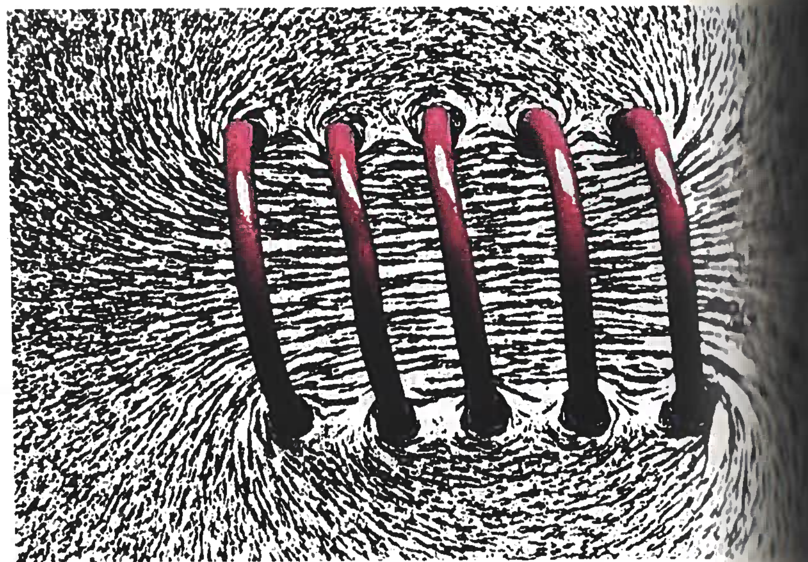
Magnets produce magnetic fields, but so do electric currents. An electric current flowing in this coil of wire produces a magnetic field which causes the tiny pieces of iron (iron "filings") to align in the field. Fig. 20–4 shows iron filings in the magnetic field produced by a magnet.



CHAPTER

20 MAGNETISM

Today it is clear that magnetism and electricity are clo
This relationship was not discovered, however, until the
century. The history of magnetism begins much earlier wi
cient civilizations in Asia Minor. It was in a region of Asia Min
as Magnesia that rocks were found that would attract each oth
rocks were called "magnets" after their place of discovery.

20–1 Magnets and Magnetic Fields

Poles of a magnet

A magnet will attract paper clips, nails, and other objects made
Any magnet, whether it is in the shape of a bar or a horsesho
ends or faces, called **poles**, which is where the magnetic effect is
If a magnet is suspended from a fine thread, it is found that on
the magnet will always point toward the north. It is not known
when this fact was discovered, but it is known that the Chinese w
ing use of it as an aid to navigation by the eleventh century and
earlier. This is, of course, the principle of a compass. A compass
simply a magnet that is supported at its center of gravity so it
freely. That pole of a freely suspended magnet which points tow
north is called the **north pole** of the magnet. The other pole point
the south and is called the **south pole**.

It is a familiar fact that when two magnets are brought near
other, each exerts a force on the other. The force can be either
or repulsive and can be felt even when the magnets don't touch

north pole of one magnet is brought near the north pole of a second magnet, the force is repulsive. Similarly, if two south poles are brought close, the force is repulsive. But when a north pole is brought near a south pole, the force is attractive. These results are shown in Fig. 20–1, and are reminiscent of the force between electric charges; like poles repel, and unlike poles attract. *But do not confuse magnetic poles with electric charge.* They are not the same thing. One important difference is that a positive or negative electric charge can easily be isolated. But the isolation of a single magnetic pole seems impossible. If a bar magnet is cut in half, you do not obtain isolated north and south poles. Instead, two new magnets are produced, Fig. 20–2. If the cutting operation is repeated, more magnets are produced, each with a north and a south pole. Physicists have tried combined means to isolate single magnetic poles (monopoles), but so far there is no firm experimental evidence for their existence.

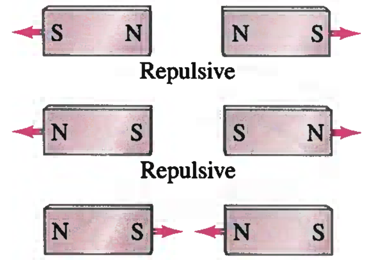
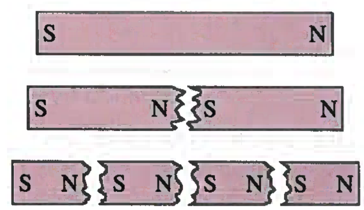


FIGURE 20–1 Like poles of a magnet repel; unlike poles attract.

Magnetic poles not found singly

FIGURE 20–2 If you break a magnet in half, you do not obtain isolated north and south poles; instead, two new magnets are produced, each with a north and a south pole.



Only iron and a few other materials such as cobalt, nickel, and gadolinium show strong magnetic effects. They are said to be **ferromagnetic** (from the Latin word *ferrum* for iron). All other materials show some slight magnetic effect, but it is extremely weak and can be detected only with delicate instruments. (We will look in more detail at ferromagnetism in Sections 20–13 and 20–15.)

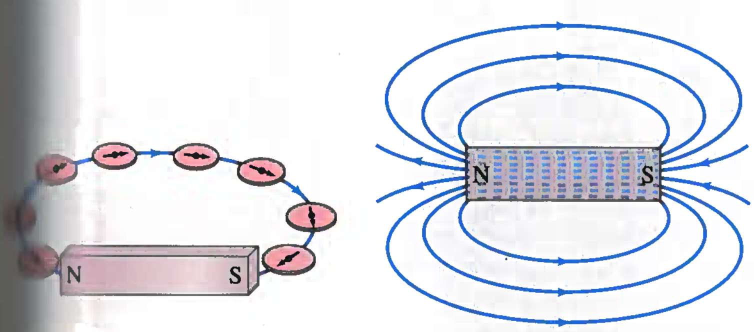
Magnetic field lines

We found it useful to speak of an electric field surrounding an electric charge. In the same way, we can imagine a **magnetic field** surrounding a magnet. The force one magnet exerts on another can then be described as the interaction between one magnet and the magnetic field of the other. Just as we drew electric field lines, we can also draw **magnetic field lines**. They can be drawn, as for electric field lines, so that (1) the direction of the magnetic field is tangent to a line at any point, and (2) the number of lines per unit area is proportional to the magnitude of the magnetic field.

The *direction* of the magnetic field at a given point can be defined as the direction that the north pole of a compass needle would point when placed at that point. Figure 20–3a shows how one magnetic field line around a bar magnet is found using compass needles. The magnetic field determined in this way for the field outside a bar magnet is shown in Fig. 20–3b. Notice that because of our definition, the lines always point from the north toward the south pole of a magnet (the north pole of a magnetic compass needle is attracted to the south pole of another magnet). Figure 20–4 shows how thin iron filings reveal the magnetic field lines by lining up like compass needles.

Magnetic field lines point from north to south magnetic poles

FIGURE 20–3 (a) Plotting a magnetic field line of a bar magnet. (b) Magnetic field lines outside of a bar magnet.



(a) (b)

FIGURE 20–4 Thin iron filings indicate the magnetic field lines around a bar magnet.

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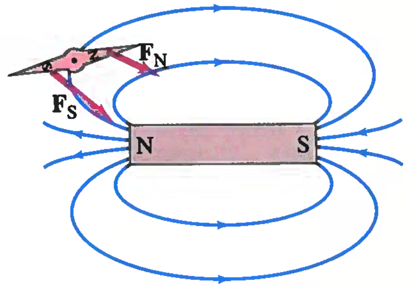


FIGURE 20–5 Forces on a compass needle that produce a torque to orient it parallel to the magnetic field lines. The net torque will be zero when the needle is parallel to the magnetic field line at that point. (Only the attractive forces are shown; try drawing in the repulsive forces and show that they produce a similar torque.)

We can define the magnetic field at any point as a vector, by the symbol **B**, whose direction is determined as discussed a compass needle. The *magnitude* of **B** can be defined in torque exerted on a compass needle when it makes a certain the magnetic field, as in Fig. 20–5. That is, the greater the greater the magnetic field strength. We can use this definition a more precise definition will be given in Section 20–3.

The Earth's magnetic field is shown in Fig. 20–6. The pattern is as if there were an (imaginary) bar magnet inside the Earth. Since pole of a compass needle points north, the magnetic pole which graphic north is actually a south pole magnetically, as indicated in the S on the schematic bar magnet inside the Earth. (Remember north pole of one magnet is attracted to the south pole of a second.) less, this pole is still often called the "north magnetic pole" simply in the north. Similarly, the Earth's south magnetic pole, near the south pole, is magnetically a north pole. The Earth's magnetic poles incide with the geographic poles (which are on the Earth's axis The north magnetic pole, for example, is in northern Canada, about from the geographic north pole. This must be taken into account a compass (Fig. 20–7). The angular difference between magnetic dicated by a compass, and true (geographical) north, is called the **declination**. In the U.S. it varies from 0° to about 25°, depending on

➥ **PHYSICS APPLIED**

Use of a compass.

Notice in Fig. 20–6 that the Earth's magnetic field is not Earth's surface at all points. The angle that the Earth's magnetic with the horizontal at any point is referred to as the **angle of dip**.

The simplest magnetic field is one that is uniform—it doesn't from one point to another. A perfectly uniform field over a not easy to produce. But the field between two flat parallel pole magnet is nearly uniform if the area of the pole faces is large their separation, as shown in Fig. 20–8. At the edges, the field somewhat and is no longer uniform. The parallel evenly spaced in the drawing indicate that the field is uniform at points not edge, much like the electric field between two parallel plates

FIGURE 20–6 The Earth acts like a huge magnet but its magnetic poles are not at the geographic poles.

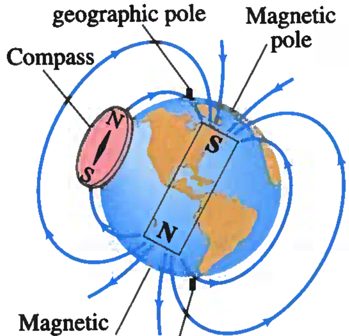
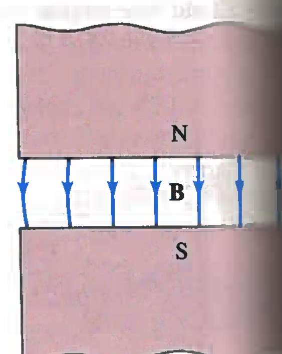


FIGURE 20–7 Using a map and compass in the wilderness. First you align the compass so the needle points away from true north (N) exactly the number of degrees of declination as stated on the (topographic) map: 15° in the case shown. Then align the map with true north, as shown, *not* with the compass needle.

FIGURE 20–8 Magnetic field between two large poles of a magnet is nearly uniform at the edges.



20–2 Electric Currents Produce Magnetism

During the eighteenth century, many natural philosophers sought to find a connection between electricity and magnetism. A stationary electric charge and a magnet were shown not to have any influence on each other. But in 1820, Hans Christian Oersted (1777–1851) found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and a current flows. As we have seen, a compass needle can be deflected by a magnetic field. What Oersted found was that **an electric current produces a magnetic field**. He had found a connection between electricity and magnetism.

A compass needle placed near a straight section of current-carrying wire aligns itself so it is tangent to a circle drawn around the wire, Fig. 20–9. Thus, the magnetic field lines produced by a current in a straight wire are in the form of circles with the wire at their center, Fig. 20–10a. The direction of these lines is indicated by the north pole of the compass in Fig. 20–9. There is a simple way to remember the direction of the magnetic field lines in this case. It is called a **right-hand rule**: you grasp the wire with your right hand so that your thumb points in the direction of the conventional (positive) current; then your fingers will encircle the wire in the direction of the magnetic field, Fig. 20–10b. The magnetic field lines due to a circular loop of current-carrying wire can be determined in a similar way using a compass. The result is shown in Fig. 20–11. Again the right-hand rule can be used, as shown in Fig. 20–12.

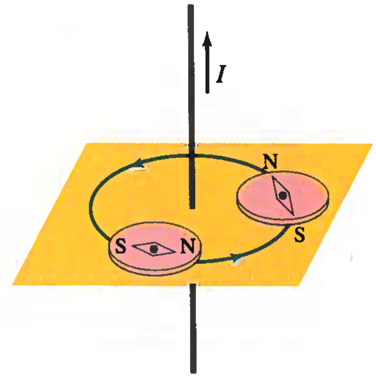


FIGURE 20–9 Deflection of a compass needle near a current-carrying wire, showing the presence and direction of the magnetic field.

FIGURE 20–10 (a) Magnetic field lines around an electric current in a straight wire. (b) Right-hand rule for remembering the direction of the magnetic field: when the thumb points in the direction of the conventional current, the fingers wrapped around the wire point in the direction of the magnetic field.

