. \qquad \textbf{(21–9)}$$

The constant of proportionality L is called the **self-inductance**, or simply the **inductance** of the coil. It, too, is measured in henrys. The magnitude of L depends on the geometry (size and shape) and on whether an iron core is present or not.

An ac circuit always contains some inductance, but often it is quite small unless the circuit contains a coil of many turns. A coil that has significant self-inductance L is called an **inductor** or a **choke coil**. It is shown on circuit diagrams by the symbol

Inductors

-ꝏꝏ- . [inductor symbol]

It can serve a useful purpose in certain circuits. Often, inductance is to be avoided in a circuit. Precision resistors are normally wire-wound and thus would have inductance as well as resistance. The inductance can be minimized by winding the insulated wire back on itself so that the current going in the two directions cancels and little magnetic flux is produced; this is called a "noninductive winding."

If an inductor has negligible resistance, it is the inductance, or the back emf, that controls the current. If a source of alternating voltage is applied to the coil, this applied voltage will just be balanced by the induced emf of the coil as given by Eq. 21–9. Thus we can see from Eq. 21–9 that, for a given $\mathscr{E}$, if the inductance L is large, the change in the current—and therefore the current itself—will be small. The greater the inductance, the less the current. An inductance thus acts something like a resistance to impede the flow of alternating current. We use the term **impedance** for th... of an inductor. We shall discuss impedance more fully in Section... 21–15, and we shall see that it depends not only on L, but also on ... quency. Here we mention one example of its importance. The resis... the primary in a transformer is usually quite small, perhaps less than ... resistance alone limited the current, tremendous currents would ... a high voltage was applied. Indeed, a dc voltage applied to a t... can burn it out. It is the impedance of the coil to an alternating cur... its "back" emf) that limits the current to a reasonable value.

DC can burn out a transformer

EXAMPLE 21–11 **Solenoid inductance.** (*a*) Determine a formu... the self-inductance L of a solenoid (a long coil—see Fig. 20–... taining N turns of wire in its length l and whose cross-sectional ar... (*b*) Calculate the value of L if $N = 100$, $l = 5.0\,\text{cm}$, $A = 0.30\,...$ the solenoid is air-filled. (*c*) Calculate L if the solenoid has an ir... with $\mu = 4000\,\mu_0$.

Calculating self-inductance of a coil

SOLUTION (*a*) According to Eq. 20–8, the magnetic field ... solenoid is $B = \mu_0 nI$, where $n = N/l$. From Eqs. 21–2 and 21... have $\mathscr{E} = -N(\Delta\Phi_B/\Delta t) = -L(\Delta I/\Delta t)$. Thus, $L = N(\Delta\Phi_B/\Delta I)$... $\Phi_B = BA = \mu_0 NIA/l$, then any change in I causes a change in ... $\Delta\Phi_B = \mu_0 NA\,\Delta I/l$. Thus

$$L = N\frac{\Delta\Phi_B}{\Delta I} = \frac{\mu_0 N^2 A}{l}.$$

(*b*) Since $\mu_0 = 4\pi \times 10^{-7}\,\text{T·m/A}$,

$$L = \frac{(4\pi \times 10^{-7}\,\text{T·m/A})(100)^2(3.0 \times 10^{-5}\,\text{m}^2)}{(5.0 \times 10^{-2}\,\text{m})} = 7.5\,\mu\text{H}$$

(*c*) Here we replace μ_0 by $\mu = 4000\mu_0$, so L will be 4000 times la... $L = 0.030\,\text{H} = 30\,\text{mH}$.

* 21–10 Energy Stored in a Magnetic Field

In Section 17–9 we saw that the energy stored in a capacitor is equal ... $\frac{1}{2}CV^2$. By using a similar argument, we can show that the energy U ... in an inductance L, carrying a current I, is

$$U = \text{energy} = \tfrac{1}{2}LI^2.$$

Just as the energy stored in a capacitor can be considered to reside in the electric field between its plates, so the energy in an inductor can be con... sidered to be stored in its magnetic field.

To write the energy in terms of the magnetic field, let us use the re... of Example 21–11 that the inductance of a solenoid is $L = \mu_0 N^2 A/l$... the magnetic field B in a solenoid is related to the current I (see Eq... by $B = \mu_0 NI/l$. Thus, $I = Bl/\mu_0 N$, and

$$U = \text{energy} = \tfrac{1}{2}LI^2 = \tfrac{1}{2}\left(\frac{\mu_0 N^2 A}{l}\right)\left(\frac{Bl}{\mu_0 N}\right)^2$$

$$= \tfrac{1}{2}\frac{B^2}{\mu_0}Al.$$

We can think of this energy as residing in the volume enclosed by th...

windings, which is Al. Then the energy per unit volume, or **energy density**, is

$$u = \text{energy density} = \frac{1}{2}\frac{B^2}{\mu_0}. \tag{21–10}$$

Energy density in magnetic field

This formula, which was derived for the special case of a solenoid, can be shown to be valid for any region of space where a magnetic field exists. If ferromagnetic material is present, μ_0 is replaced by μ. This equation is analogous to that for an electric field, $\frac{1}{2}\epsilon_0 E^2$, Section 17–9.

21–11 *LR* Circuit

Any inductor will have some resistance. We represent this situation by drawing the inductance L and the resistance R separately, as in Fig. 21–29. The resistance R could also include a separate resistor connected in series. Now we ask, what happens when a dc source is connected in series to such an LR circuit? At the instant the switch connecting the battery is closed, the current starts to flow. It is, of course, opposed by the induced emf in the inductor. However, as soon as current starts to flow, there is a voltage drop across the resistance. Hence, the voltage drop across the inductance is reduced and there is then less impedance to the current flow from the inductance. The current thus rises gradually, as shown in Fig. 21–30a, and approaches the steady value $I_{max} = V/R$ when all the voltage drop is across the resistance. The actual shape of the curve for I as a function of time is

$$I = \left(\frac{V}{R}\right)(1 - e^{-t/\tau}),$$

where e is the exponential function (see Section 19–7) and $\tau = L/R$ is called the **time constant** of the circuit. When $t = \tau$, then we have $(1 - e^{-1}) = 0.63$ so we see that τ is the time required for the current to reach $0.63I_{max}$.

If the battery is suddenly removed from the circuit (dashed line in Fig. 21–29), the current drops off as shown in Fig. 21–30b. This is an exponential decay curve given by $I = I_{max}e^{-t/\tau}$. The time constant τ is the time for the current to drop to 37 percent of the original value, and again equals L/R.

These graphs show that there is always some "reaction time" when an electromagnet, for example, is turned on or off. We also see that an LR circuit has properties similar to an RC circuit (Section 19–7). Unlike the capacitor case, however, the time constant here is *inversely* proportional to R.

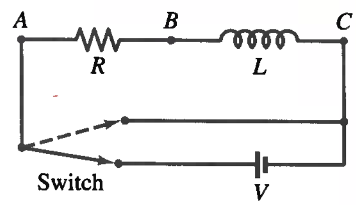


FIGURE 21–29 *LR* circuit.

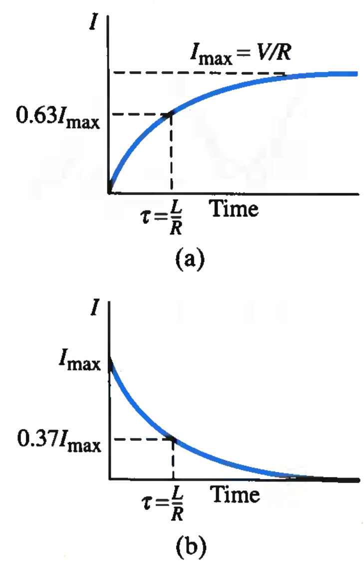


FIGURE 21–30 (a) Growth of current in an *LR* circuit when connected to a battery; (b) decay of current when the *LR* circuit is shorted out (battery is out of the circuit).

EXAMPLE 21–12 Solenoid time constant. A solenoid has an inductance of 87.5 mH and a resistance of 0.250 Ω. Find (*a*) the time constant for this circuit, and (*b*) how long it would take for the current to go from zero to half its final (maximum) value when connected to a battery of voltage V.

SOLUTION (*a*) By definition, $\tau = L/R$, so

$$\tau = \frac{L}{R} = \frac{87.5 \times 10^{-3}\ \text{H}}{0.250\ \Omega} = 0.350\ \text{s}.$$

(*b*) $I = (V/R)(1 - e^{-t/\tau})$ and we wish to find t such that I ... where $V/R = I_{max}$. So

$$\tfrac{1}{2}\frac{V}{R} = \frac{V}{R}(1 - e^{-t/\tau})$$

$$\tfrac{1}{2} = 1 - e^{-t/\tau}$$

$$e^{-t/\tau} = \tfrac{1}{2}$$

$$e^{t/\tau} = 2.$$

The inverse operation to the exponential e is the natural log, ln ...

$$\ln(e^{t/\tau}) = \frac{t}{\tau} = \ln 2,$$

so

$$t = \tau \ln 2 = (0.350\text{ s})(0.693) = 0.243\text{ s}.$$

* 21–12 AC Circuits and Impedance

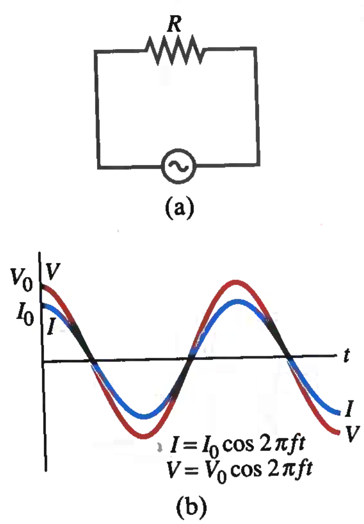


FIGURE 21–31 Resistor connected to an ac source. Current is in phase with the voltage.