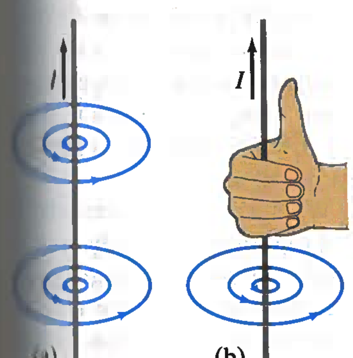


FIGURE 20–11 Magnetic field due to a circular loop of wire.

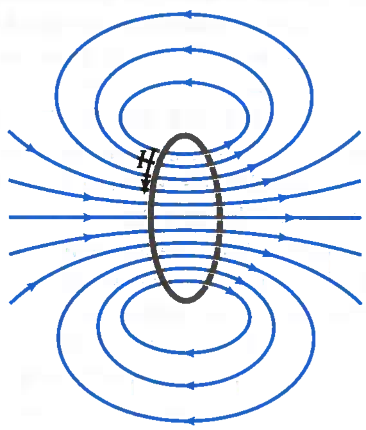
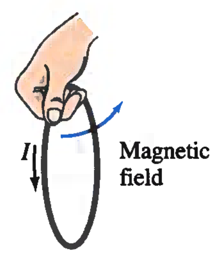


FIGURE 20–12 Right-hand rule for determining the direction of the magnetic field relative to the current.



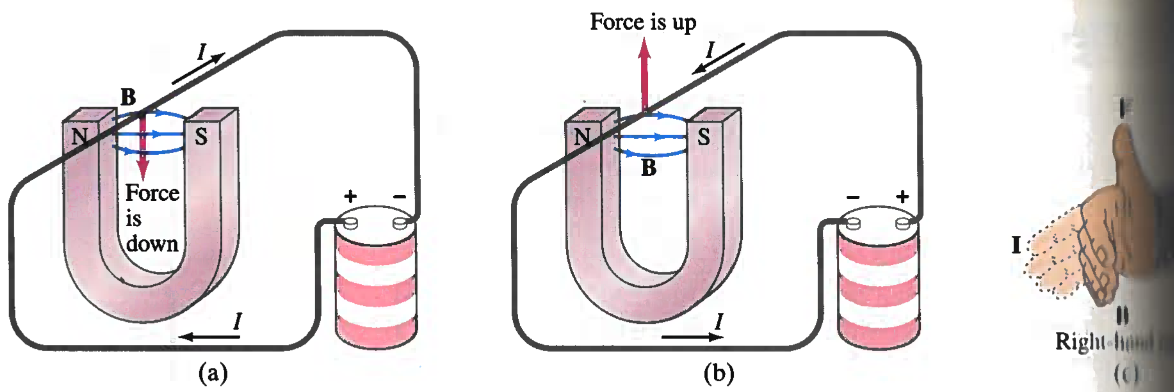


FIGURE 20–13 (a) Force on a current-carrying wire placed in a magnetic field **B**; (b) same, but current reversed; (c) right-hand rule for setup in (b).

20–3 Force on an Electric Current in a Magnetic Field; Definition of B

Magnet exerts a force on an electric current

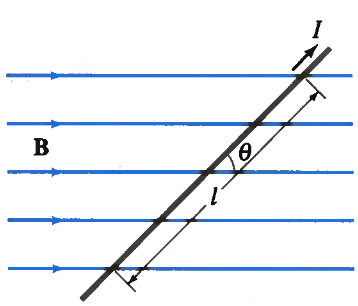
In Section 20–2 we saw that an electric current exerts a force on a ma such as a compass needle. By Newton's third law, we might expe verse to be true as well: we should expect that *a magnet exerts a* *current-carrying wire*. Experiments indeed confirm this effect, was first observed by Oersted.

Let us look at the force exerted on a wire in detail. Suppose wire is placed between the poles of a horseshoe magnet as Fig. 20–13. When a current flows in the wire, a force is exerted on th But this force is *not* toward one or the other poles of the magnet the force is directed at right angles to the magnetic field direction current is reversed in direction, the force is in the opposite direc found that *the direction of the force is always perpendicular to the of the current and also perpendicular to the direction of the magne* **B**. This statement does not completely describe the direction, howe force could be either up or down in Fig. 20–13b and still be perpen to both the current and to **B**. Experimentally, the direction of the given by another *right-hand rule*, as illustrated in Fig. 20–13c. First ent your right hand so that the outstretched fingers point in the di of the (conventional) current; from this position, bend your fingers they point in the direction of the magnetic field lines (which poin the N toward the S pole outside a magnet); you may have to rota hand and arm about the wrist until they do point along **B** when membering that straightened fingers must point along the direction current first. When your hand is oriented in this way, then the thumb points in the direction of the force on the wire.

Right-hand rule for force on current due to **B**

This describes the direction of the force. What about its magnit is found experimentally that the magnitude of the force is directly tional to the current I in the wire, to the length l of wire in the m field (assumed uniform), and to the magnetic field B. The force pends on the angle θ between the current direction and the magne (Fig. 20–14). When the current is perpendicular to the field lines, th is strongest. When the wire is parallel to the magnetic field lines, the force at all. At other angles, the force is proportional to sin θ (Fig.

FIGURE 20–14 Current-carrying wire in a magnetic field. Force on the wire is directed into the page.



Thus we have

$$F \propto IlB \sin\theta.$$

Up to now we have talked of magnetic field strengths in terms of the torque exerted by the field on a compass needle (Fig. 20–5). But we have not defined the magnetic field strength precisely. In fact, the magnetic field B is defined in terms of the above proportion so that the proportionality constant is precisely 1. Thus we have

FORCE ON ELECTRIC CURRENT IN A MAGNETIC FIELD

$$F = IlB \sin\theta. \qquad \textbf{(20–1)}$$

If the direction of the current is perpendicular to the field ($\theta = 90°$), then the force is

$$F_{max} = IlB. \qquad [\mathbf{I} \perp \mathbf{B}] \qquad \textbf{(20–2)}$$

If the current is parallel to the field ($\theta = 0°$), the force is zero.†

Definition of magnetic field

In summary, the magnetic field vector **B** is defined as follows. The direction of **B** in a region of space is the direction that a straight section of current-carrying wire would have when placed in the field and the force on it is zero ($\theta = 0°$ in Eq. 20–1), and consistent with the right-hand rule when the wire is oriented in another direction. The magnitude of **B** is defined (from Eq. 20–2) as

$$B = \frac{F_{max}}{Il},$$

where F_{max} is the magnitude of the force on a straight length l of wire carrying a current I when the wire is perpendicular to **B**.

The tesla and the gauss (units)

The SI unit for magnetic field B is the **tesla** (T). From Eq. 20–1, it is clear that $1\,T = 1\,N/A{\cdot}m$. An older name for the tesla is the "weber per meter squared" ($1\,Wb/m^2 = 1\,T$). Another unit commonly used to specify magnetic field is a cgs unit, the **gauss** (G): $1\,G = 10^{-4}\,T$. A field given in gauss should always be changed to teslas before using with other SI units. To get a "feel" for these units, we note that the magnetic field of the Earth at its surface is about $\frac{1}{2}$ G or $0.5 \times 10^{-4}\,T$. On the other hand, strong electromagnets can produce fields on the order of 2 T and superconducting magnets over 10 T.

EXAMPLE 20–1 Magnetic force on a current-carrying wire. A wire carrying a 30-A current has a length $l = 12$ cm between the pole faces of a magnet at an angle $\theta = 60°$ (Fig. 20–14). The magnetic field is approximately uniform at 0.90 T. We ignore the field beyond the pole pieces. What is the force on the wire?

SOLUTION We use Eq. 20–1 and find that

$$F = IlB \sin\theta$$
$$= (30\,A)(0.12\,m)(0.90\,T)(0.866) = 2.8\,N.$$

On a diagram, when we want to represent a magnetic field that is pointing out of the page (toward us) or into the page, we use ⊙ or ×. The ⊙ is meant to

† In our discussion, we have assumed that the magnetic field is uniform. If it is not, then B in Eqs. 20–1 and 20–2 is the average field over the length l of the wire. In practical cases, we can consider a wire as made up of many short segments Δl and the force on each segment is proportional to the length Δl of that segment and to the magnetic field B at that segment. The total force is the vector sum of the individual forces.

resemble the tip of an arrow pointing directly toward the read..., ... the × or ⊗ resembles the tail of an arrow going away. (See Fig...

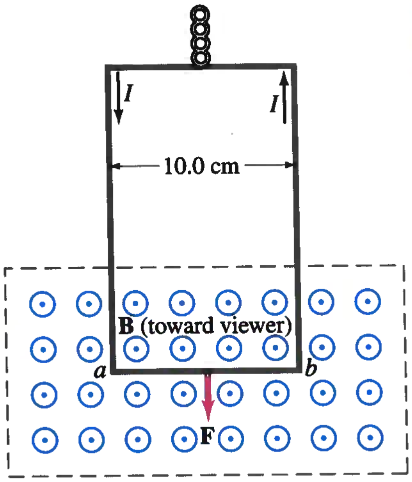


FIGURE 20–15 Measuring a magnetic field **B**. Example 20–2.

EXAMPLE 20–2 **Measuring a magnetic field.** A rectangul... wire hangs vertically as shown in Fig. 20–15. A magnetic field **B** i... horizontally, perpendicular to the wire, and points out of the pa... points as represented by the symbol ⊙. The magnetic field **B** is ve... uniform along the horizontal portion of wire *ab* (length l ... which is near the center of a large magnet producing the field. ... portion of the wire loop is free of the field. The loop hangs from a ... which measures a downward force (in addition to the gravitation... of $F = 3.48 \times 10^{-2}\,\mathrm{N}$ when the wire carries a current $I = 0.245$ A. ... the magnitude of the magnetic field B at the center of the magne...?

SOLUTION The magnetic forces on the two vertical sections ... wire loop point to the left and right, respectively. They are equal ... opposite directions and so add up to zero. Hence, the net magneti... on the loop is that on the horizontal section *ab* whose leng... $l = 0.100\,\mathrm{m}$ (and $\theta = 90°$ so $\sin\theta = 1$); thus

$$B = \frac{F}{Il} = \frac{3.48 \times 10^{-2}\,\mathrm{N}}{(0.245\,\mathrm{A})(0.100\,\mathrm{m})} = 1.42\,\mathrm{T}.$$

This technique is a highly precise means of determining magnetic fi...

20–4 Force on an Electric Charge Moving in a Magnetic Field

We have seen that a current-carrying wire experiences a force when ... in a magnetic field. Since a current in a wire consists of moving ... charges, we might expect that freely moving charged particles (... wire) would also experience a force when passing through a magnetic ... Indeed, this is the case.

From what we already know, let us determine the force on ... moving electric charge. If N such particles of charge q pass by a ... point in time t, they constitute a current $I = Nq/t$. We let t be the ... a charge q to travel a distance l in a magnetic field B; then $l = vt$, ... is the velocity of the particle. Thus, the force on these N particles ... Eq. 20–1, $F = IlB\sin\theta = (Nq/t)(vt)B\sin\theta$. The force on *one* of the ... cles is found by dividing by N:

FORCE ON MOVING CHARGE IN MAGNETIC FIELD

$$F = qvB\sin\theta.$$

This equation gives the magnitude of the force on a particle of ... q moving with velocity v in a magnetic field of strength B, where θ ... angle between **v** and **B**. The force is greatest when the particle mov... pendicular to **B** ($\theta = 90°$):

$$F_{max} = qvB. \qquad [\mathbf{v} \perp \mathbf{B}]$$

The force is *zero* if the particle moves *parallel* to the field lines ($\theta = 0°$) ... *direction* of the force is perpendicular to the magnetic field **B** and to the ... locity **v** of the particle. It is again given by a *right-hand rule*: you orient ... right hand so that your outstretched fingers point along the direction of ... tion of the particle (**v**), and when you bend your fingers they mus...

Right-hand rule

the direction of **B**; then your thumb will point in the direction of the force. This is true only for *positively* charged particles, and will be "down" for the situation shown in Fig. 20–16. For negatively charged particles, the force is in exactly the opposite direction ("up" in Fig. 20–16).

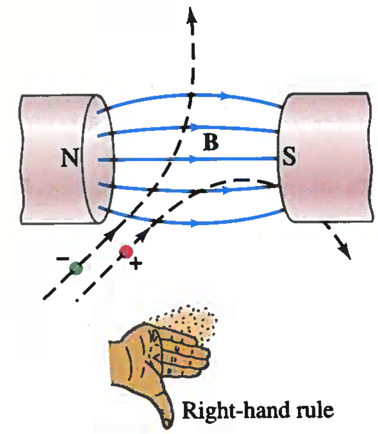


FIGURE 20–16 Force on charged particles due to a magnetic field is perpendicular to the magnetic field direction.