We have previously discussed circuits that contain combinations of ... tor, capacitor, and inductor, but only when they are connected ... source of emf or to no source (as in the discharge of a capacitor ... circuit). Now we discuss these circuit elements when they are conn... a source of alternating emf.

First we examine, one at a time, how a resistor, a capacitor, and ... ductor behave when connected to a source of alternating emf, re... by the symbol •⊙•, which produces a sinusoidal voltage of freq... We assume in each case that the emf gives rise to a current

$$I = I_0 \cos 2\pi ft,$$

where t is time and I_0 is the peak current. We must remember (Section ... that $V_{rms} = V_0/\sqrt{2}$ and $I_{rms} = I_0/\sqrt{2}$ (Eq. 18–8).

Resistor: current and voltage are in phase

Resistor. When an ac source is connected to a resistor as in Fig. ... the current increases and decreases with the alternating emf acc... Ohm's law, $I = V/R$. Figure 21–31b shows the voltage (red curve) and ... current (blue curve). Because the current is zero when the voltage is ... and the current reaches a peak when the voltage does, we say that the ... rent and voltage are **in phase**. Energy is transformed into heat ... tion 18–8), at an average rate $\overline{P} = \overline{IV} = I^2_{rms}R = V^2_{rms}/R$.

Inductor. In Fig. 21–32a an inductor of inductance L, represented by the symbol –ᴖᴖᴖᴖ–, is connected to the ac source. We ignore any resist... it might have (it is usually small). The voltage applied to the induct... be equal to the "back" emf generated in the inductor by the changing ... rent as given by Eq. 21–9. This is because the sum of the emfs around ... closed circuit must be zero, as Kirchhoff's rule tells us. Thus

$$V - L\frac{\Delta I}{\Delta t} = 0 \qquad \text{or} \qquad V = L\frac{\Delta I}{\Delta t},$$

where V is the sinusoidally varying voltage of the source and $L\,\Delta I/\Delta t$ is the voltage induced in the inductor. According to this equation, ...

changing most rapidly when V has its maximum value, $V = V_0$. And I will be decreasing most rapidly when $V = -V_0$. These two instants correspond to points d and b on the graph of voltage versus time in Fig. 21–32b. At points a and c, $V = 0$. The equation above tells us that $\Delta I/\Delta t = 0$ at these points, so I is not changing and these points correspond to the maximum and minimum values of the current I (the slope of I versus t is zero), as confirmed by points a and c on the graph. By going point by point in this manner, the curve of I versus t as compared to that for V versus t can be constructed, and they are shown by the blue and red lines, respectively, in Fig. 21–32b. Notice that the current reaches its peaks (and troughs) one quarter of a cycle after the voltage does. We say that

in an inductor, the current lags the voltage by 90°.

Inductor: current lags voltage

Remember that 360° corresponds to a full cycle, so 90° is a quarter cycle. Alternatively, we can say that the voltage leads the current by 90°. Since we originally chose $I = I_0 \cos 2\pi ft$, we see from the graph that V must vary as $V = -V_0 \sin 2\pi ft$.

Because the current and voltage are out of phase by 90°, no energy is transformed in an inductor on the average; and no energy is dissipated as thermal energy. This can be seen as follows from Fig. 21–32b. From point c to d the voltage is increasing from zero to its maximum. The current, however, is in the opposite direction to the voltage and is approaching zero. The average power over this interval, VI, is negative. From d to e, however, both V and I are positive so VI is positive; this contribution just balances the negative contribution of the previous quarter cycle. Similar considerations apply to the rest of the cycle. Thus, the average power transformed over one or many cycles is zero. We can see that energy from the source passes into the magnetic field of the inductor, where it is stored temporarily. Then the field decreases and the energy is transferred back to the source. None is dissipated in this process. Compare this to a resistor where the current is always in the same direction as the voltage and energy is transferred out of the source and never back into it. (The product VI is never negative.) This energy is not stored in the resistor, but is transformed to thermal energy.

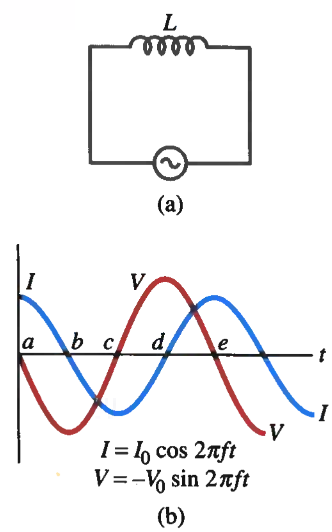


FIGURE 21–32 Inductor connected to an ac source. Current lags voltage by a quarter cycle, or 90°.

As mentioned in Section 21–9, the back emf of an inductor impedes the flow of an ac current. Indeed, it is found that the magnitude of the current in an inductor is directly proportional to the applied ac voltage at a given frequency. We can therefore write an equation for a pure inductor (no resistance) that is the equivalent of Ohm's law:

$$V = IX_L, \qquad \textbf{(21–11a)}$$

where X_L is called the **inductive reactance**, or **impedance**, of the inductor. Normally, we use the term "reactance" to refer solely to the inductive properties. We then reserve the term "impedance" to include the total "impeding" qualities of the coil—its inductance plus any resistance it may have (more on this in the next section). In the absence of resistance, the impedance is the same as the reactance.

Reactance and impedance

The quantities V and I in Eq. 21–11a can refer either to rms for both, or to peak values for both. Note, however, that although this equation can relate the peak values, the peak current and voltage are not reached at the same time; so Eq. 21–11a is *not* valid at a particular instant, as is the case for Ohm's law.

From the fact that $V = L\,\Delta I/\Delta t$, we see that the larger L is, the [illegible] be the change in current ΔI during the time Δt. Hence I itself will [illegible] er at any instant for a given frequency, and we expect $X_L \propto L$. [illegible] tance X_L also depends on the frequency. The greater the frequency [illegible] more rapidly the magnetic flux changes in the inductor. If the [illegible] by this field is to remain equal to the source emf, as it must, the [illegible] of the current must then be less. Hence the greater the frequency [illegible] greater is the reactance X_L, and we expect $X_L \propto fL$. This is also [illegible] with the fact that if the frequency f is zero (so the current is dc), there [illegible] back emf and no impedance to the flow of charge. Careful calculation [illegible] calculus), as well as experiment, shows that the constant of proportionality [illegible] 2π. Thus

$$X_L = 2\pi f L.$$

EXAMPLE 21–13 **Reactance of a coil.** A coil has a resistance [illegible] and an inductance of 0.300 H. Determine the current in the coil if (a) [illegible] is applied to it; (b) 120 V ac (rms) at 60.0 Hz is applied.

SOLUTION (a) There is no inductive impedance ($X_L = 0$ since [illegible] so we apply Ohm's law for the resistance:

$$I = \frac{V}{R} = \frac{120\ \text{V}}{1.00\ \Omega} = 120\ \text{A}.$$

(b) The inductive reactance in this case is

$$X_L = 2\pi f L = (6.28)(60.0\ \text{s}^{-1})(0.300\ \text{H}) = 113\ \Omega.$$

In comparison to this, the resistance can be ignored. Thus,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_L} = \frac{120\ \text{V}}{113\ \Omega} = 1.06\ \text{A}.$$

[It might be tempting to say that the total impedance is 113 Ω + 1 Ω [illegible] This might imply that about 1 percent of the voltage drop is across the [illegible] tor, or about 1 V; and that across the inductance is 119 V. Although the [illegible] across the resistor is correct, the other statements are not true because [illegible] alteration in phase in an inductor. This will be discussed in the next [illegible]

Capacitor. When a capacitor is connected to a battery, the [illegible] plates quickly acquire equal and opposite charges; but no steady [illegible] flows in the circuit. A capacitor prevents the flow of a dc current [illegible] as the capacitor is not "leaky," in which case a leakage current would [illegible] across the gap). However, if a capacitor is connected to an alternating [illegible] source of voltage, as in Fig. 21–33a, an alternating current will flow [illegible] uously. This can happen because when the ac voltage is first turned [illegible] charge begins to flow so that one plate acquires a negative charge [illegible] other a positive charge. But when the voltage reverses itself, the [illegible] flow in the opposite direction. Thus, for an alternating applied voltage [illegible] ac current is present in the circuit continuously.

Let us look at this process in more detail. First, we recall that the applied voltage must equal the voltage across the capacitor: $V = Q/C$, where C is the capacitance and Q the charge on the plates. Thus the charge Q on the plates follows the voltage; when the voltage is zero, the charge is zero; when the voltage is a maximum, the charge is a maximum. But what about the current I? At point a in Fig. 21–33b, when the voltage starts increasing, the charge on the plates is zero. Thus charge flows readily toward the plates and the current is large. As the voltage approaches its maximum of V_0 (point b), the charge that has accumulated on the plates tends to prevent more charge from flowing, so the current drops to zero at point b. The charge that has accumulated now starts to flow off the plates and the magnitude of current again increases (blue curve), but in the opposite direction; it reaches a maximum (negatively) when the voltage is at point c. Thus the current follows the blue curve in Fig. 21–33b. Like an inductor, the voltage and current are out of phase by 90°. But for a capacitor, the current reaches its peaks $\frac{1}{4}$ cycle before the voltage does, so we say that the

current leads the voltage by 90°.

(Or the voltage lags the current by 90°.) This is the opposite of what happens for an inductor. Again we have chosen $I = I_0 \cos 2\pi ft$ and we see from the graph that $V = V_0 \sin 2\pi ft$.

Because the current and voltage are out of phase, the average power dissipated is zero, just as for an inductor. Energy from the source is fed to the capacitor, where it is stored in the electric field between its plates. As the field decreases, the energy returns to the source. Thus, in an ac circuit, *only a resistance will dissipate energy* to thermal energy.

A relationship between the applied voltage and the current in a capacitor can be written just as for an inductance:

$$V = IX_C, \tag{21–12a}$$

where X_C is the **capacitive reactance** (or **impedance**) of the capacitor. This equation relates the rms or peak values for the voltage and current but is not valid at a particular instant because I and V are out of phase. X_C depends on both the capacitance C and the frequency f. The larger the capacitance, the more charge it can handle, so the less it will retard the flow of an alternating current. Hence, we expect X_C will be inversely proportional to C. It is also inversely proportional to the frequency f, since, when the frequency is higher, there is less time per cycle for the charge to build up on the plates and impede the flow. Again there is a factor of 2π, and

$$X_C = \frac{1}{2\pi fC}. \tag{21–12b}$$

Note that for dc conditions, $f = 0$ and X_C becomes infinite. This is as it should be, since a capacitor does not pass dc current.

It is interesting to note that the reactance of an inductor increases with frequency, but that of a capacitor decreases with frequency.

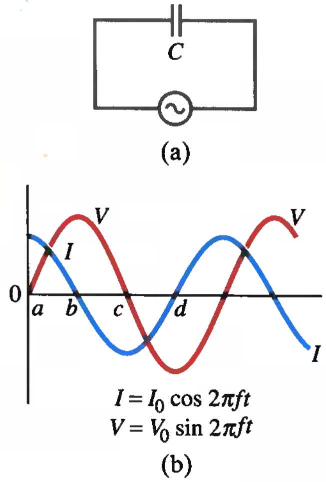


FIGURE 21–33 Capacitor connected to an ac source. Current leads voltage by a quarter cycle, or 90°.

Capacitor: current leads voltage

Only R (not C or L) dissipates energy

EXAMPLE 21–14 **Capacitor reactance.** What are the peak and currents in the circuit of Fig. 21–33a if $C = 1.0\,\mu F$ and V_{rms} = 1... Calculate for (*a*) $f = 60$ Hz, and then for (*b*) $f = 6.0 \times 10^5$ Hz.

SOLUTION (*a*) $V_0 = \sqrt{2}V_{rms} = 170$ V. Then

$$X_C = \frac{1}{2\pi fC} = \frac{1}{(6.28)(60\,\text{s}^{-1})(1.0 \times 10^{-6}\,\text{F})} = 2.7\,\text{k}\Omega$$

Thus

$$I_0 = \frac{V_0}{X_C} = \frac{170\,\text{V}}{2.7 \times 10^3\,\Omega} = 63\,\text{mA},$$

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120\,\text{V}}{2.7 \times 10^3\,\Omega} = 44\,\text{mA}.$$

(*b*) For $f = 6.0 \times 10^5$ Hz, X_C will be $0.27\,\Omega$, $I_0 = 630$ A, and I_{rms} = ... The dependence on f is dramatic.

➥ **PHYSICS APPLIED**

Capacitors as filters

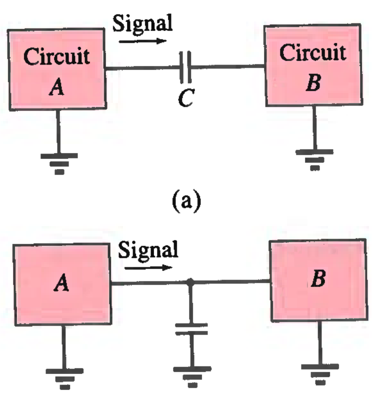


FIGURE 21–34 Two common uses for a capacitor.

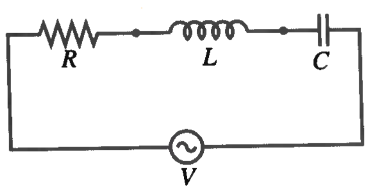
Capacitors are used for a variety of purposes, some of which have already been described. Two other applications are illustrated in Fig. 21–34. In Fig. 21–34a, circuit A is said to be capacitively coupled to circuit B. The purpose of the capacitor is to prevent a dc voltage from passing from A to B but allowing an ac signal to pass relatively unimpeded. If C is sufficiently large, the ac signal will not be significantly attenuated, whereas dc is filtered out. The capacitor in Fig. 21–34b also passes ac but not dc. In this case, a dc voltage can be maintained between circuits A and B. If the capacitance C is large enough, the capacitor offers little impedance to an ac signal leaving A. Such a signal then passes to ground instead of into B. The capacitor in Fig. 21–34b acts like a *filter* when a constant dc voltage is required; any sharp variation in voltage will pass to ground instead of into circuit B. Capacitors used in these two ways are very common in circuits.

* 21–13 *LRC* Series AC Circuit; Problem Solving

We now examine a circuit containing all three elements in series, a resistor R, an inductor L, and a capacitor C (Fig. 21–35). If a given circuit contains only two of these elements, we can still use the results of this section by setting $R = 0$, $L = 0$, or $C = \infty$ (infinity) as needed. We let V_R, V_L, and V_C represent the voltage across each element at a given instant in time, and V_{R0}, V_{L0}, and V_{C0} represent the *maximum* (peak) values of these voltages. The voltage across each of the elements will follow the phase relations discussed in the last section. That is, V_R will be in phase with the current, V_L will lead the current by 90°, and V_C will lag behind the current by 90°. Also, at any instant the total voltage V supplied by the source will be $V = V_R + V_L + V_C$. However, because the various voltages are not in phase, the rms voltages (which is what ac voltmeters usually measure) will not simply add up to give the rms voltage of the source. And V_0 will not equal $V_{R0} + V_{L0} + V_{C0}$. Let us now examine the circuit in detail. What we would like to find in particular is the impedance of the circuit as a whole, the rms current that flows through the circuit, and the phase difference between the source voltage and the current.

First we note that the current at any instant must be the same at all points in the circuit. Thus, *the currents in each element are in phase with each other, although the voltages are not.* We choose our origin in time ($t = 0$) so that the current I at any time t is $I = I_0 \cos 2\pi ft$.

FIGURE 21–35 An *LRC* circuit.



It is convenient to analyze an *LRC* circuit using a **phasor diagram.** Arrows (acting like vectors) are drawn in an *xy* coordinate system to represent each voltage. (These "vectors" are not "real." Phasors are just a useful analytical tool.) *The length of each arrow represents the magnitude of the peak voltage across each element:*

$$V_{R0} = I_0 R, \quad V_{L0} = I_0 X_L, \quad \text{and} \quad V_{C0} = I_0 X_C.$$

V_{R0} is in phase with the current and is initially ($t = 0$) drawn along the positive x axis, as is the current. Since V_{L0} leads the current by 90°, it leads V_{R0} by 90°, so is initially drawn along the positive y axis. V_{C0} lags the current by 90°, so lags V_{R0} by 90°; hence, V_{C0} is drawn initially along the negative y axis. Such a diagram is shown in Fig. 21–36a. If we let the vector diagram rotate counterclockwise at frequency f, we get the diagram shown in Fig. 21–36b; after a time t, each arrow has rotated through an angle $2\pi ft$. Then the *projections of each arrow on the x axis represent the voltages across each element at the instant t* (see Fig. 21–36c). For example, the projection of V_{R0} on the x axis is $V_{R0}\cos 2\pi ft$ (as in Fig. 21–31); and the projections of V_{L0} and V_{C0} on the x axis are $-V_{L0}\sin 2\pi ft$ and $V_{C0}\sin 2\pi ft$, respectively, as in Figs. 21–32b and 21–33b. Maintaining the 90° angle between each arrow ensures the correct phase relations. Although these results show the validity of a phasor diagram, what we are really interested in is how to add the voltages.

The sum of the projections of the three vectors on the x axis is equal to the projection of their sum. But the sum of the projections represents the instantaneous voltage across the whole circuit, V (equal to the source voltage). Therefore, the vector sum of these vectors will be the vector that represents the peak source voltage, V_0. This is shown in Fig. 21–37, where it is seen that V_0 makes an angle ϕ with I_0 and V_{R0}. As time passes, V_0 rotates with the other vectors, so the instantaneous voltage V (projection of V_0 on the x axis) is (see Fig. 21–37)

$$V = V_0 \cos(2\pi ft + \phi).$$

The voltage V across the whole circuit must, of course, equal the source voltage (Fig. 21–35). Thus we see that the voltage from the source is out of phase† with the current by an angle ϕ.

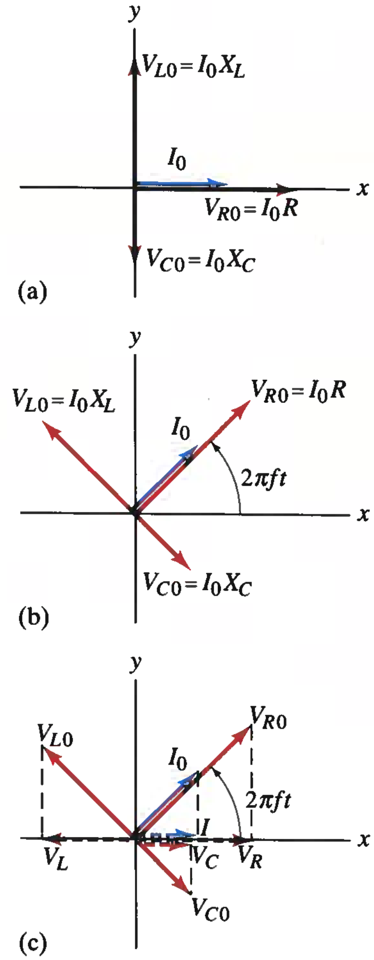


FIGURE 21–36 Phasor diagram for a series *LRC* circuit.