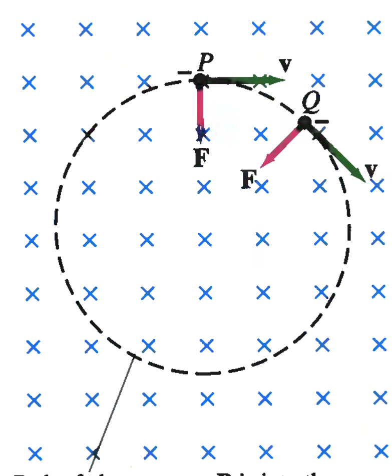
EXAMPLE 20–3 **Magnetic force on a proton.** A proton having a speed of 5.0×10^6 m/s in a magnetic field feels a force of 8.0×10^{-14} N toward the west when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and direction of the magnetic field in this region? (The charge on a proton is $q = +e = 1.6 \times 10^{-19}$ C.)

SOLUTION Since the proton feels no force when moving north, the field must be in a north–south direction. The right-hand rule tells us that **B** must point toward the north in order to produce a force to the west when the proton moves upward. (Your thumb points west and the outstretched fingers of your right hand point upward only when your bent fingers point north.) The magnitude of **B**, from Eq. 20–3 with $\theta = 90°$, is

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14}\,\text{N}}{(1.6 \times 10^{-19}\,\text{C})(5.0 \times 10^6\,\text{m/s})} = 0.10\,\text{T}.$$

The path of a charged particle moving in a plane perpendicular to a uniform magnetic field is a circle (or the arc of a circle if the particle later passes out of the magnetic field region). See Fig. 20–17, where the magnetic field is directed *into* the paper, as represented by ×'s. An electron at point P is moving to the right, and the force on it at this point is downward as shown (use the right-hand rule and reverse the direction for negative charge). The electron is thus deflected downward. A moment later, say when it reaches point Q, the force is still perpendicular to the velocity and is in the direction shown. Since the force is always perpendicular to **v**, the magnitude of **v** does not change but the particle changes direction and moves in a circular path, with a centripetal acceleration (we demonstrate this in Example 20–4). The force is directed toward the center of this circle at all points. Note that the electron moves clockwise in Fig. 20–17. A positive particle would feel a force in the opposite direction and would thus move counterclockwise.

FIGURE 20–17 Force exerted by a uniform magnetic field on a moving charged particle (in this case, an electron) produces a circular path.



EXAMPLE 20–4 **Electron's path in a uniform magnetic field.** An electron travels at 2.0×10^7 m/s in a plane perpendicular to a 0.010-T magnetic field. Describe its path.

SOLUTION The electron moves at constant speed in a curved path whose radius of curvature is found using Newton's second law, $F = ma$. We have a centripetal acceleration $a = v^2/r$ (Eq. 5–1). The force is given by Eq. 20–4, $F = qvB$, so we have

$$F = ma$$

$$qvB = \frac{mv^2}{r}.$$

We solve for r and find

$$r = \frac{mv}{qB}.$$

Since **F** is perpendicular to **v**, the magnitude of **v** doesn't change. From this equation we see that if **B** = constant, then r = constant, and the curve must be a circle as we claimed above.

To get r, we put in the numbers:

$$r = \frac{(9.1 \times 10^{-31}\,\text{kg})(2.0 \times 10^{7}\,\text{m/s})}{(1.6 \times 10^{-19}\,\text{C})(0.010\,\text{T})} = 1.1 \times 10^{-2}\,\text{m},$$

or 1.1 cm.

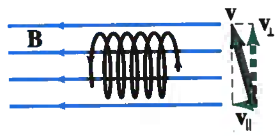


FIGURE 20–18 Example 20–5.

CONCEPTUAL EXAMPLE 20–5 **A spiral path.** What is the pa[...] charged particle if its velocity is *not* perpendicular to the magne[...]

RESPONSE The velocity vector can be broken down into com[...] parallel and perpendicular to the field. The velocity component [...] to the field lines experiences no force, and so this component [...] constant. The velocity component perpendicular to the field [...] circular motion about the field lines. Putting these two motions [...] gether produces a helical (spiral) motion around the field lines as [...] in Fig. 20–18.

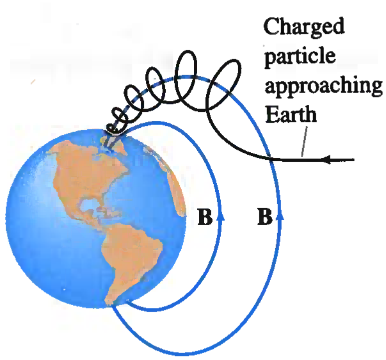
PHYSICS APPLIED

The aurora borealis

CONCEPTUAL EXAMPLE 20–6 **Aurora borealis.** Charged [...] proach the Earth from the Sun (the "solar wind") and are drawn [...] the poles, sometimes causing a phenomenon called the *aurora bo[...]* "northern lights" in northern latitudes. Why toward the poles?

RESPONSE A glance at Fig. 20–19 (see also Fig. 20–18) provi[...] answer. Imagine a stream of charged particles approaching the [...] shown. The velocity component perpendicular to the field for each [...] cle becomes a circular orbit around the field lines, whereas the ve[...] component parallel to the field carries the particle along the field lin[...] ward the poles. The high concentration of charged particles ioniz[...] air, and the recombining of electrons with atoms emits light (Chap[...] which is the aurora, especially during periods of high sun spot activity [...] the solar wind is greater.

FIGURE 20–19 (a) Diagram showing a charged particle approaching the Earth which is "captured" by the magnetic field of the Earth. Such particles follow the field lines toward the poles as shown. (b) Photo of aurora borealis.



(a)

(b)

20–5 Magnetic Field Due to a Straight Wire

We saw in Section 20–2, Fig. 20–10, that the magnetic field due to the electric current in a long straight wire is such that the field lines are circles with the wire at the center (Fig. 20–20). You might expect that the field strength at a given point would be greater if the current flowing in the wire were greater; and that the field would be less at points farther from the wire. This is indeed the case. Careful experiments show that the magnetic field B at a point near a long straight wire is directly proportional to the current I in the wire and inversely proportional to the distance r from the wire:

$$B \propto \frac{I}{r}.$$

This relation is valid as long as r, the perpendicular distance to the wire, is much less than the distance to the ends of the wire (i.e., the wire is long).

The proportionality constant is written[†] as $\mu_0/2\pi$; thus,

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}. \qquad \text{[outside a long straight wire]} \qquad \textbf{(20–5)}$$

Magnetic field due to current in straight wire

The value of the constant μ_0, which is called the **permeability of free space**, is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A}$.

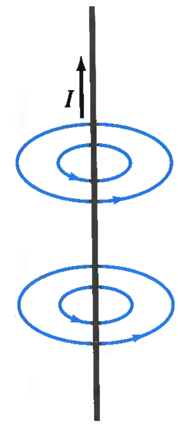


FIGURE 20–20 Same as Fig. 20–10a, magnetic field lines around a long straight wire carrying an electric current I.

EXAMPLE 20–7 Calculation of B near a wire. A vertical electric wire in the wall of a building carries a dc current of 25 A upward. What is the magnetic field at a point 10 cm due north of this wire (Fig. 20–21)?

SOLUTION According to Eq. 20–5:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\,\mathrm{T{\cdot}m/A})(25\,\mathrm{A})}{(2\pi)(0.10\,\mathrm{m})} = 5.0 \times 10^{-5}\,\mathrm{T},$$

or 0.50 G. By the right-hand rule (Fig. 20–10b), the field points to the west (into the page in Fig. 20–21) at this point. Since this field has about the same magnitude as Earth's, a compass would not point north but in a northwesterly direction.

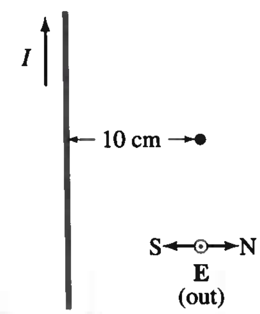


FIGURE 20–21 Example 20–7.

PHYSICS APPLIED

A compass, near a current, may not point North

20–6 Force Between Two Parallel Wires

We have seen that a wire carrying a current produces a magnetic field (magnitude given by Eq. 20–5 for a long straight wire), and furthermore that such a wire feels a force when placed in a magnetic field (Section 20–3, Eq. 20–1). Thus, we expect that two current-carrying wires would exert a force on each other.

[†] The constant is chosen in this complicated way so that Ampère's law (Section 20–8), which is considered more fundamental, will have a simple and elegant form.

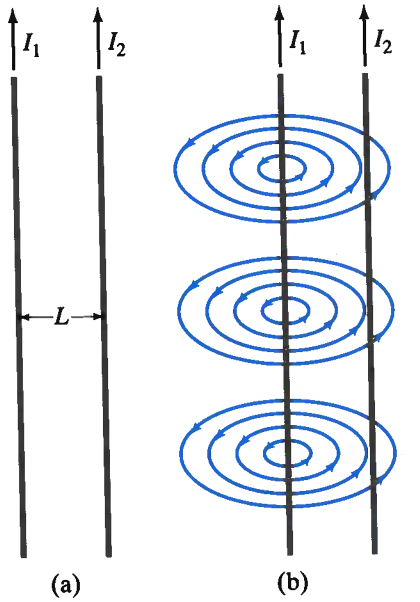
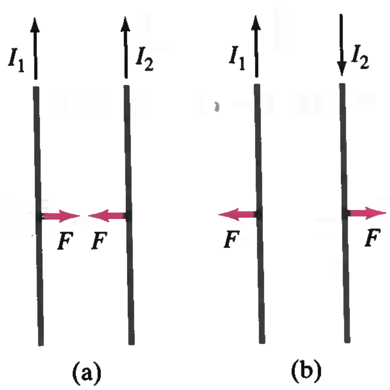


FIGURE 20–22 (a) Two parallel conductors carrying currents I_1 and I_2. (b) Magnetic field produced by I_1. (Field produced by I_2 is not shown.)

FIGURE 20–23 (a) Parallel currents in the same direction exert attractive force on each other. (b) Antiparallel currents (in opposite directions) exert repulsive force on each other.



Consider two long parallel conductors separated by a distance L, as in Fig. 20–22a. They carry currents I_1 and I_2, respectively. Each current produces a magnetic field that is "felt" by the other so that each must exert a force on the other, as Ampère first pointed out. For example, the magnetic field B_1 produced by I_1 is given by Eq. 20–5. At the location of the second conductor, the magnitude of this field is

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{L}.$$

See Fig. 20–22b where the field due *only* to I_1 is shown. According to Eq. 20–2, the force F per unit length l on the conductor carrying current I_2 is

$$\frac{F}{l} = I_2 B_1.$$

Note that the force on I_2 is due only to the field produced by I_1. Of course I_2 also produces a field, but it does not exert a force on itself. We substitute in the above formula for B_1 and find

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{L}.$$

If we use the right-hand rule of Fig. 20–10b, we see that the lines of B_1 are as shown in Fig. 20–22b. Then using the right-hand rule of Fig. 20–11 we see that the force exerted on I_2 will be to the left in the figure. That is, I_1 exerts an attractive force on I_2 (Fig. 20–23a). This is true as long as the currents are in the same direction. If I_2 is in the opposite direction, the right-hand rule indicates that the force is in the opposite direction. That is, I_1 exerts a repulsive force on I_2 (Fig. 20–23b). Reasoning similar to that above shows that the magnetic field produced by I_2 exerts an equal but opposite force on I_1. We expect this to be true also, of course, from Newton's third law. Thus, as shown in Fig. 20–23, parallel currents in the same direction attract each other, whereas parallel currents in opposite directions repel.

EXAMPLE 20–8 **Force between two current carrying wires.** The two wires of a 2.0-m-long appliance cord are 3.0 mm apart and carry a current of 8.0 A dc. Calculate the force between these wires.

SOLUTION Equation 20–6 gives us

$$F = \frac{(2.0 \times 10^{-7}\,\mathrm{T{\cdot}m/A})(8.0\,\mathrm{A})^2(2.0\,\mathrm{m})}{(3.0 \times 10^{-3}\,\mathrm{m})} = 8.5 \times 10^{-3}\,\mathrm{N}$$

where we have writen $\mu_0/2\pi = 2.0 \times 10^{-7}\,\mathrm{T{\cdot}m/A}$. Since the currents are in opposite directions, the force would tend to spread them apart.

EXAMPLE 20–9 **Suspending a current with a current.** A horizontal wire carries a current $I_1 = 80\,\text{A}$ dc. A second parallel wire 20 cm below it (Fig. 20–24) must carry how much current I_2 so that it doesn't fall due to gravity? The lower wire has a mass of 0.12 g per meter of length.