From this analysis we can now draw some useful conclusions. First, we determine the total **impedance** Z of the circuit, which is defined by the relation

Impedance

$$V_{rms} = I_{rms}Z, \quad \text{or} \quad V_0 = I_0 Z. \tag{21–13}$$

From Fig. 21–37 we see, using the Pythagorean theorem (V_0 is the hypotenuse of a right triangle), that

$$\begin{aligned} V_0 &= \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} \\ &= \sqrt{I_0^2 R^2 + (I_0 X_L - I_0 X_C)^2} \\ &= I_0\sqrt{R^2 + (X_L - X_C)^2}. \end{aligned}$$

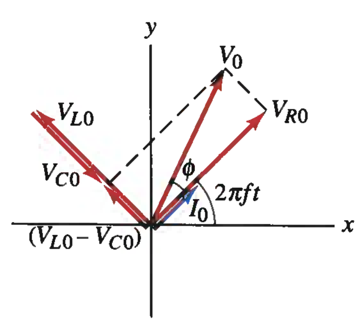
Thus, from Eq. 21–13, and then Eqs. 21–11b and 21–12b,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{21–14a}$$

$$= \sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}. \tag{21–14b}$$

This gives the total impedance of the circuit. Also from Fig. 21–37, we can

FIGURE 21–37 Phasor diagram for a series *LRC* circuit showing the sum vector, V_0.



†As a check, note that if $R = X_C = 0$, then $\phi = 90°$, and V_0 would lead the current by 90°, as it must for an inductor alone. Similarly, if $R = L = 0$, $\phi = -90°$ and V_0 would lag the current by 90°, as it must for a capacitor alone.

find the phase angle ϕ:

$$\tan\phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{X_L - X_C}{R}$$

or

$$\cos\phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}.$$

Finally, we can determine the power dissipated in the circuit. W... earlier that power is only dissipated by a resistance; none is dissi... inductance or capacitance. Therefore, the average power $\overline{P} = I$... from Eq. 21–15b, $R = Z\cos\phi$. Therefore,

$$\overline{P} = I_{\text{rms}}^2 Z\cos\phi$$
$$= I_{\text{rms}} V_{\text{rms}} \cos\phi.$$

The factor $\cos\phi$ is referred to as the **power factor** of the circuit. Fo... resistor, $\cos\phi = 1$ and $\overline{P} = I_{\text{rms}}V_{\text{rms}}$. For a capacitor or inductor, ϕ ... or $+90°$, respectively, so $\cos\phi = 0$ and no power is dissipated.

The test of this analysis is, of course, in experiment; and experi... in full agreement with these results.

EXAMPLE 21–15 **LRC Circuit.** Suppose that $R = 25.0\ \Omega$, $L = $ ... and $C = 12.0\ \mu\text{F}$ in Fig. 21–35, and that they are connected to a 90... (rms) 500-Hz source. Calculate (*a*) the current in the circuit, (*b*) the ... meter readings (rms) across each element, (*c*) the phase angle ... (*d*) the power dissipated in the circuit.

SOLUTION (*a*) First, we find the individual impedances at $f = 5$...

$$X_L = 2\pi f L = 94.2\ \Omega,$$

$$X_C = \frac{1}{2\pi f C} = 26.5\ \Omega.$$

Then

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(25.0\ \Omega)^2 + (94.2\ \Omega - 26.5\ \Omega)^2} = 72.2\ \Omega.$$

From Eq. 21–13,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{90.0\ \text{V}}{72.2\ \Omega} = 1.25\ \text{A}.$$

(*b*) The rms voltage across each element is

$$(V_R)_{\text{rms}} = I_{\text{rms}} R = (1.25\ \text{A})(25.0\ \Omega) = 31.2\ \text{V}$$
$$(V_L)_{\text{rms}} = I_{\text{rms}} X_L = (1.25\ \text{A})(94.2\ \Omega) = 118\ \text{V}$$
$$(V_C)_{\text{rms}} = I_{\text{rms}} X_C = (1.25\ \text{A})(26.5\ \Omega) = 33.1\ \text{V}.$$

Notice that these do *not* add up to give the source voltage, 90.0 V (r... deed, the rms voltage across the inductance *exceeds* the source volta... can happen because the different voltages are out of phase with each ...

and at any instant one voltage can be negative, to compensate for a large positive voltage of another. The rms voltages, however, are always positive by definition. Although the rms voltages do not have to add up to the source voltage, the instantaneous voltages at any time do add up, of course.
(c) Since

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{94.2\ \Omega - 26.5\ \Omega}{25.0\ \Omega} = 2.71,$$

then $\phi = 69.7°$.
(d) $P = I_{rms}V_{rms}\cos\phi = (1.25\text{ A})(90.0\text{ V})(25.0\ \Omega/72.2\ \Omega) = 39.0\text{ W}.$

21–14 Resonance in AC Circuits; Oscillators

The rms current in an LRC series circuit is given by (see Eqs. 21–13 and 21–14):

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}. \qquad \textbf{(21–17)}$$

Because the impedance of inductors and capacitors depends on the frequency f of the source, the current in an LRC circuit will depend on frequency. From Eq. 21–17 we can see that the current will be maximum at a frequency such that

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

We solve this for f, and call the solution f_0:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}. \qquad \textbf{(21–18)}$$

Resonant frequency

This is the **resonant frequency** of the circuit. At this frequency, $X_C = X_L$, so the impedance is purely resistive and $\cos\phi = 1$. A graph of I_{rms} versus f is shown in Fig. 21–38 for particular values of R, L, and C. For smaller R compared to X_L and X_C, the resonance peak will be higher and sharper.

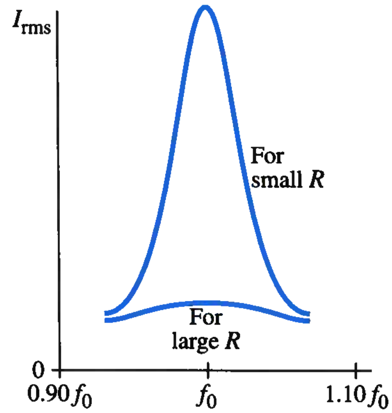
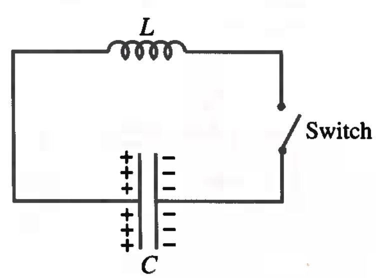


FIGURE 21–38 Current in an LRC circuit as a function of frequency, showing resonance peak at $f = f_0 = (1/2\pi)\sqrt{1/LC}$.

LC circuit

When R is very small, we speak of an ***LC* circuit**. The energy in an LC circuit oscillates, at frequency f_0, between the inductor and the capacitor, with some being dissipated in R (some resistance is unavoidable). To see this in more detail, consider a perfect LC circuit in which $R = 0$. Assume at $t = 0$ that the capacitor C is charged and the switch is closed (Fig. 21–39). The capacitor immediately begins to discharge. As it does so, the current I through the inductor increases. At every instant, the potential difference across the capacitor, $V = Q/C$ (where Q is the charge on the capacitor at that instant), must equal the potential difference across the inductor, which is equal to its emf $-L(\Delta I/\Delta t)$. At the instant when the charge on the capacitor reaches zero ($Q = 0$), the current I in the inductor has reached its maximum value, but at this instant I is not changing ($-L\,\Delta I/\Delta t = Q/C = 0$). At this moment, the magnetic field B in the inductor is also a maximum. The current next begins to decrease as the flowing charge starts to accumulate on the opposite plate of the capacitor. When the current has dropped to zero, the capacitor has again

FIGURE 21–39 A pure LC circuit.



attained its maximum charge. The capacitor then begins to discha[...] with the current now flowing in the opposite direction. This proce[...] charge flowing back and forth from one plate of the capacitor to the [...] through the inductor, continues to repeat itself. This is called an *LC* os[...] or an **electromagnetic oscillation**. Not only does the charge oscillate [...] forth, but so does the energy, which oscillates between being stored [...] electric field of the capacitor and in the magnetic field of the inductor.

EM oscillations

Electric resonance is used in many electronic devices. Radio and [...] sion sets, for example, use resonant circuits for tuning in a station. M[...] quencies reach the circuit from the antenna, but a significant curre[...] only for frequencies at or near the resonant frequency. Either L or C [...] able so that different stations can be tuned in (more on this in Chapter [...] *LC* circuits are also used in **oscillators**, which are devices that put out [...] cillating signal of particular frequency. Since some resistance is alwa[...] sent, electrical oscillators generally need a periodic input of pow[...] compensate for the energy converted to thermal energy in the resista[...]

Electrical resonance is analogous to mechanical resonance, wh[...] discussed in Chapter 11 (see Fig. 11–18). The energy transferred [...] system is a maximum at resonance whether it is electrical resonan[...] oscillation of a spring, or pushing a child on a swing (Section 11–[...]). [...] this is true in the electrical case can be seen from Eqs. 21–14, 21–1[...] 21–16. At resonance, $Z = R$, $\cos\phi = 1$, and I_{rms} is a maximum. F[...] stant V_{rms}, the power is then a maximum at resonance. A graph of p[...] versus frequency looks much like that for the current (Fig. 21–38).

An *LRC* circuit can have the elements arranged in parallel inst[...] in series. Resonance will occur in this case, too, but the analysis of su[...] cuits is more involved.

* 21–15 Impedance Matching

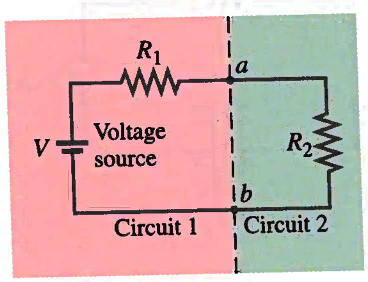
It is common to connect one electric circuit to a second circuit. For ex[...] ple, a TV antenna is connected to a TV set; an FM tuner is connected to [...] amplifier; the output of an amplifier is connected to a speaker; ele[...] for an ECG or EEG (electrocardiogram and electroencephalogra[...] electrical traces of heart and brain signals) are connected to an ampl[...] or a recorder. In many cases it is important that the maximum pow[...] transferred from one to the other, with a minimum of loss. This ca[...] achieved when the output impedance of the one device matches the in[...] impedance of the second.

PHYSICS APPLIED

Maximum power transferred when impedances match

To show why this is true, we consider simple circuits that cont[...] resistance. In Fig. 21–40, the source in circuit 1 could represent a p[...] supply, the output of an amplifier, or the tiny signal from an ante[...] laboratory probe, or electrodes. R_1 represents the resistance of this d[...] and includes the internal resistance of the source. R_1 is called the ou[...] impedance (or resistance) of circuit 1. The output of circuit 1 is acro[...] terminals a and b, which are connected to the input of circuit 2. Ci[...] may be very complicated. By combining the various resistors, we can [...] an equivalent resistance. This is represented by R_2, the "input resist[...]" (or input impedance) of circuit 2.

FIGURE 21–40 Output of the circuit on the left is input to the circuit on the right.



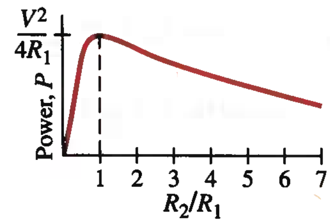
The power delivered to circuit 2 is $P = I^2R_2$, where $I = V/(R_1 + R_2)$. Thus

$$P = I^2R_2 = \frac{V^2R_2}{(R_1 + R_2)^2}.$$

We divide the top and bottom of the right side by R_1 and find

$$P = \frac{V^2}{R_1}\frac{\left(\frac{R_2}{R_1}\right)}{\left(1 + \frac{R_2}{R_1}\right)^2}. \qquad \textbf{(21–19)}$$

The question is, if the resistance of the source is R_1, what value should R_2 be so that the maximum power is transferred to circuit 2? To determine this, we plot a graph of P versus (R_2/R_1). This is shown in Fig. 21–41, where representative values are given in the table. For example, for $R_2/R_1 = 1$, Eq. 21–19 gives $P = V^2/4R_1$; for $R_2/R_1 = 3$, $P = 3V^2/16R_1 = 0.19V^2/R_1$; and so on. As can be seen from the graph, P is a maximum when $R_2 = R_1$. Thus, the maximum power is transmitted when the *output impedance* of one device *equals the input impedance* of the second. This is called **impedance matching.**



$\frac{R_2}{R_1}$	P
0	0
0.5	$0.22\ V^2/R_1$
1.0	$0.25\ V^2/R_1$
2.0	$0.22\ V^2/R_1$
5.0	$0.14\ V^2/R_1$
100	$0.01\ V^2/R_1$

FIGURE 21–41 Power transferred is at its maximum when $R_2 = R_1$.

Impedance matching

In an ac circuit that contains capacitors and inductors, the different phases are important and the analysis is more complicated. However, the same result holds: To maximize power transfer it is important to match impedances ($Z_2 = Z_1$). In addition, one must be aware that it is possible to seriously distort a signal. For example, when a second circuit is connected, it may put the first circuit into resonance, or take it out of resonance for a certain frequency. Without proper consideration of the impedances involved, one can make measurements that are completely meaningless. These considerations are normally examined by engineers when designing an integrated set of apparatus. It has happened that researchers have connected several components to one another without regard for impedance matching, and made a "new discovery" that later was found, embarrassingly, to be due to impedance mismatch rather than the natural phenomenon they had thought.

PHYSICS APPLIED

Impedance mismatch errors

In some cases, a transformer is used to alter an impedance, so it can be matched to that of a second circuit. If Z_S is the secondary impedance and Z_P the primary impedance, then $V_S = I_SZ_S$ and $V_P = I_PZ_P$ (I and V are either peak or rms values of current and voltage). Hence,

$$\frac{Z_P}{Z_S} = \frac{V_PI_S}{V_SI_P} = \left(\frac{N_P}{N_S}\right)^2,$$

where we have used Eqs. 21–6 and 21–7 for a transformer. Thus the impedance can be changed with a transformer. A transformer is used for this purpose in some audio amplifiers, which may have several taps corresponding to 4 Ω, 8 Ω, and 16 Ω so the output can be matched to the impedance of any loudspeaker.

Some instruments, such as oscilloscopes, require only a signal voltage but very little power. Maximum power transfer is then not important and such instruments can have a high input impedance. This has the advantage that the instrument draws very little current and disturbs the original circuit as little as possible. This is often desirable in laboratory experiments.

SUMMARY

The **magnetic flux** passing through a loop is equal to the product of the area of the loop times the perpendicular component of the magnetic field strength: $\Phi_B = B_\perp A = BA\cos\theta$.

If the magnetic flux through a coil of wire changes in time, an emf is induced in the coil; the magnitude of the induced emf equals the time rate of change of the magnetic flux through the loop times the number N of loops in the coil:

$$\mathscr{E} = -N\frac{\Delta\phi_B}{\Delta t}.$$

This is **Faraday's law of induction.**

The induced emf produces a current whose magnetic field opposes the original change in flux (**Lenz's law**).

Faraday's law also tells us that a changing magnetic field produces an electric field; and that a straight wire of length l moving with speed v perpendicular to a magnetic field of strength B has an emf induced between its ends equal to:

$$\mathscr{E} = Blv.$$

An electric **generator** changes mechanical energy into electrical energy. Its operation is based on Faraday's law: a coil of wire is made to rotate uniformly by mechanical means in a magnetic field, and the changing flux through the ... duces a sinusoidal current, which is the ... the generator.

A motor, which operates in the ... generator, acts like a generator in that ... **emf** is induced in its rotating coil ... counter emf opposes the input voltage, ... limit the current in a motor coil.

Similarly, a generator acts somewh... motor in that a **counter torque** acts on ... ing coil.

A **transformer**, which is a device to ... magnitude of an ac voltage, consists of ... and a secondary coil. The changing flux ... ac voltage in the primary induces an ac ... the secondary. In a 100 percent effici... former, the ratio of output to input ... (V_S/V_P) equals the ratio of the number of ... in the secondary to the number N_P in the ...

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}.$$

The ratio of secondary to primary current ... inverse ratio of turns:

$$\frac{I_S}{I_P} = \frac{N_P}{N_S}.$$

QUESTIONS

1. What would be the advantage, in Faraday's experiments (Fig. 21–1), of using coils with many turns?
2. What is the difference between magnetic flux and magnetic field? Discuss in detail.
3. Suppose you are holding a circular piece of wire and suddenly thrust a magnet, south pole first, toward the center of the circle. Is a current induced in the wire? Is a current induced when the magnet is held steady within the loop? Is a current induced when you withdraw the magnet? In each case, if your answer is yes, specify the direction.
4. In what direction will the current flow in Fig. 21–9 if the rod moves to the left, which decreases the area of the loop to the left?
5. Two loops of wire are moving in the vicinity of a very long straight wire carrying a steady current as shown in Fig. 21–42. Find the direction of the induced current in each loop.

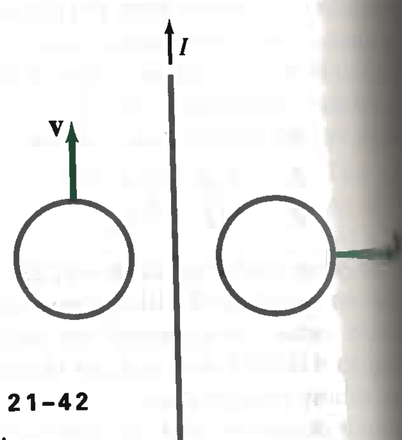


FIGURE 21–42 Question 5.

6. In situations where a small signal must travel ... distance, a "shielded cable" is used in which the ... wire is surrounded by an insulator and then ... by a cylindrical conductor. Why is a "shield" ne...

7. What is the advantage of placing the two insulated electric wires carrying ac close together or even twisted about each other?

8. In some early automobiles, the starter motor doubled as a generator to keep the battery charged once the car was started. Explain how this might work.

9. Explain why, exactly, the lights may dim briefly when a refrigerator motor starts up. When an electric heater is turned on, the lights may stay dimmed as long as it is on. Explain the difference.

10. Explain what is meant by the statement "a motor acts as a motor and generator at the same time." Can the same be said for a generator?

11. Use Fig. 21–12 and the right-hand rules to show why the counter torque in a generator *opposes* the motion.

12. Will an eddy current brake (Fig. 21–19) work on a copper or aluminum wheel, or must it be ferromagnetic?

13. It has been proposed that eddy currents be used to help sort solid waste for recycling. The waste is first ground into tiny pieces and iron removed with a dc magnet. The waste then is allowed to slide down an incline over permanent magnets. How will this aid in the separation of nonferrous metals (Al, Cu, Pb, brass) from nonmetallic materials?

14. The pivoted metal bar with slots in Fig. 21–43 falls much more quickly through a magnetic field than does a solid bar. Explain in detail.

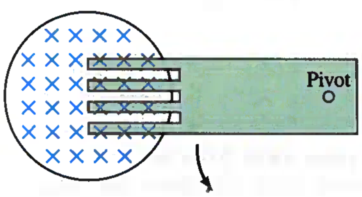


FIGURE 21–43 Question 14.

15. If an aluminum sheet is held between the poles of a large bar magnet, it requires some force to pull it out of the magnetic field even though the sheet is not ferromagnetic and does not touch the pole faces. Explain.

16. A bar magnet falling inside a vertical metal tube reaches a terminal velocity even if the tube is evacuated so that there is no air resistance. Explain.

17. A metal bar, pivoted at one end, oscillates freely in the absence of a magnetic field; but in a magnetic field, its oscillations are quickly damped out. Explain. (This *magnetic damping* is used in a number of practical devices.)