SOLUTION The force of gravity on the lower wire is downward and for each meter of length has magnitude

$$\frac{F}{l} = \frac{mg}{l} = \frac{(0.12 \times 10^{-3}\,\text{kg})(9.8\,\text{m/s}^2)}{1.0\,\text{m}} = 1.18 \times 10^{-3}\,\text{N/m}.$$

The magnetic force on wire 2 must be upward (hence I_2 must have the same direction as I_1) and with $L = 0.20\,\text{m}$ and $I_1 = 80\,\text{A}$ has magnitude

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{L}.$$

We solve for I_2 and find

$$I_2 = \frac{2\pi L}{\mu_0 I_1}\left(\frac{F}{l}\right) = \frac{2\pi\,(0.20\,\text{m})}{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})(80\,\text{A})}(1.18 \times 10^{-3}\,\text{N/m}) = 15\,\text{A}.$$

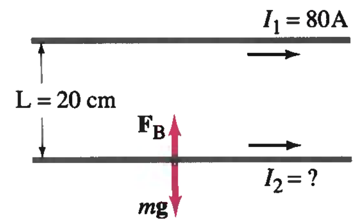


FIGURE 20–24 Example 20–9.

20–7 Definition of the Ampere and the Coulomb

You may have wondered how the constant μ_0 in Eq. 20–5 could be exactly $4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$. Here is how it happened. With an older definition of the ampere, μ_0 was measured experimentally to be very close to this value. Today, however, μ_0 is *defined* to be exactly $4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A}$. This, of course, could not be done if the ampere were defined independently. The ampere, the unit of current, is now defined in terms of the magnetic field it produces using the defined value of μ_0.

In particular, we use the force between two parallel current-carrying wires, Eq. 20–6, to define the ampere precisely. If $I_1 = I_2 = 1\,\text{A}$ exactly, and the two wires are exactly 1 m apart, then

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7}\,\text{T}\cdot\text{m/A})}{(2\pi)}\frac{(1\,\text{A})(1\,\text{A})}{(1\,\text{m})} = 2 \times 10^{-7}\,\text{N/m}.$$

Thus, *one* **ampere** *is defined as that current flowing in each of two long parallel conductors 1 m apart, which results in a force of exactly 2×10^{-7} N/m of length of each conductor.*

Definitions of ampere and coulomb

This is the precise definition of the ampere. The **coulomb** is then defined as being *exactly* one ampere-second: $1\,\text{C} = 1\,\text{A}\cdot\text{s}$. The value of k or in Coulomb's law (Section 16–5) is obtained from experiment.

*20–8 Ampère's Law

In Section 20–5, we saw that Eq. 20–5 gives the relation between the current in a long straight wire and the magnetic field it produces. This equation is valid only for a long straight wire. The following important question arises: Is there a general relation between a current in a wire of whatever shape and the magnetic field around it? The answer is yes: the French scientist André Marie Ampère (1775–1836) proposed such a relation shortly after Oersted's discovery. Consider any (arbitrary) closed path ar... current, as shown in Fig. 20–25, and imagine this path as being mad... short segments each of length Δl. First, we take the product of the ... of each segment times the component of **B** parallel to that segment. I... now sum all these terms, according to Ampère, the result will be e... μ_0 times the net current I that passes through the surface enclosed b... path. This is known as **Ampère's law** and can be written mathemati...

AMPÈRE'S LAW

$$\sum B_{\parallel}\,\Delta l = \mu_0 I.$$

The symbol Σ means "the sum of" and $B_{\parallel}$ means the component of B ... lel to that particular Δl. The lengths Δl are chosen so that $B_{\parallel}$ is e... constant on each length. The sum must be made over a closed path an... the net current passing through the surface bounded by this closed p...

We can check Ampère's law by applying it to the simple case of ... straight wire carrying a current I, which we have already examin... which served as an inspiration for Ampère himself. Suppose that we w... find the magnitude of B at point A, a distance r from the wire in Fig... We know that the magnetic field lines are circles with the wire at th... ter. We then choose a path to be used in Eq. 20–7: we choose a circl... dius r (the choice of path is ours—so we choose one that will be conv... We choose this circular path because at any point on this path, **B** will b... gent to this circle. Thus, for any short segment of the circle (Fig. 20... will be parallel to that segment, so $B_{\parallel} = B$. Suppose that we break th... lar path down into 100 segments.† Then Ampère's law states that

$$(B\,\Delta l)_1 + (B\,\Delta l)_2 + (B\,\Delta l)_3 + \cdots + (B\,\Delta l)_{100} = \mu_0 I$$

The dots represent all the terms we did not write down. Since all th... ments are the same distance from the wire, we expect B to be the ... each segment. We can then factor out B from the sum:

$$B(\Delta l_1 + \Delta l_2 + \Delta l_3 + \cdots + \Delta l_{100}) = \mu_0 I.$$

The sum of the segment lengths is just the circumference of the circl... Thus we have

$$B(2\pi r) = \mu_0 I,$$

or

$$B = \frac{\mu_0 I}{2\pi r}.$$

This is just Eq. 20–5 for the field near a long straight wire, as discussed ...

Ampère's law thus works for this simple case. A great many ... ments indicate that Ampère's law is valid in general. However, it ... used to calculate the magnetic field mainly for simple cases. Its ... tance is that it relates the magnetic field to the current in a dir... mathematically elegant way. Ampère's law is thus considered one ... basic laws of electricity and magnetism. It is valid for any situation ... the currents and fields are not changing in time.

We now can see why the constant in Eq. 20–5 is written $\mu_0/2\pi$ ... done so that only μ_0 appears in Eq. 20–7 (rather than, say, $2\pi k$ if we ... used k in Eq. 20–5). In this way, the more fundamental equation, Am... père's law, has the simpler form.

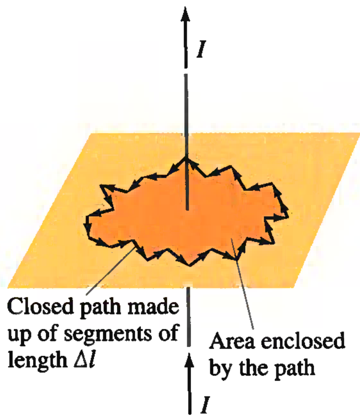


FIGURE 20–25 Arbitrary path enclosing a current, for Ampère's law. The path is broken down into segments of equal length Δl.

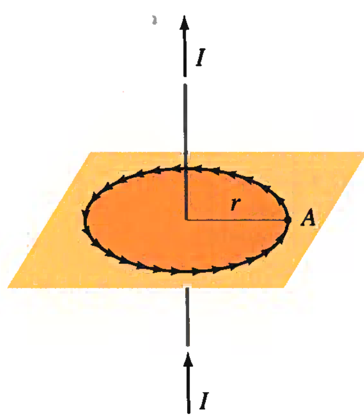


FIGURE 20–26 Circular path of radius r.

†Actually, Ampère's law is precisely accurate when there is an infinite number of inf... mally short segments, but that leads into calculus.

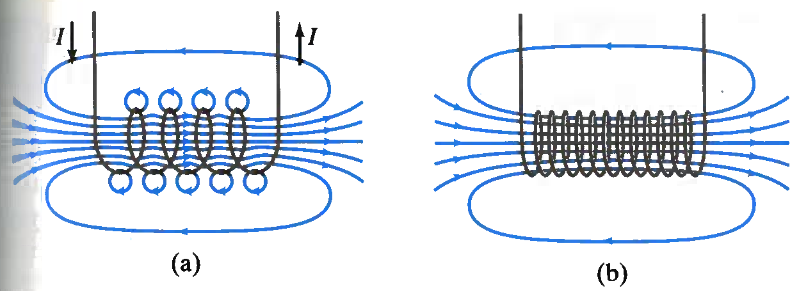


FIGURE 20–27 (a) Magnetic field due to several loops of a solenoid. (b) If the coils are closely spaced, the field is very nearly uniform.

We now use Ampère's law to calculate the magnetic field inside a long coil of wire with many loops, as shown in Fig. 20–27, which is known as a solenoid. Each coil produces a magnetic field as shown in Fig. 20–11, and the total field inside the solenoid will be the sum of the fields due to each current loop as shown in Fig. 20–27a. If the coils of the solenoid are very closely spaced, the field inside will be essentially parallel to the axis except at the ends, as shown in Fig. 20–27b. For applying Ampère's law, we choose the path *abcd* shown in Fig. 20–28, far from either end. We will consider this path as made up of four segments, the sides of the rectangle: *ab*, *bc*, *cd*, *da*. Then the left side of Eq. 20–7 becomes

$$(B_{\parallel}\,\Delta l)_{ab} + (B_{\parallel}\,\Delta l)_{bc} + (B_{\parallel}\,\Delta l)_{cd} + (B_{\parallel}\,\Delta l)_{da}.$$

The first term in this sum will be very small since the field outside the solenoid is so small as to be negligible compared to the field inside (the same number of lines inside the solenoid spread throughout space outside). Thus the first term will be zero. Furthermore, **B** is perpendicular to the segments *bc* and *da*, so these terms are zero, too. Therefore, the left side of Eq. 20–7 is simply $(B_{\parallel}\,\Delta l)_{cd} = Bl$, where B is the field inside the solenoid, and l is the length *cd*. Now we determine the current enclosed by our chosen rectangular loop, to use for the right side of Eq. 20–7. If a current I flows in the wire of the solenoid, the total current enclosed by our path *abcd* is NI, where N is the number of loops our path encircles (five in Fig. 20–28). Thus Ampère's law gives us

$$Bl = \mu_0 NI.$$

If we let $n = N/l$ be the *number of loops per unit length*, then

$$B = \mu_0 nI. \qquad \text{[solenoid]} \qquad \textbf{(20–8)}$$

Magnetic field inside a solenoid

This is the magnitude of the magnetic field within a solenoid. Note that B depends only on the number of loops per unit length, n, and the current I. The field does not depend on the position within the solenoid, so B is uniform. This is strictly true only for an infinite solenoid, but it is a good approximation for real ones for points not close to the ends.

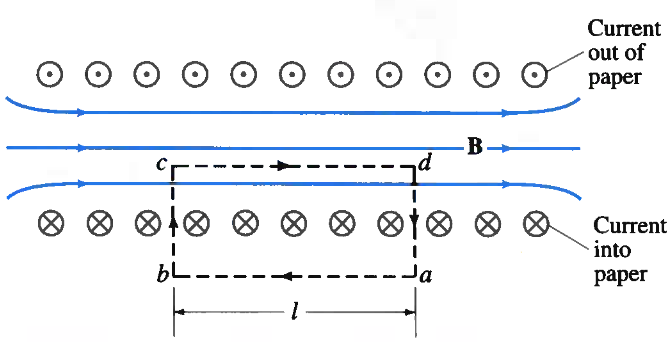


FIGURE 20–28 Magnetic field inside a solenoid is straight except at the ends. Dashed lines indicate the path chosen for use in Ampère's law.

EXAMPLE 20–10 **Field inside a solenoid.** A thin 10-cm-long solenoid has a total of 400 turns of wire and carries a current of 2.0 A. Calculate the field inside near the center.

SOLUTION The number of turns per unit length is $n = 400/0.10\text{ m} =$ $4.0 \times 10^3\,\text{m}^{-1}$. From Eq. 20–8:

$$B = \mu_0 nI = (12.57 \times 10^{-7}\,\text{T}\cdot\text{m/A})(4.0 \times 10^3\,\text{m}^{-1})(2.0\,\text{A})$$
$$= 1.0 \times 10^{-2}\,\text{T}.$$

➡ **PHYSICS APPLIED**

Coaxial cable (shielding)

CONCEPTUAL EXAMPLE 20–11 **Coaxial cable.** A *coaxial cable* is a single wire surrounded by a cylindrical metallic braid, as shown in Fig. 20–29. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (*a*) in the space between the conductors, and (*b*) outside the cable.

RESPONSE (*a*) In the space between the conductors, we can apply Ampère's law for a circular path around the center wire, just as we did for the case shown in Fig. 20–26 and the magnitude is as given by Eq. 20–5. The current in the outer conductor has no bearing on this result. (Ampère's law uses only the current enclosed *inside* the path; as long as the currents outside the path don't affect the symmetry of the field, they do not contribute to the field along the path at all).

(*b*) Outside the cable, we can draw a similar circular path, for we expect the field to have the same circular symmetry. Now, however, there are two currents enclosed by the path, and they add up to zero. The field outside the cable is zero.

The nice feature of coaxial cables is that they are self-shielding: no stray magnetic fields escape outside the cable. The outer cylindrical conductor also shields external electric fields from coming in (see also Example 16–…). This makes them ideal for carrying signals near sensitive equipment. Audiophiles use coaxial cables between stereo equipment components and even to the loudspeakers.

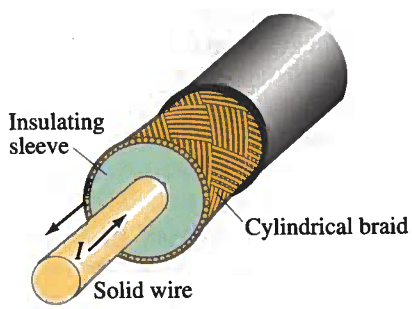


FIGURE 20–29 Coaxial cable. Example 20–11.