18. An enclosed transformer has four wire leads coming from it. How could you determine the ratio of turns on the two coils without taking the transformer apart? How would you know which wires paired with which?

19. The use of higher-voltage lines in homes, say 600 V or 1200 V, would reduce energy waste. Why are they not used?

*** 20.** Since a magnetic microphone is basically like a loudspeaker, could a loudspeaker (Section 20–10) actually serve as a microphone? That is, could you speak into a loudspeaker and obtain an output signal that could be amplified? Explain. Discuss, in light of your response, how a microphone and loudspeaker differ in construction.

*** 21.** The primary of a transformer on a telephone pole has a resistance of 0.10 Ω and the input voltage is 2400 V ac. Can you estimate the current that will flow? Will it be 24,000 A? Explain.

*** 22.** A transformer designed for a 120-V ac input will often "burn out" if connected to a 120-V dc source. Explain. [*Hint*: The resistance of the primary coil is usually very low.]

*** 23.** How would you arrange two flat circular coils so that their mutual inductance was (*a*) greatest, (*b*) least (without separating them by a great distance)?

*** 24.** If you are given a fixed length of wire, how would you shape it to obtain the greatest self-inductance? The least?

*** 25.** Does the emf of the battery in Fig. 21–29a affect the time needed for the *LR* circuit to reach (*a*) a given fraction of its maximum possible current, (*b*) a given value of current?

*** 26.** Can you tell whether the current in an *LRC* circuit leads or lags the applied voltage from a knowledge of the power factor, cos ϕ?

*** 27.** Under what conditions is the impedance in an *LRC* circuit a minimum?

*** 28.** An *LC* resonance circuit is often called an *oscillator* circuit. What is it that oscillates?

*** 29.** Compare the oscillations of an *LC* circuit to the vibration of a mass m on a spring. What do L and C correspond to in the mechanical system?

PROBLEMS

SECTIONS 21–1 TO 21–4

1. (I) A 9.2-cm-diameter loop of wire is initially oriented perpendicular to a 1.5-T magnetic field. It is rotated so that its plane is parallel to the field direction in 0.20 s. What is the average induced emf in the loop?
2. (I) A 16-cm-diameter circular loop of wire is in a 1.10-T magnetic field. It is removed from the field in 0.15 s. What is the average induced emf?

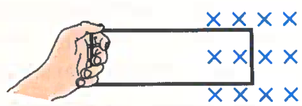


FIGURE 21–44
Problem 3.

3. (I) The rectangular loop shown in Fig. 21–44 is pushed into the magnetic field which points inward as shown. In what direction is the induced current?
4. (I) The north pole of the magnet in Fig. 21–45 is being inserted into the coil. In which direction is the induced current flowing through the resistor R?

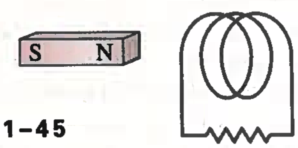


FIGURE 21–45
Problem 4.

5. (I) The magnetic flux through a coil of wire containing two loops changes from −30 Wb to +38 Wb in 0.42 s. What is the emf induced in the coil?
6. (I) A 7.2-cm-diameter wire coil is initially oriented so that its plane is perpendicular to a magnetic field of 0.63 T pointing up. During the course of 0.15 s, the field is changed to one of 0.25 T pointing down. What is the average induced emf in the coil?
7. (II) (*a*) If the resistance of the resistor in Fig. 21–46 is slowly increased, what is the direction of the current induced in the small circular loop inside the larger loop? (*b*) What would it be if the small loop were placed outside the larger one, to the left?

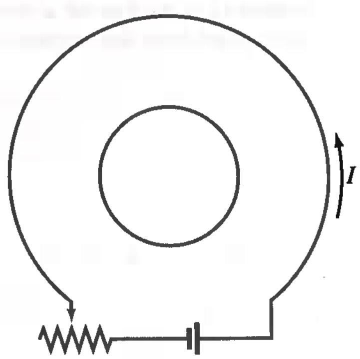


FIGURE 21–46
Problem 7.

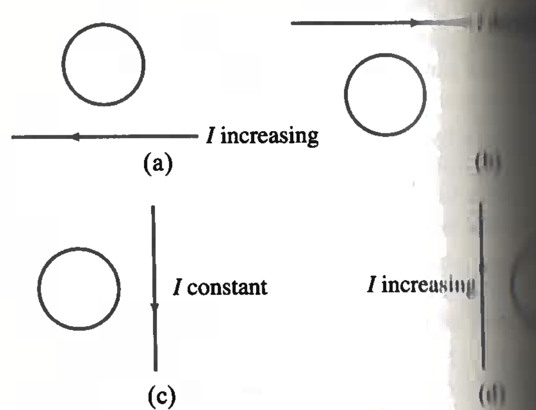


FIGURE 21–47 Problem 8

8. (II) What is the direction of the induced the circular loop due to the current part of Fig. 21–47?
9. (II) If the solenoid in Fig. 21–48 is away from the loop shown, in what direction induced current in the loop?

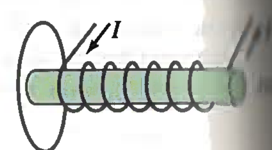


FIGURE 21–48
Problem 9.

10. (II) The magnetic field perpendicular loop of wire 20 cm in diameter is +0.52 T to −0.45 T in 180 ms, where field points away from an observer and the observer. (*a*) Calculate the induced what direction does the induced current
11. (II) The moving rod in Fig. 21–9 is 12.0 cm moves with a speed of 15.0 cm/s. If the field is 0.800 T, calculate (*a*) the emf develop (*b*) the electric field in the rod.
12. (II) A circular loop in the plane of the paper a 0.75 T magnetic field pointing into the the loop's diameter changes from 20.0 cm in 0.50 s, (*a*) what is the direction of the current, (*b*) what is the magnitude of the induced emf, and (*c*) if the coil resistance what is the average induced current?
13. (II) The moving rod in Fig. 21–9 is 13.2 cm generates an emf of 100 mV while moving in magnetic field. (*a*) What speed is it moving (*b*) What is the electric field in the rod?

[illegible] (II) In Fig. 21–9, the rod moves with a speed of [illegible] m/s, is 30.0 cm long, and has a resistance of [illegible] Ω. The magnetic field is 0.75 T, and the resistance of the U-shaped conductor is 25.0 Ω at a given instant. Calculate (*a*) the induced emf, (*b*) the current flowing in the circuit, and (*c*) the external force necessary to ensure that the rod is moving at constant velocity at that instant.

[illegible] (II) A single rectangular loop of wire with dimensions shown in Fig. 21–49 is situated so that part is inside a region of uniform magnetic field of 0.450 T and part is outside the field. The total resistance of the loop is 0.230 Ω. Calculate the force required to pull the loop from the field (to the right) at a constant velocity of 3.40 m/s. Neglect gravity.

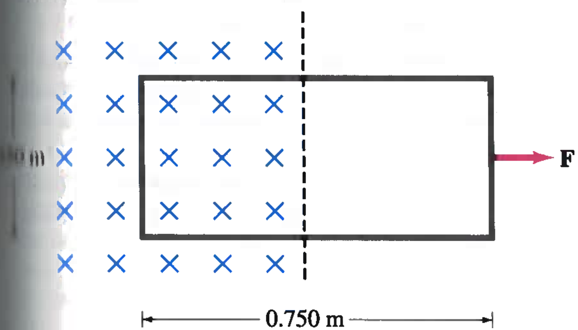


FIGURE 21–49 Problem 15.

[illegible] (II) A 31.0-cm-diameter coil consists of 20 turns of circular copper wire 2.6 mm in diameter. A uniform magnetic field, perpendicular to the plane of the coil, changes at a rate of 8.65×10^{-3} T/s. Determine (*a*) the current in the loop, and (*b*) the rate at which thermal energy is produced.

[illegible] (III) If the U-shaped conductor in Fig. 21–9 has resistivity ρ, whereas that of the moving rod is negligible, derive a formula for the current I as a function of time. Assume the rod has length l, starts at the bottom of the U at $t = 0$, and moves with uniform speed v in the magnetic field B. The cross-sectional area of the rod and all parts of the U is A.

[illegible] (III) The magnetic field perpendicular to a single 13.2-cm-diameter circular loop of copper wire decreases uniformly from 0.750 T to zero. If the wire is 2.25 mm in diameter, how much charge moves past a point in the coil during this operation?

SECTION 21–5

[illegible] (I) The generator of a car idling at 1000-rpm produces 12.4 V. What will the output be at a rotation speed of 2500 rpm assuming nothing else changes?

[illegible] (I) A simple generator is used to generate a peak output voltage of 24.0 V. The square armature consists of windings that are 6.0 cm on a side and rotates in a field of 0.420 T at a rate of 60 rev/s. How many loops of wire should be wound on the square armature?

21. (I) Show that the rms output of an ac generator is $V_{rms} = NAB\omega/\sqrt{2}$.

22. (II) A simple generator has a 720-loop square coil 21.0 cm on a side. How fast must it turn in a 0.650-T field to produce a 120-V peak output?

23. (II) A 450-loop circular armature coil with a diameter of 10.0 cm rotates at 60 rev/s in a uniform magnetic field of strength 0.75 T. What is the rms voltage output of the generator? What would you do to the rotation frequency in order to double the rms voltage output?

*SECTION 21–6

* 24. (I) A motor has an armature resistance of 3.75 Ω. If it draws 9.20 A when running at full speed and connected to a 120-V line, how large is the counter emf?

* 25. (I) The counter emf in a motor is 72 V when operating at 1800 rpm. What would be the counter emf at 2500 rpm if the magnetic field is unchanged?

* 26. (II) The counter emf in a motor is 100 V when the motor is operating at 1000 rpm. How would you change the motor's magnetic field if you wanted to reduce the counter emf to 65 V when the motor was running at 2500 rpm?

* 27. (II) What will be the current in the motor of Example 21–7 if the load causes it to run at half speed?

* 28. (II) The magnetic field of a "shunt-wound" dc motor is produced by field coils placed in parallel to the armature coils. Suppose that the field coils have a resistance of 66.0 Ω and the armature coils 5.00 Ω. The back emf at full speed is 105 V when the motor is connected to a 115-V line. (*a*) Draw the equivalent circuit for the situations when the motor is just starting and when it is running full speed. (*b*) What is the total current drawn by the motor at start up? (*c*) What is the total current drawn when the motor runs at full speed?

29. (II) A dc generator is rated at 10 kW, 200 V, and 50 A when it rotates at 1000 rpm. The resistance of the armature windings is 0.40Ω. (*a*) Calculate the "no-load" voltage at 1000 rpm (when there is no circuit hooked up to the generator). (*b*) Calculate the full-load voltage (i.e. at 50 A) when the generator is run at 800 rpm. Assume that the magnitude of the magnetic field remains constant.

SECTION 21–7

30. (I) A transformer is designed to change 120 V into 10,000 V, and there are 5000 turns in the primary. How many turns are in the secondary, assuming 100 percent efficiency?

31. (I) A transformer has 420 turns in the primary and 120 in the secondary. What kind of transformer is this and, assuming 100 percent efficiency, by what factor does it change the voltage? By what factor does it change the current?

32. (I) A step-up transformer increases 16 V to 120 V. What is the current in the secondary as compared to the primary? Assume 100 percent efficiency.

33. (I) Neon signs require 12 kV for their operation. To operate from a 220-V line, what must be the ratio of secondary to primary turns of the transformer? What would the voltage output be if the transformer were connected backward?

34. (II) A model-train transformer plugs into 120 V ac, and draws 0.75 A while supplying 15 A to the train. (*a*) What voltage is present across the tracks? (*b*) Is the transformer step-up or step-down?

35. (II) The output voltage of a 100-W transformer is 12 V and the input current is 20 A. (*a*) Is this a step-up or a step-down transformer? (*b*) By what factor is the voltage multiplied?

36. (II) High-intensity desk lamps are rated at 40 W but require only 12 V. They contain a transformer that converts 120 V household voltage. (*a*) Is the transformer step-up or step-down? (*b*) What is the current in the secondary when the lamp is on? (*c*) What is the current in the primary? (*d*) What is the resistance of the bulb when on?

37. (II) A transformer has 330 primary turns and 1240 secondary turns. The input voltage is 120 V and the output current is 15.0 A. What is the output voltage and input current assuming 100 percent efficiency?

38. (II) If 30 MW of power at 45 kV (rms) arrives at a town from a generator via 4.0-Ω transmission lines, calculate (*a*) the emf at the generator end of the lines, and (*b*) the fraction of the power generated that is lost in the lines.

39. (II) Show that the power loss in transmission lines, P_L, is given by $P_L = (P_T)^2 R_L / V^2$, where P_T is the power transmitted to the user, V is the delivered voltage, and R_L is the resistance of the power lines.

40. (II) If 50 kW is to be transmitted over two 0.100-Ω lines, estimate how much power is saved if the voltage is stepped up from 120 V to 1200 V and then down again, rather than simply transmitting at 120 V. Assume the transformers are each 99 percent efficient.

41. (III) Design a dc transmission line that can transmit 300 MW of electricity 200 km with only a 2 percent loss. The wires are to be made of aluminum and the voltage is 600 kV.

*SECTION 21–9

* 42. (I) If the current in a 120-mH coil changes steadily from 25.0 A to 10.0 A in 350 ms, what is the direction and magnitude of the induced emf?

* 43. (I) What is the inductance L of a 0.60-m-long air-filled coil 2.9 cm in diameter containing 10,000 loops?

* 44. (I) What is the inductance of a coil if it ... emf of 8.50 V when the current in it ... −28.0 mA to +31.0 mA in 42.0 ms?

* 45. (I) How many turns of wire would be ... make a 100-mH inductance out of a ... air-filled coil with a diameter of 5.2 cm?

* 46. (II) An air-filled cylindrical inductor has ... and it is 2.5 cm in diameter and 28.2 cm long ... its inductance? How many turns would you ... generate the same inductance if the core ... filled instead? Assume the magnetic ... of iron is about 1000 times that of free ...

* 47. (II) A coil has 2.25-Ω resistance and 440 ... tance. If the current is 3.00 A and is increasing ... rate of 3.50 A/s, what is the potential ... across the coil at this moment?

* 48. (II) The wire of a tightly wound solenoid ... wound and used to make another tightly ... lenoid of twice the diameter. By what ... the inductance change?

* 49. (II) (*a*) Show that the self-inductance L ... (Figs. 20–43 and 20–57) of radius R ... loops each of radius r is

$$L \approx \frac{\mu_0 N^2 r^2}{2R}$$

if $R \gg r$. Assume the field is uniform ... torus; is this actually true? Is this result ... with L for a solenoid? Should it be? (*b*) ... the inductance L of a large toroid if the ... the coils is 1.5 cm and the diameter of the ... ring is 40 cm. Assume the field inside the ... uniform. There are a total of 1000 loops of ...

* 50. (II) (*a*) Ignoring any mutual inductance, what ... equivalent inductance of two inductors ... series? (*b*) What if they are connected in ... (*c*) How does their mutual inductance (their ... rical relationship to each other) affect the ...

* 51. (II) A long thin solenoid of length l and ... sectional area A contains N_1 closely packed ... wire. Wrapped tightly around it is an insul... of N_2 turns, Fig. 21–50. Assume all the ... coil 1 (the solenoid) passes through coil 2, ... culate the mutual inductance.

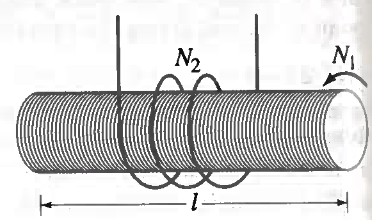


FIGURE 21–50 Problem 51.

(III) A 30-cm-long coil with 1500 loops is wound on an iron core ($\mu = 3000\ \mu_0$) along with a second coil of 800 loops. The loops of each coil have a radius of [illegible]0 cm. If the current in the first coil drops uniformly from 3.0 A to zero in 8.0 ms, determine (*a*) the emf induced in the second coil, and (*b*) the mutual inductance *M*.

(III) The potential difference across a given coil is [illegible]2.5 V at an instant when the current is 860 mA and is increasing at a rate of 3.40 A/s. At a later instant, the potential difference is 16.2 V whereas the current is 700 mA and is decreasing at a rate of 1.80 A/s. Determine the inductance and resistance of the coil.

SECTION 21–10

(I) The magnetic field inside an air-filled solenoid [illegible]6 cm long and 2.0 cm in diameter is 0.80 T. Approximately how much energy is stored in this field?

(II) At a given instant the current through an inductor is 50.0 mA and is increasing at the rate of 100 mA/s. What is the initial energy stored in the inductor if the inductance is known to be 60.0 mH, and how long does it take for the energy to increase by a factor of 10 from the initial value?

(II) Assuming the Earth's magnetic field averages about 0.50×10^{-4} T near the surface of the Earth, estimate the total energy stored in this field in the first 10 km above the Earth's surface.

SECTION 21–11

(II) It takes 7.20 ms for the current in an *LR* circuit to reach 80 percent of its maximum value. Determine (*a*) the time constant of the circuit, and (*b*) the inductance of the circuit if $R = 250\ \Omega$.

(II) Determine $\Delta I/\Delta t$ at $t = 0$ (when the battery is connected) for the *LR* circuit of Fig. 21–29 and show that if *I* continued to increase at this rate, it would reach its maximum value in one time constant.

(II) After how many time constants does the current in Fig. 21–29 reach within (*a*) 10 percent, (*b*) 1.0 percent, and (*c*) 0.1 percent of its maximum value?

(III) Two tightly wound solenoids have the same length and circular cross-sectional area. But solenoid 1 uses wire that is half as thick as solenoid 2. (*a*) What is the ratio of their inductances? (*b*) What is the ratio of their inductive time constants (assuming no other resistance in the circuits)?

SECTION 21–12

(I) At what frequency will a 160-mH inductor have a reactance of 1.5 kΩ?

(I) A 9.20-μF capacitor is measured to have a reactance of 250 Ω. At what frequency is it being driven?

* 63. (I) Plot a graph of the impedance of a 1.0-μF capacitor as a function of frequency from 10 to 1000 Hz.

* 64. (I) Plot a graph of the impedance of a 1.0-mH inductor as a function of frequency from 100 to 10,000 Hz.

* 65. (II) Calculate the impedance of, and rms current in, a 160-mH radio coil connected to a 240-V (rms) 10.0-kHz ac line. Ignore resistance.

* 66. (II) An inductance coil operates at 240 V and 60 Hz. It draws 12.8 A. What is the coil's inductance?

* 67. (II) (*a*) What is the impedance of a well-insulated 0.030-μF capacitor connected to a 2.0-kV (rms) 700-Hz line? (*b*) What will be the peak value of the current?

* 68. (II) A capacitor is placed in parallel across a load, as in Fig. 21–34b, to filter out stray high-frequency signals, but to allow ordinary 60-Hz ac to pass through with little loss. Suppose that circuit *B* in the figure is a resistance $R = 300\ \Omega$ connected to ground, and that $C = 0.60\ \mu$F. What percent of the incoming current will pass through *C* rather than *R* if (*a*) it is 60 Hz and (*b*) it is 60,000 Hz?