*20–9 Torque on a Current Loop; Magnetic Moment

When an electric current flows in a closed loop of wire placed in a magnetic field, as shown in Fig. 20–30, the magnetic force on the current can produce a torque. This is the basic principle behind a number of important practical devices, including meters and motors. (We discuss these applications in the next Section.) The interaction between a current and a magnetic field is important in other areas as well, including atomic physics.

When current flows through the loop in Fig. 20–30a, whose face we assume is parallel to **B** and is rectangular, the magnetic field exerts a force on both vertical sections of wire as shown, $\mathbf{F}_1$ and $\mathbf{F}_2$ (see also top view, Fig. 20–30b). Notice that, by the right-hand rule (Fig. 20–13c), the direction of the force on the upward current on the left is in the opposite direction from the equal magnitude force $\mathbf{F}_2$ on the descending current on the right. These forces give rise to a net torque that tends to rotate the coil about its vertical axis.

Let us calculate the magnitude of this torque. From Eq. 20–2, the force $F = IaB$, where a is the length of the vertical arm of the coil. The lever arm for each force is $b/2$, where b is the width of the coil and the "axis" is at the midpoint. The total torque is the sum of the torques due to each of the forces, so

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB,$$

where $A = ab$ is the area of the coil. If the coil consists of N loops of wire, the current is then NI, so the torque becomes

$$\tau = NIAB. \qquad \textbf{(20–9a)}$$

If the coil makes an angle θ with the magnetic field, as shown in Fig. 20–30c, the forces are unchanged, but each lever arm is reduced from $\frac{1}{2}b$ to $\frac{1}{2}b \sin\theta$. Note that the angle θ is chosen to be the angle between **B** and the perpendicular to the face of the coil, Fig. 20–30c. So the torque becomes

$$\tau = NIAB \sin\theta. \qquad \textbf{(20–9b)}$$

This formula, derived here for a rectangular coil, is valid for any shape of flat coil.

The quantity NIA is called the **magnetic dipole moment** of the coil and is considered a vector:

$$\mathbf{M} = NI\mathbf{A}, \qquad \textbf{(20–10)}$$

where the direction of **A** (and therefore of **M**) is *perpendicular* to the plane of the coil (the black arrow in Fig. 20–30c).

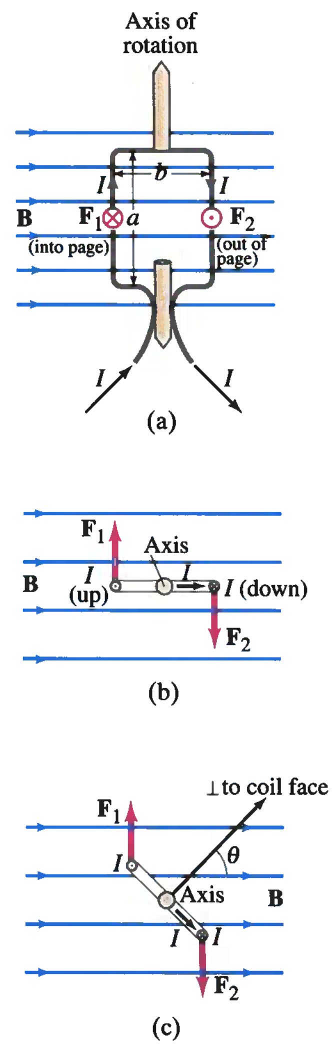


FIGURE 20–30 Calculating the torque on a current loop in a magnetic field **B**. (a) Loop face parallel to **B** field lines; (b) Top view; (c) Loop makes an angle to **B**, reducing the torque since the lever arm is reduced.

EXAMPLE 20–12 **Torque on a coil.** A circular coil of wire h... eter of 20.0 cm and contains 10 loops. The current in each loop i... and the coil is placed in a 2.00-T magnetic field. Determine th... mum and minimum torque exerted on the coil by the field.

SOLUTION Equation 20–9 is valid for any shape of coil, includ... cular, where the area is

$$A = \pi r^2 = \pi(0.100\text{ m})^2 = 3.14 \times 10^{-2}\text{ m}^2.$$

The maximum torque occurs when the coil's face is parallel to th... netic field, so $\theta = 90°$ in Fig. 20–30c, and $\sin\theta = 1$ in Eq. 20–9b

$$\tau = NIAB\sin\theta = (10)(3.00\text{ A})(3.14 \times 10^{-2}\text{ m}^2)(2.00\text{ T})(1) = \ldots$$

The minimum torque occurs if $\sin\theta = 0$, for which $\theta = 0°$, and th... from Eq. 20–9b.

* 20–10 Applications: Galvanometers, Motors, Loudspeakers

PHYSICS APPLIED

Galvanometer

The basic component of most meters, including ammeters, voltmeters, ... ohmmeters, is a galvanometer. We have already seen how these met... designed (Section 19–10), and now we can examine how the cru... ment, a galvanometer, itself works. As shown in Fig. 20–31, a **galvano...** consists of a coil of wire (with attached pointer) suspended in the ma... ic field of a permanent magnet. When current flows through the lo... wire, which is usually rectangular, the magnetic field exerts a torque o... loop, as given by Eq. 20–9b, $\tau = NIAB\sin\theta$. This torque is oppos... spring which exerts a torque τ_s approximately proportional to the an... through which it is turned (Hooke's law). That is,

$$\tau_s = k\phi,$$

where k is the stiffness constant of the spring. Thus the coil and th... tached pointer will rotate only to the point where the spring torque ba... ances the torque due to the magnetic field. From Eq. 20–9b we then ha... $k\phi = NIAB\sin\theta$ or

$$\phi = \frac{NIAB\sin\theta}{k}.$$

Thus the deflection of the pointer, ϕ, is directly proportional to the cu... rent I flowing in the coil. But it also depends on the angle θ the coil m... with **B**. For a useful meter we need ϕ to depend only on I, independ... θ. To solve this problem, curved pole pieces are used and the galvano... coil is wrapped around a cylindrical iron core as shown in Fig. 20–32. Th... iron tends to concentrate the magnetic field lines so that **B** always p... parallel to the face of the coil at the wire outside the core. The f... then always perpendicular to the face of the coil and the torque will ... vary with angle. Thus ϕ will be proportional to I, as required.

A **chart recorder**, in which a pen graphs a signal such as an E... (electrocardiogram–Section 17–11) on a moving roll of paper, is ba...

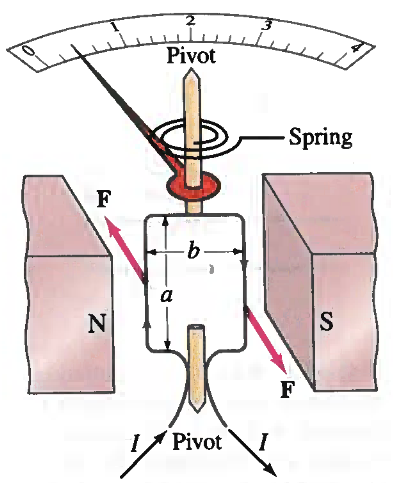
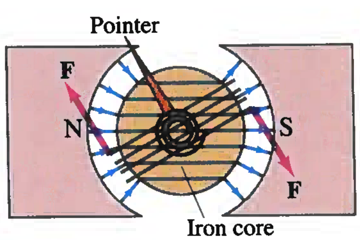


FIGURE 20–31 Galvanometer.

FIGURE 20–32 Galvanometer coil wrapped on an iron core.



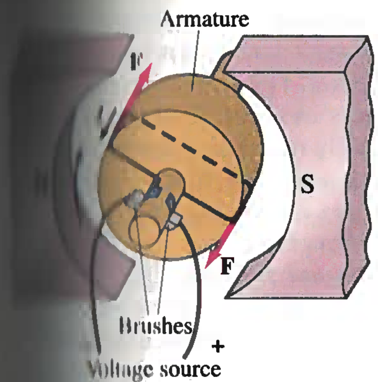


FIGURE 20–33 Diagram of a simple dc motor.

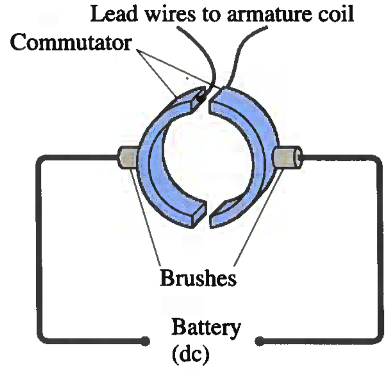


FIGURE 20–34 The commutator-brush arrangement in a dc motor assures alternation of the current in the armature to keep rotation continuous. The commutators are attached to the motor shaft and turn with it, whereas the brushes remain stationary.

galvanometer. The pen is attached to an arm, which is connected to the galvanometer coil. The instrument could record either voltage or current, just as any galvanometer can be connected as a voltmeter or ammeter.

Electric motor

An **electric motor** changes electric energy into (rotational) mechanical energy. A motor works on the same principle as a galvanometer, except that there is no spring so the coil can rotate continuously in one direction. The coil is larger and is mounted on a large cylinder called the **rotor** or armature, Fig. 20–33. Actually, there are several coils, although only one is indicated in the figure. The armature is mounted on a shaft or axle. At the moment shown in Fig. 20–33, the magnetic field exerts forces on the current in the loop as shown. However, when the coil, which is rotating clockwise in Fig. 20–33, passes beyond the vertical position the forces would then act to return the coil back to vertical if the current remained the same. But if the current could somehow be reversed at that critical moment, the forces would reverse, and the coil would continue rotating in the same direction. Thus, alternation of the current is necessary if a motor is to turn continuously in one direction. This can be achieved in a **dc** motor with the use of **commutators** and **brushes**: as shown in Fig. 20–34, brushes are stationary contacts that rub against the conducting commutators mounted on the motor shaft. At every half revolution, each commutator changes its connection to the other brush. Thus the current in the coil reverses every half revolution as required for continuous rotation. Most motors contain several coils, called "windings," each located in a different place on the armature, Fig. 20–35. Current flows through each coil only during a small part of a revolution, at the time when its orientation results in the maximum torque. In this way, a motor produces a much smoother torque than can be obtained from a single coil. An **ac motor**, with ac current as input, can work without commutators since the current itself alternates. Many motors use wire coils to produce the magnetic field (electromagnets) instead of a permanent magnet. Indeed the design of most practicial motors is more complex than described here, but the general principles remain the same.

➥ PHYSICS APPLIED

DC motor

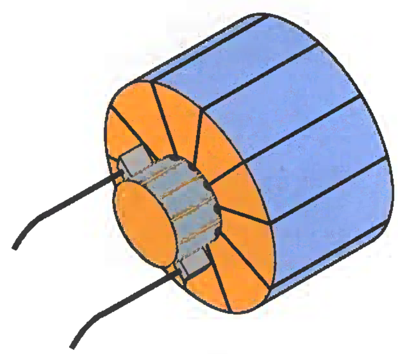


FIGURE 20–35 Motor with many windings.

➥ PHYSICS APPLIED

AC motor

PHYSICS APPLIED

Loudspeakers

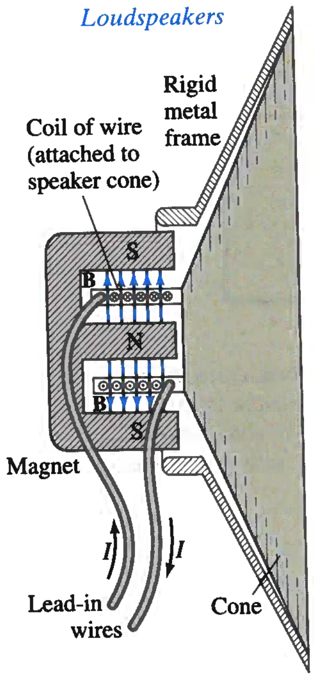


FIGURE 20–36
Loudspeaker.

A **loudspeaker** also works on the principle that a magnet ex... force on a current-carrying wire. The electrical output of a radio or ... is connected to the wire leads of the speaker. The speaker leads are ... nected internally to a coil of wire, which is itself attached to the ... cone, Fig. 20–36. The speaker cone is usually made of stiffened card... and is mounted so that it can move back and forth freely. A perm... magnet is mounted directly in line with the coil of wire. When the ... nating current of an audio signal flows through the wire coil, the ... the attached speaker cone experience a force due to the magnetic ... the magnet. As the current alternates at the frequency of the audio ... the speaker cone moves back and forth at the same frequency, ca... ternate compressions and rarefactions of the adjacent air, and ... waves are produced. A speaker thus changes electrical energy in... energy, and the frequencies and intensities of the emitted sound w... be an accurate reproduction of the electrical input.

* 20–11 The Hall Effect

When a current-carrying conductor is held firmly in a magnetic fi... field exerts a sideways force on the charges moving in the condu... example, if electrons move to the right in the rectangular conduct... in Fig. 20–37a, the inward magnetic field will exert a downward f... the electrons $F_B = ev_dB$, where v_d is the drift velocity of the el... (Section 18–9). So the electrons will tend to move nearer face S th... R. There will thus be a potential difference between faces R and ... conductor. This potential difference builds up until the electric field ... produces exerts a force, eE_H, on the moving charges that is equal ... posite to the magnetic force. This effect is called the **Hall effect** aft... Hall, who discovered it in 1879. The difference of potential produ... called the **Hall emf**.

The electric field due to the separation of charge is called the ... *field*, $\mathbf{E}_H$, and points downward in Fig. 20–37a, as shown. In equilibri... force due to this electric field is balanced by the magnetic force ev_dB ...

$$eE_H = ev_dB.$$

Hence $E_H = v_dB$. The Hall emf is then (assuming the conductor ... and thin so E_H is uniform)

$$\mathscr{E}_H = E_Hl = v_dBl,$$

where l is the width of the conductor.

A current of negative charges moving to the right is equivalent to p... itive charges moving to the left, at least for most purposes. But the H... fect can distinguish these two. As can be seen in Fig. 20–37b, p... particles moving to the left are deflected downward, so that the b... surface is positive relative to the top surface. This is the reverse of p... Indeed, the direction of the emf in the Hall effect first revealed th... negative particles that move in metal conductors. In some semicond... however, the Hall effect reveals that the carriers of current are p... (more on this in Chapter 29).

The magnitude of the Hall emf is proportional to the strength of ... magnetic field. The Hall effect can thus be used to measure magnet...

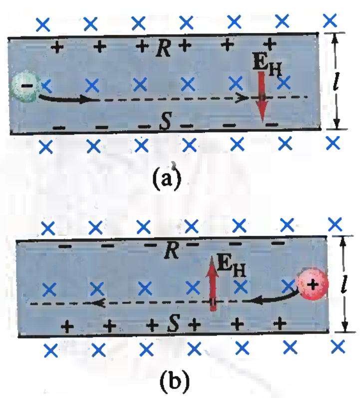


FIGURE 20–37 The Hall effect. (a) Negative charges moving to the right as the current. (b) Positive charges moving to the left as the current.

lengths. First the conductor, called a *Hall probe*, is calibrated with known magnetic fields. Then, for the same current, its emf output will be a measure of B. Hall probes can be made very small and are convenient and accurate to use.

The Hall effect can also be used to measure the drift velocity of charge carriers when the external magnetic field B is known. Such a measurement also allows us to determine the density of charge carriers in the material.

EXAMPLE 20–13 Drift velocity using the Hall effect. A long copper strip 1.8 cm wide and 1.0 mm thick is placed in a 1.2-T magnetic field as in Fig. 20–37a. When a steady current of 15 A passes through it, the Hall emf is measured to be 1.02 μV. Determine the drift velocity of the electrons and the density of free (conducting) electrons (number per unit volume) in the copper.

SOLUTION The drift velocity (Eq. 20–11) is

$$v_d = \frac{\mathscr{E}_H}{Bl} = \frac{1.02 \times 10^{-6}\,\text{V}}{(1.2\,\text{T})(1.8 \times 10^{-2}\,\text{m})} = 4.7 \times 10^{-5}\,\text{m/s}.$$

The density of charge carriers n is obtained from Eq. 18–10, $I = nev_dA$, where A is the cross-sectional area through which the current I flows. Then

$$n = \frac{I}{ev_dA} = \frac{15\,\text{A}}{(1.6 \times 10^{-19}\,\text{C})(4.7 \times 10^{-5}\,\text{m/s})(1.8 \times 10^{-2}\,\text{m})(1.0 \times 10^{-3}\,\text{m})}$$
$$= 11 \times 10^{28}\,\text{m}^{-3}.$$

This value for the density of free electrons in copper, $n = 11 \times 10^{28}$ per m^3, is the experimentally measured value. It represents *more* than one free electron per atom, which as we saw in Example 18–13 is $8.4 \times 10^{28}\,\text{m}^{-3}$.

*20–12 Mass Spectrometer

Various methods were developed in the early part of this century to measure the masses of atoms. One of the most accurate was the **mass spectrometer**† of Fig. 20–38. Ions are produced by heating, or by an electric current, in the source S. Those that pass through slit S_1 enter a region where there are both electric and magnetic fields: the magnetic field points out of the page in Fig. 20–38, and the electric field points up (from the + plate toward the − plate). Ions will follow a straight-line path in this region, as shown, if the electric force qE (upward on a positive ion) is just balanced by the magnetic force qvB (downward on a positive ion): that is, if

$$qE = qvB$$

$$v = \frac{E}{B}.$$

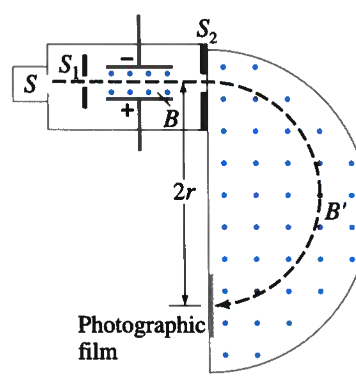
In other words, those ions (and only those) whose speed is $v = E/B$ will pass through undeflected and emerge through slit S_2. (This arrangement is called a *velocity selector*.) In the second region, after S_2, there is only a

† The term *mass spectrograph* is also used.

➡ **PHYSICS APPLIED**

The mass spectrometer

FIGURE 20–38 Bainbridge mass spectrometer. The magnetic fields B and B' point out of the paper (indicated by the dots).



magnetic field B' so the ions follow a circular path. The radius of the path can be measured because the ions darken the photographic film where they strike. Since $qvB' = mv^2/r$ and $v = E/B$, then we have

$$m = \frac{qB'r}{v} = \frac{qBB'r}{E}.$$

All the quantities on the right can be measured, and thus m can be determined. Note that for ions of the same charge, the mass of each is proportional to the radius of its path.

The masses of many atoms were measured in this way. When a pure substance was used, it was sometimes found that two or more closely spaced marks would appear on the film. For example, neon produced marks whose radii corresponded to atoms of mass 20 and 22 atomic mass units (u). Impurities were ruled out and it was concluded that there must be two types of neon with different masses. These different forms were called **isotopes.** It was soon found that most elements are mixtures of isotopes. We shall see in Chapter 30 that the difference in mass is due to different numbers of neutrons.

Mass spectrometers can be used to separate not only different elements and isotopes, but different molecules as well. They are used in physics, chemistry, and in biological and biomedical laboratories.

EXAMPLE 20–14 **Mass spectrometry.** Carbon atoms of atomic mass 12.0 u are found to be mixed with another, unknown, element. In a mass spectrometer, the carbon traverses a path of radius 22.4 cm and the unknown's path has a 26.2 cm radius. What is the unknown element? Assume they have the same charge.

SOLUTION Since mass is proportional to the radius, we have

$$\frac{m_x}{m_C} = \frac{26.2\text{ cm}}{22.4\text{ cm}} = 1.17.$$

Thus $m_x = 1.17 \times 12.0\text{ u} = 14.0\text{ u}$. The other element is probably nitrogen (see the periodic table, inside the back cover). However, it could also be an isotope of carbon or oxygen. Further physical or chemical analysis would be needed.

* 20–13 Ferromagnetism; Domains

We saw in Section 20–1 that iron (and a few other materials) can be made into strong magnets. These materials are said to be **ferromagnetic**. We now look more deeply into the sources of ferromagnetism.

A bar magnet, with its two opposite poles at either end, resembles an electric dipole (equal-magnitude positive and negative charges separated by a distance). Indeed, a bar magnet is sometimes referred to as a "magnetic dipole." There are opposite "poles" separated by a distance. And the magnetic field lines of a bar magnet form a pattern much like that for the electric field of an electric dipole: compare Fig. 16–29a with Fig. 20–

Domains in iron

Microscopic examination reveals that a magnet is actually made up of tiny regions known as **domains**, which are at most about 1 mm in length pole. Each domain behaves like a tiny magnet with a north and a south pole. In an unmagnetized piece of iron, these domains are arranged randomly, as shown in Fig. 20–39a. The magnetic effects of the domains cancel each other out, so this piece of iron is not a magnet. In a magnet, the domains are preferentially aligned in one direction as shown in Fig. 20–39b (downward in this case). A magnet can be made from an unmagnetized piece of iron by placing it in a strong magnetic field. (You can make a needle magnetic, for example, by stroking it with one pole of a strong magnet.) Careful observations show in this case that the magnetization of domains may actually rotate slightly so as to be more nearly parallel to the external field. Or, more commonly, the borders of domains move so that those domains whose magnetic orientation is parallel to the external field grow in size at the expense of other domains. This can be seen by comparing Fig. 20–39a and b. This explains how a magnet can pick up unmagnetized pieces of iron like paper clips or bobby pins. The magnet's field causes a slight alignment of the domains in the unmagnetized object so that the object becomes a temporary magnet with its north pole facing the south pole of the permanent magnet, and vice versa; thus, attraction results. In the same way, elongated iron filings will arrange themselves in a magnetic field just as a compass needle does, and will reveal the shape of the magnetic field, Fig. 20–40.

An iron magnet can remain magnetized for a long time, and thus it is referred to as a "permanent magnet." However, if you drop a magnet on the floor or strike it with a hammer, you may jar the domains into randomness. The magnet can thus lose some or all of its magnetism. Heating a magnet too can cause a loss of magnetism, for raising the temperature increases the random thermal motion of the atoms which tends to randomize the domains. Above a certain temperature known as the **Curie temperature** (1043 K for iron), a magnet cannot be made at all.†

There is a striking similarity between the fields produced by a bar magnet and by a loop of electric current or a solenoid (compare Fig. 20–3b with Figs. 20–11 and 20–27). This suggests that the magnetic field produced by a current may have something to do with ferromagnetism, an idea proposed by Ampère in the nineteenth century. According to modern atomic theory, the atoms that make up any material can be roughly visualized as containing electrons that orbit around a central nucleus. Since the electrons are charged, they constitute an electric current and therefore produce a magnetic field. But if there is no external field, the electron orbits in different atoms are arranged randomly, so the magnetic fields due to the many orbits in all the atoms in a material cancel out. However, electrons produce an additional magnetic field, almost as if they and their electric charge were spinning about their own axes. It is the magnetic field due to electron spin‡ that is believed to produce ferromagnetism.

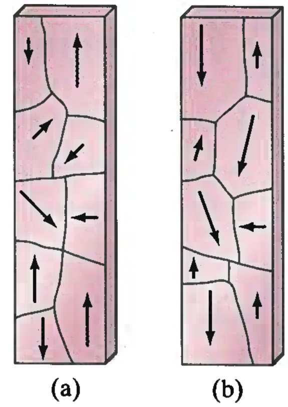
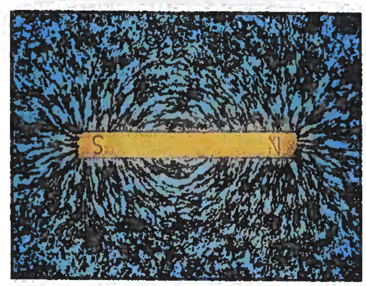


FIGURE 20–39 (a) An unmagnetized piece of iron is made up of domains that are randomly arranged. Each domain is like a tiny magnet; the arrows represent the magnetization direction, with the arrowhead being the N pole. (b) In a magnet, the domains are preferentially aligned in one direction, and may be altered in size by the magnetization process.

FIGURE 20–40 Iron filings line up along magnetic field lines.



† Iron, nickel, cobalt, gadolinium, and certain alloys are ferromagnetic at room temperature; several other elements and alloys have low Curie temperature and thus are ferromagnetic only at low temperatures.

‡ The name "spin" comes from the early suggestion that the additional magnetic field arises from the electron "spinning" on its axis (as well as "orbiting" the nucleus) and this additional motion of the charge was supposed to produce the extra field. However this view of a spinning electron is oversimplified: see Chapter 28.

In most materials, the magnetic fields due to electron spin cancel ... in iron and other ferromagnetic materials, a complicated ... mechanism seems to operate. The result is that the electrons ... to the ferromagnetism in a domain "spin" in the same direction ... tiny magnetic fields due to each of the electrons add up to give the ... netic field of a domain. And when the domains are aligned, ... seen, a strong magnet results.

It is believed possible today that *all* magnetic fields are ... electric currents. This would explain why no single magnetic pole has been found: there is no way to divide up a current and obtain a ... magnetic pole. Of course if an isolated pole is found, we will have to ... the idea that all magnetic fields are produced by currents.

The lack of single magnetic poles means that magnetic field ... closed loops, unlike electric field lines which begin on positive charges ... end on negative charges.

* 20–14 Electromagnets and Solenoids

A long coil of wire consisting of many loops of wire, as discussed in Section 20–8, is called a solenoid. The magnetic field within a solenoid ... fairly large since it will be the sum of the fields due to the current in ... loop (see Fig. 20–41). The solenoid acts like a magnet; one end can be considered the north pole and the other the south pole, depending on the direction of the current in the loops (use the right-hand rule). ... magnetic field lines leave the north pole of a magnet, the north pole of ... solenoid in Fig. 20–41 is on the right.