* 69. (II) Suppose that circuit *B* in Fig. 21–34a is a resistance $R = 500\ \Omega$, connected to ground, and the capacitance $C = 2.0\ \mu$F. Will this capacitor act to eliminate 60 Hz ac but pass a high-frequency signal of frequency 60,000 Hz? To check this, determine the voltage drop across *R* for a 50-mV signal of frequency (*a*) 60 Hz, and (*b*) 60,000 Hz.

*SECTION 21–13

* 70. (I) A 30-kΩ resistor is in series with a 0.50-H inductor and an ac source. Calculate the impedance of the circuit if the source frequency is (*a*) 60 Hz, and (*b*) 3.0×10^4 Hz.

* 71. (I) A 2.5-kΩ resistor and a 4.0-μF capacitor are connected in series to an ac source. Calculate the impedance of the circuit if the source frequency is (*a*) 100 Hz, and (*b*) 10,000 Hz.

* 72. (I) For a 120-V rms 60-Hz voltage, a current of 70 mA passing through the body for 1.0 s could be lethal. What would be the impedance of the body for this to occur?

* 73. (II) A 2.5-kΩ resistor in series with a 420-mH inductor is driven by an ac power supply. At what frequency is the impedance double that of the impedance at 60 Hz?

* 74. (II) (*a*) What is the rms current in an *RC* circuit if $R = 28.8$ kΩ, $C = 0.80\ \mu$F, and the rms applied voltage is 120 V at 60 Hz? (*b*) What is the phase angle between voltage and current? (*c*) What is the power dissipated by the circuit? (*d*) What are the voltmeter readings across *R* and *C*?

* **75.** (II) (*a*) What is the rms current in an RL circuit when a 60-Hz 120-V rms ac voltage is applied, where $R = 1.80\,\text{k}\Omega$, and $L = 900\,\text{mH}$? (*b*) What is the phase angle between voltage and current? (*c*) How much power is dissipated? (*d*) What are the rms voltage readings across R and L?

* **76.** (II) What is the total impedance, phase angle, and rms current in an LRC circuit connected to a 10.0-kHz, 300-V (rms) source if $L = 22.0\,\text{mH}$, $R = 8.70\,\text{k}\Omega$, and $C = 5000\,\text{pF}$?

* **77.** (II) What is the resistance of a coil if its impedance is $35\,\Omega$ and its reactance is $30\,\Omega$?

* **78.** (II) A voltage $V = 4.8 \sin 754t$ is applied to an LRC circuit. If $L = 3.0\,\text{mH}$, $R = 1.40\,\text{k}\Omega$, and $C = 3.0\,\mu\text{F}$, how much power is dissipated in the circuit?

* **79.** (II) A circuit consists of a 250-Ω resistor in series with a 40.0-mH inductor and a 50.0-V ac generator. The power dissipated by the circuit is 9.50 W. What is the frequency of the generator?

* **80.** (II) Show that for the LRC circuit of Fig. 21–35, if we have $I = I_0 \cos \omega t$, then

$$V_R = I_0 R \cos \omega t,$$

$$V_L = I_0 \omega L \cos(\omega t + \pi/2),$$

and

$$V_C = (I_0/\omega C) \cos(\omega t - \pi/2),$$

where $\omega = 2\pi f$.

*SECTION 21–14

* **81.** (I) A 3500-pF capacitor is connected [illegible] coil of resistance $3.0\,\Omega$. What is the [illegible] quency of this circuit?

* **82.** (I) The variable capacitor in the tuner [illegible] radio has a capacitance of 2800 pF when [illegible] is tuned to a station at 580 kHz. (*a*) What [illegible] the capacitance for a station at 1600 kHz? [illegible] is the inductance (assumed constant)?

* **83.** (II) An LRC circuit has $L =$ [illegible] $R = 4.4\,\Omega$. (*a*) What value must C have [illegible] resonance at 3600 Hz? (*b*) What will be [illegible] mum current at resonance if the peak [illegible] age is 50 V?

* **84.** (II) A 3000-pF capacitor is charged to [illegible] then quickly connected to an inductor [illegible] quency of oscillation is observed to be [illegible] termine (*a*) the inductance, (*b*) the peak [illegible] the current, and (*c*) the maximum energy [illegible] the magnetic field of the inductor.

*SECTION 21–15

* **85.** (I) An audio amplifier has output conn[illegible] $4\,\Omega$, $8\,\Omega$, and $16\,\Omega$. If two 8-Ω speakers [illegible] connected in parallel, to which output [illegible] should they be connected?

* **86.** (I) The output of an amplifier has an imp[illegible] $30\,\text{k}\Omega$. It is to be connected to an 8.0-Ω [illegible] er through a transformer. What should be the [illegible] ratio of the transformer?

GENERAL PROBLEMS

87. Suppose you are looking along a line through the centers of two circular (but separate) wire loops, one behind the other. A battery is suddenly connected to the front loop, establishing a clockwise current. (*a*) Will a current be induced in the second loop? (*b*) If so, when does this current start? (*c*) When does it stop? (*d*) In what direction is this current? (*e*) Is there a force between the two loops? (*f*) If so, in what direction?

88. Suppose you are looking at two current loops in the plane of the page as shown in Fig. 21–51. When the switch is thrown in the left-hand coil, (*a*) what is the direction of the induced current in the other loop? (*b*) What is the situation after a "long" time? (*c*) What is the direction of the induced current in the second loop if the second loop is quickly pulled horizontally to the right?

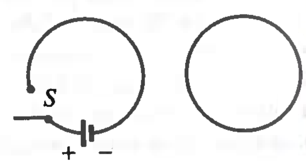


FIGURE 21–51 Problem 88.

89. A square loop 24.0 cm on a side has a re[illegible] $6.50\,\Omega$. It is initially in a 0.755-T magne[illegible] with its plane perpendicular to **B**, but is [illegible] from the field in 40.0 ms. Calculate the el[illegible] ergy dissipated in this process.

90. Two conducting rails 30 cm apart rest on a [illegible] They are joined at the bottom by a 0.60-Ω [illegible] and at the top a copper bar of mass 0.040 kg [illegible] across the rails. The whole apparatus is [illegible] in a vertical 0.55 T field. What is the [illegible] (steady) velocity of the bar as it slides fricti[illegible] down the rails?

A **search coil** for measuring B (also called a *flip coil*) is a small coil with N turns, each of cross-sectional area A. It is connected to a so-called **ballistic galvanometer**, which is a device to measure the total charge Q that passes through it in a short time. The flip coil is placed in the magnetic field to be measured with its face perpendicular to the field. It is then quickly rotated 180°. Show that the total charge Q that flows in the induced current during this short "flip" time is proportional to the magnetic field B; in particular, show that B is given by

$$B = \frac{QR}{2NA}$$

where R is the total resistance of the circuit, including that of the coil and that of the ballistic galvanometer which measures the charge Q.

(*a*) Show that the power $P = Fv$ needed to move the conducting rod to the right in Fig. 21–9 is equal to $B^2l^2v^2/R$, where R is the total resistance of the circuit. (*b*) Show that this equals the power dissipated in the resistance, I^2R.

The primary windings of a transformer, which has an 80% efficiency, are connected to 110 V ac. The secondary windings are connected across a 2.4 Ω, 58 W lightbulb. (*a*) Calculate the current through the primary windings of the transformer. (*b*) Calculate the ratio of the number of primary windings of the transformer to the number of secondary windings of the transformer.

A pair of power transmission lines each have a 0.80-Ω resistance and carry 700 A over 9.0 km. If the rms input voltage is 42 kV, calculate (*a*) the voltage at the other end, (*b*) the power input, (*c*) power loss in the lines, and (*d*) the power output.

Calculate the peak output voltage of a simple generator whose square armature windings are 6.60 cm on a side if the armature contains 125 loops and rotates in a field of 0.200 T at a rate of 120 rev/s.

A small electric car overcomes a 250-N friction force when traveling 30 km/h. The electric motor is powered by ten 12-V batteries connected in series and is coupled directly to the wheels whose diameters are 50 cm. The 300 armature coils are rectangular, 10 cm by 15 cm, and rotate in a 0.60-T magnetic field. (*a*) How much current does the motor draw to produce the required torque? (*b*) What is the back emf? (*c*) How much power is dissipated in the coils? (*d*) What percent of the input power is used to drive the car?

* **97.** What is the inductance L of the primary of a transformer whose input is 220 V at 60 Hz and the current drawn is 5.8 A? Assume no current in the secondary.

* **98.** A 230-mH coil, whose resistance is 18.5 Ω, is connected to a capacitor C and a 3360-Hz source voltage. If the current and voltage are to be in phase, what value must C have?

* **99.** A circuit contains two elements, but it is not known if they are L, R, or C. The current in this circuit when connected to a 120-V 60-Hz source is 5.6 A and lags the voltage by 50°. What are the two elements and what are their values?

* **100.** A resonant circuit using a 220-pF capacitor is to resonate at 48.0 MHz. The air-core inductor is to be a solenoid with closely packed coils made from 14.0 m of insulated wire 1.1 mm in diameter. How many loops will the inductor contain?

* **101.** An inductance coil draws 2.5 A dc when connected to a 36-V battery. When connected to a 60-Hz 120-V (rms) source, the current drawn is 3.8 A (rms). Determine the inductance and resistance of the coil.

* **102.** The **Q factor** of a resonance circuit is defined as the ratio of the voltage across the capacitor (or inductor) to the voltage across the resistor, at resonance. The larger the Q factor, the sharper the resonance curve will be and the sharper the tuning. (*a*) Show that the Q factor is given by the equation $Q = (1/R)\sqrt{L/C}$. (*b*) At a resonant frequency $f_0 = 1.0$ MHz, what must be the value of L and R to produce a Q factor of 1000? Assume that $C = 0.010\ \mu$F.

* **103.** In a series LRC circuit, the inductance is 20 mH, the capacitance is 50 nF, and the resistance is 200 Ω. At what frequencies is the power factor equal to 0.17?