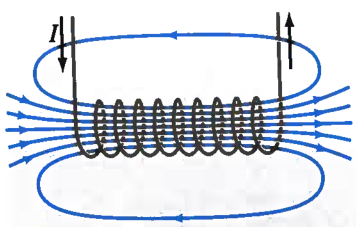


FIGURE 20–41 Magnetic field of a solenoid. The north pole of this solenoid, thought of as a magnet, is on the right, and the south pole is on the left.

➥ PHYSICS APPLIED

Electromagnets and solenoids

If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the domains of the iron are aligned by the magnetic field produced by the current. The resulting magnetic field is the ... of that due to the current and that due to the iron, and can be hundreds or thousands of times that due to the current alone (see Section 20–...). ... arrangement is called an **electromagnet**. The iron used in electromagnets acquires and loses its magnetism quite readily when the current is ... on or off, and so is referred to as "soft iron." (It is "soft" only in a ... ic sense.) Iron that holds its magnetism even when there is no external applied field is called "hard iron." Hard iron is used in permanent magnets. Soft iron is usually used in electromagnets so that the field ... turned on and off readily. Whether iron is hard or soft depends on ... treatment and other factors.

Electromagnets find use in many practical applications, from ... motors and generators to producing large magnetic fields for research. ... cause the current flows continuously, a great deal of waste heat ... power) is often produced. Cooling coils, which are tubes carrying ... must be used to absorb the heat in bigger installations. For some ... tions, superconducting magnets are coming into use. The current ... wires are made of superconducting material (Section 18–5) kept ... the transition temperature. No electric power is needed to maintain ... current, which means large savings of electricity. Of course, energy is ... ed to keep the superconducting coils at the necessary low temperature.

Another useful device consists of a solenoid into which a rod of iron is partially inserted. This combination is also referred to as a solenoid. One simple use is as a doorbell (Fig. 20–42). When the circuit is closed by pushing the button, the coil effectively becomes a magnet and exerts a force on the iron rod. The rod is pulled into the coil and strikes the bell. A larger solenoid is used in the starters of cars; when you engage the starter, you are closing a circuit that not only turns the starter motor, but activates a solenoid that first moves the starter into direct contact with the engine. Solenoids are used as switches in many other devices, such as tape recorders. They have the advantage of moving mechanical parts quickly and accurately.

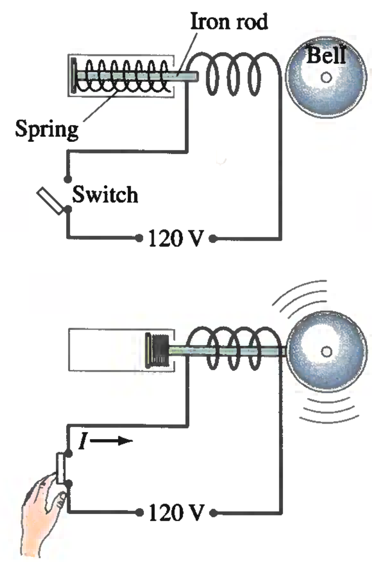


FIGURE 20–42 Solenoid used as a doorbell.

20–15 Magnetic Fields in Magnetic Materials; Hysteresis

The field of a long solenoid is directly proportional to the current. Indeed, Eq. 20–8 tells us that the field B_0 inside a solenoid is given by

$$B_0 = \mu_0 n I.$$

This is valid if there is only air inside the coil. If we put a piece of iron or other ferromagnetic material inside the solenoid, the field will be greatly increased, often by hundreds or thousands of times. This occurs because the domains in the iron become preferentially aligned by the external field. The resulting magnetic field is the sum of that due to the current and that due to the iron. It is sometimes convenient to write the total field in this case as a sum of two terms:

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_M. \tag{20–12}$$

Here, $\mathbf{B}_0$ refers to the field due only to the current in the wire (the "external field"). It is the field that would be present in the absence of a ferromagnetic material. Then $\mathbf{B}_M$ represents the additional field due to the ferromagnetic material itself; often $\mathbf{B}_M \gg \mathbf{B}_0$.

The total field inside a solenoid in such a case can also be written by replacing the constant μ_0 in Eq. 20–8 by another constant, μ, characteristic of the material inside the coil:

$$B = \mu n I; \tag{20–13}$$

μ is called the **magnetic permeability** of the material. For ferromagnetic materials, μ is much greater than μ_0. For all other materials, its value is very close to μ_0.† The value of μ, however, is not constant for ferromagnetic materials; it depends on the value of the external field B_0, as the following experiment shows.

†All materials are slightly magnetic. Nonferromagnetic materials fall into two principal classes: **paramagnetic**, in which μ is very slightly larger than μ_0; and **diamagnetic**, in which μ is very slightly less than μ_0. Paramagnetic materials apparently contain atoms that have a net magnetic dipole moment due to orbiting electrons, and these become slightly aligned with an external field just as the galvanometer coil in Fig. 20–31 experiences a torque that tends to align it. Atoms of diamagnetic materials have no net dipole moment. However, in the presence of an external field, electrons revolving in one direction are caused to increase in speed slightly, whereas those revolving in the opposite direction are reduced in speed. The result is a slight net magnetic effect that actually opposes the external field.

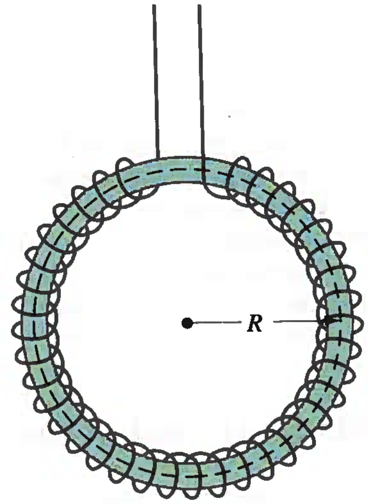


FIGURE 20–43 Iron-core torus.

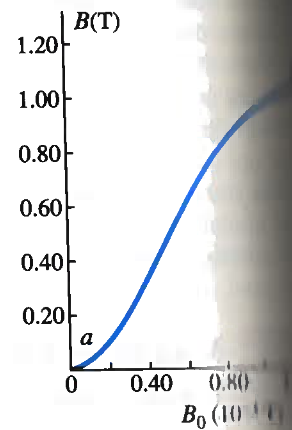


FIGURE 20–44 Total ... field B in an iron-core tor... function of the external fie... is caused by the current I in th...

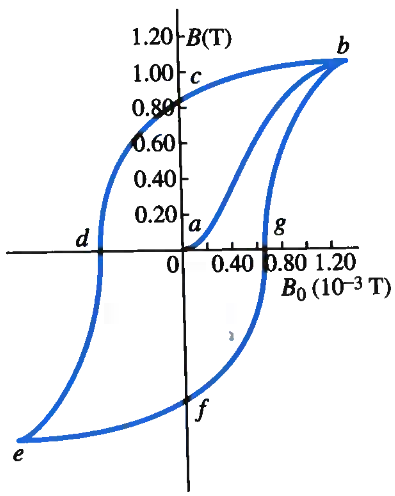


FIGURE 20–45 Hysteresis curve.

Measurements on magnetic materials are generally done ... torus, which is essentially a long solenoid bent into the shape of a ... (Fig. 20–43), so that practically all the lines of **B** remain within th... Suppose the torus has an iron core that is initially unmagnetized ... is no current in the windings of the torus. Then the current I is ... creased, and B_0 increases linearly with I. The total field B also in... but follows the curved line shown in the graph of Fig. 20–44. (Note th... ferent scales: $B \gg B_0$.) Initially, point a, the domains (Section 20... randomly oriented. As B_0 increases, the domains become more ... aligned until at point b, nearly all are aligned. The iron is said to ... proaching **saturation**. Point b is typically 70 percent of full saturati... B_0 is increased further, the curve continues to rise very slowly, and ... 98 percent saturation only when B_0 reaches a value about a thous... above that at point b; the last few domains are very difficult to ... Next, suppose the external field B_0 is reduced by decreasing the curre... the coils. As the current is reduced to zero, point c in Fig. 20–45, th... mains do not become completely random. Some permanent magnetis... mains. If the current is then reversed in direction, enough domains ... turned around so $B = 0$ (point d). As the reverse current is increased ... ther, the iron approaches saturation in the opposite direction (point ... nally, if the current is again reduced to zero and then increased in ... original direction, the total field follows the path $efgb$, again approach... saturation at point b.

Notice that the field did not pass through the origin (point a) ... cycle. The fact that the curves do not retrace themselves on the same ... is called **hysteresis**. The curve $bcdefgb$ is called a **hysteresis loop**. In ... cycle, much energy is transformed to thermal energy (friction) due to ... aligning of the domains. It can be shown that the energy dissipated ... way is proportional to the area of the hysteresis loop.

Hysteresis

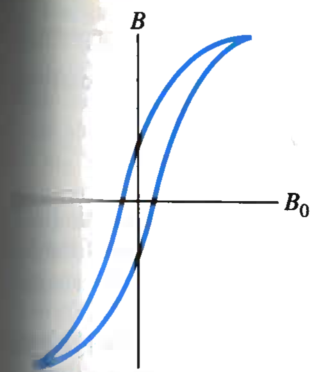


FIGURE 20–46 Hysteresis curve for soft iron.

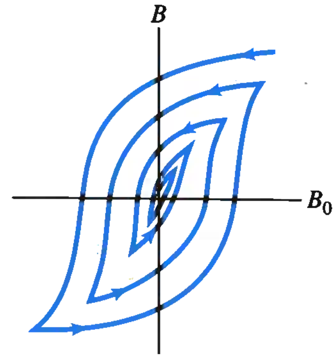


FIGURE 20–47 Successive hysteresis loops during demagnetization.

At points *c* and *f*, the iron core is magnetized even though there is no current in the coils. These points correspond to a permanent magnet. For a permanent magnet, it is desired that *ac* and *af* be as large as possible. Materials for which this is true are said to have high **retentivity**, and may be referred to as "hard." On the other hand, a hysteresis curve such as that in Fig. 20–46 occurs for so-called "soft iron" (soft from a magnetic point of view). This is preferred for *electromagnets* (Section 20–14) since the field can be more readily switched off, and the field can be reversed with less loss of energy.

Demagnetizing

A ferromagnetic material can be demagnetized—that is, made unmagnetized. This can be done by reversing the magnetizing current repeatedly while decreasing its magnitude. This results in the curve of Fig. 20–47. The heads of a tape recorder are demagnetized in this way. The alternating magnetic field acting at the heads due to a demagnetizer is strong when the demagnetizer is placed near the heads and decreases as it is moved slowly away. (Cassette tapes themselves can be erased and ruined by a magnetic field.)

PROBLEM SOLVING Magnetic Fields

Magnetic fields are somewhat analogous to the electric fields of Chapter 16, but there are several important differences to recall:

1. The force experienced by a charged particle moving in a magnetic field is *perpendicular* to the direction of the magnetic field (and to the direction of the velocity of the particle), whereas the force exerted by an electric field is *parallel* to the direction of the field (and unaffected by the velocity of the particle).
2. The *right-hand rule*, in its many forms, is intended to help you determine the directions of magnetic field, and the forces they exert, and/or the directions of electric current or charged particle velocity. The right-hand rules are specifically designed to deal with the "perpendicular" nature of these quantities.
3. Note that the equations in this chapter are generally not printed as vector equations, but involve magnitudes only. The right-hand rule is to be used to find directions of vector quantities.

SUMMARY

A magnet has two **poles**, north and south. The north pole is that end which points toward the north when the magnet is freely suspended. Unlike poles of two magnets attract each other, whereas like poles repel.

We can imagine that a **magnetic field** surrounds every magnet. The SI unit for magnetic field is the **tesla** (T). The force one magnet exerts on another is said to be an interaction between one magnet and the magnetic field produced by the other.

Electric currents produce magnetic fields. For example, the lines of magnetic field due to a current in a straight wire form circles around the wire and the field exerts a force on magnets.

The magnitude of the magnetic field a distance r from a long straight wire carrying a current I is given by

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}.$$

A magnetic field exerts a force on an electric current. For a straight wire of length l [illegible] current I, the force has magnitude

$$F = IlB \sin\theta,$$

where θ is the angle between the magnet[illegible] strength B and the wire. The direction [illegible] is perpendicular to the wire and to the [illegible] field, and is given by the right-hand rule.

Similarly, a magnetic field exerts a [illegible] charge q moving with velocity v of magn[illegible]

$$F = qvB \sin\theta,$$

where θ is the angle between **v** and **B**. [illegible] tion of **F** is perpendicular to **v** and to **B**. [illegible] of a charged particle moving perpendic[illegible] uniform magnetic field is a circle.

The force exerted on a current-carrying [illegible] a magnetic field is the basis for operation [illegible] devices, such as meters, motors, and louds[illegible]

QUESTIONS

1. A compass needle is not always balanced parallel to the Earth's surface but one end may dip downward. Explain.
2. Draw the magnetic field lines around a straight section of wire carrying a current horizontally to the left.
3. In what direction are the magnetic field lines surrounding a straight wire carrying a current that is moving directly toward you?
4. The magnetic field due to current in wires in your home can affect a compass. Discuss the problem in terms of currents, including if they are ac or dc.
5. What kind of field or fields surround a moving electric charge?
6. Will a magnet attract any metallic object, or only those made of iron? (Try it and see.) Why is this so?
7. Two iron bars attract each other no matter which ends are placed close together. Are both magnets? Explain.

*8. Note that the pattern of magnetic field lines surrounding a bar magnet is similar to that of the electric field around an electric dipole. From this fact, predict how the magnetic field will change with distance (*a*) when near one pole of a very long bar magnet, and (*b*) when far from a magnet as a whole.

9. Suppose you have three iron rods, two of which are magnetized but the third is not. How would you determine which two are the magnets without using any additional objects?
10. How can you make a compass without [illegible] other ferromagnetic material?
11. A horseshoe magnet is held vertically with [illegible] pole on the left and south pole on the [illegible] passes between the poles, equidistant from [illegible] carries a current directly away from you. [illegible] direction is the force on the wire?
12. Can you set a resting electron into motion with [illegible] netic field? With an electric field?
13. A charged particle is moving in a circle und[illegible] fluence of a uniform magnetic field. If [illegible] field that points in the same direction as the [illegible] ic field is turned on, describe the path the [illegible] particle will take.
14. Each of the right-hand rules you learned in [illegible] ter can be changed to *left-hand rules* if you [illegible] fying the direction of movement of *negative* [illegible] such as electrons in a wire. Show, for each [illegible] rule, that the same operations using the left [illegible] the same results if the direction of charge [illegible] negative charges.
15. A charged particle moves in a straight line [illegible] particular region of space. Could there be [illegible] magnetic field in this region? If so, give two [illegible] situations.
16. If a moving charged particle is deflected side[illegible] some region of space, can we conclude [illegible] that $\mathbf{B} \neq 0$ in that region?

17. If a negatively charged particle enters a region of uniform magnetic field which is perpendicular to the particle's velocity, will the kinetic energy of the particle increase, decrease, or stay the same. Explain your answer. (Neglect gravity.)

18. In Fig. 20–48, charged particles move in the vicinity of a current carrying wire. For each charged particle the arrow indicates the direction of motion of the particle and the + or − indicates the sign of the charge. For each of the particles, indicate the direction of the magnetic force due to the magnetic field produced by the wire.

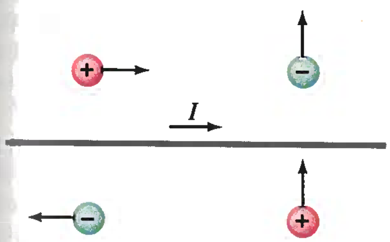


FIGURE 20–48 Question 18.

19. Explain why a strong magnet held near a television screen causes the picture to become distorted. Also, explain why the picture sometimes goes completely black where the field is the strongest.

20. In a particular region of space there is a uniform magnetic field **B**. Outside this region, $B = 0$. Can you inject an electron into the field perpendicularly so it will move in a closed circular path in the field?

21. How could you tell whether moving electrons in a certain region of space are being deflected by an electric field or by a magnetic field (or by both)?

22. A beam of electrons is directed perpendicularly toward a horizontal wire carrying a current from left to right. In what direction are the electrons deflected?

23. Two long wires carrying equal currents I are at right angles to each other, but don't quite touch. Describe the magnetic force one exerts on the other.

24. A horizontal current-carrying wire, free to move, is suspended directly above a second, parallel, current-carrying wire. (*a*) In what direction is the current in the lower wire? (*b*) Can the upper wire be held in stable equilibrium due to the magnetic force of the lower wire? Explain.

25 What factors determine the sensitivity of a galvanometer?

26. A rectangular piece of semiconductor is inserted in a magnetic field and a battery is connected to its ends as shown in Fig. 20–49. When a sensitive voltmeter is connected between points a and b, it is found that point a is at a higher potential than b. What is the sign of the charge carriers in this semiconductor material?

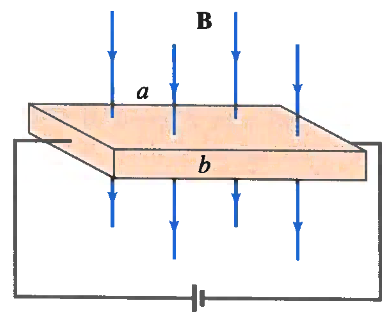


FIGURE 20–49 Question 26.

* 27. Two ions have the same mass, but one is singly ionized and the other is doubly ionized. How will their positions on the film of the mass spectrograph of Fig. 20–38 differ?

* 28. Why will either pole of a magnet attract an unmagnetized piece of iron?

* 29. An unmagnetized nail will not attract an unmagnetized paper clip. However, if one end of the nail is in contact with a magnet, the other end *will* attract a paper clip. Explain.

* 30. Another type of magnetic switch similar to a solenoid is a **relay**. A relay is an electromagnet (the iron rod inside the coil does not move) that, when activated, attracts a piece of soft iron on a pivot. Design a relay (*a*) to make a doorbell, and (*b*) to close an electrical switch. A relay is used in the latter case when you need to switch on a circuit carrying a very large current but you do not want that large current flowing through the main switch. For example, the starter switch of a car is connected to a relay so that the large currents needed for the starter do not pass to the dashboard switch.

PROBLEMS

SECTIONS 20–3 AND 20–4

1. (I) (*a*) What is the force per meter on a wire carrying a 9.80-A current when perpendicular to a 0.80-T magnetic field? (*b*) What if the angle between the wire and field is 45.0°?

2. (I) A 1.5-m length of wire carrying 6.5 A of current is oriented horizontally. At that point on the Earth's surface, the dip angle of the Earth's magnetic field makes an angle of 40° to the wire. Estimate the magnetic force on the wire due to the Earth's magnetic field of 5.5×10^{-5} T at this point.

3. (I) How much current is flowing in a wire 4.20 m long if the maximum force on it is 0.900 N when placed in a uniform 0.0800-T field?

4. (I) The force on a wire carrying 25.0 A is a maximum of 4.14 N when placed between the pole faces of a magnet. If the pole faces are 22.0 cm in diameter, what is the approximate strength of the magnetic field?

5. (I) Determine the magnitude and direction of the force on an electron traveling 3.58×10^6 m/s horizontally to the west in a vertically upward magnetic field of strength 1.30 T.

6. (I) Describe the path of an electron that is projected vertically upward with a speed of 1.80×10^6 m/s into a uniform magnetic field of 0.250 T that is directed away from the observer.

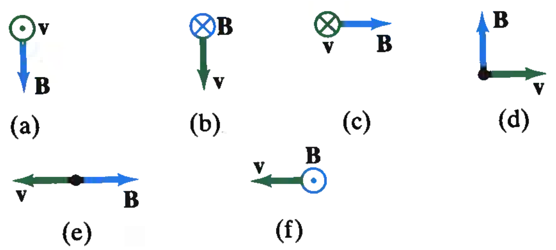


FIGURE 20–50 Problem 7.

7. (I) Find the direction of the force on a negative charge for each diagram shown in Fig. 20–50, where **v** is the velocity of the charge and **B** is the direction of the magnetic field. (⊗ means the vector points inward. ⊙ means it points outward, toward the viewer.)

8. (I) Determine the direction of **B** for each case in Fig. 20–51, where **F** represents the force on a positively charged particle moving with velocity **v**.

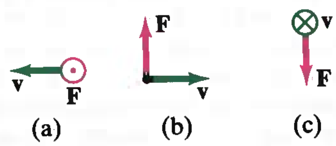


FIGURE 20–51 Problem 8.

9. (I) Alpha particles of charge $q = +2e$ and mass $m = 6.6 \times 10^{-27}$ kg are emitted from a radioactive source at a speed of 1.6×10^7 m/s. What magnetic field strength would be required to bend these into a circular path of radius $r = 0.25$ m?

10. (II) An electron experiences the greatest force as it travels 1.8×10^6 m/s in a magnetic field when it is moving southward. The force is upward and of magnitude 2.2×10^{-12} N. What is the magnitude and direction of the magnetic field?

11. (II) The magnetic force per meter on a wi[illegible] sured to be only 45 percent of its maxim[illegible] value. Sketch the relationship of the wi[illegible] field if the force were a maximum, and s[illegible] lationship as it actually is, calculating th[illegible] tween the wire and the magnetic field.

12. (II) The force on a wire is a maximum [illegible] when placed between the pole faces of [illegible] The current flows horizontally to the [illegible] magnetic field is vertical. The wire is [illegible] "jump" toward the observer when the [illegible] turned on. (*a*) What type of magnetic pol[illegible] pole face? (*b*) If the pole faces have a di[illegible] 10.0 cm, estimate the current in the wire [illegible] is 0.15 T. (*c*) If the wire is tipped so [illegible] makes an angle of 10° with the horizontal, [illegible] will it now feel?

13. (II) A proton moves in a circular path pe[illegible] to a 1.15-T magnetic field. The radius of [illegible] 8.40 mm. Calculate the energy of the prot[illegible]

14. (II) For a particle of mass m and charge q [illegible] a circular path in a magnetic field B, show [illegible] netic energy is proportional to r^2, the squ[illegible] radius of curvature of its path.

15. (II) A particle of charge q moves in a circ[illegible] of radius r in a uniform magnetic field B. [illegible] its momentum is $p = qBr$.

16. (II) For a particle of mass m and charge q [illegible] a circular orbit in a uniform magnetic field [illegible] that its angular momentum is given by L [illegible]

17. (II) A sort of "projectile launcher" is [illegible] Fig. 20–52. A large current moves in a c[illegible] composed of fixed rails, a power supply, [illegible] light, almost frictionless bar touching the rail[illegible] netic field is perpendicular to the plane of [illegible] If the bar has a length of 20 cm, a mass of 1[illegible] placed in a field of 1.7 T, what constant curr[illegible] needed in order for it to accelerate to 30 m[illegible] tance of 1.0 m? In what direction must the fi[illegible]

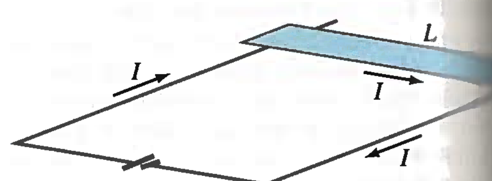


FIGURE 20–52 Problem 17

18. (III) A 3.80-g bullet moves with a speed o[illegible] perpendicular to the Earth's magnetic fi[illegible] 5.00×10^{-5} T. If the bullet possesses a net ch[illegible] 8.10×10^{-9} C, by what distance will it be d[illegible] from its path due to the magnetic field aft[illegible] traveled 1.00 km?

SECTIONS 20–5 AND 20–6

(I) Jumper cables used to start a stalled vehicle often carry a 15-A current. How strong is the magnetic field 15 cm away? What percentage of the Earth's magnetic field is this?

(I) If a magnetic field no larger than that of the Earth (0.55×10^{-4} T) is to be allowed 30 cm from an electrical wire, what is the maximum current the wire can carry?

(I) What is the magnitude and direction of the force between two parallel wires 45 m long and 6.0 cm apart, each carrying 35 A in the same direction?

(I) A vertical straight wire carrying an upward 12-A current exerts an attractive force per unit length of 8.8×10^{-4} N/m on a second parallel wire 7.0 cm away. What current (magnitude and direction) flows in the second wire?

(II) What is the maximum current that a wire can carry if an experimenter is performing an experiment 1.0 m away that deals with the Earth's magnetic field, which she wishes to measure to ± 1 percent?

(II) What is the acceleration (in g's) of a 175-g model airplane charged to 18.0 C and traveling at 1.8 m/s as it passes within 8.6 cm of a wire, nearly parallel to its path, carrying a 30-A current?

(II) A horizontal compass is placed 20 cm due south from a straight vertical wire carrying a 30-A current downward. In what direction does the compass needle point at this location? Assume the horizontal component of the Earth's field at this point is 0.45×10^{-4} T and the magnetic declination is 0°.

(II) A long horizontal wire carries 12.0 A of current due north. What is the net magnetic field 20.0 cm due west of the wire if the Earth's field there points downward, 40° below the horizontal, and has magnitude 5.0×10^{-5} T?

(II) A stream of protons passes a given point in space at a rate of 10^9 protons/s. What magnetic field do they produce 2.0 m from the beam?

(II) Determine the magnetic field midway between two long straight wires 2.0 cm apart in terms of the current I in one when the other carries 15 A. Assume these currents are (a) in the same direction, and (b) in opposite directions.

(II) A long pair of wires serves to conduct 25.0 A of dc current to (and from) an instrument. If the wires are of negligible diameter but are 2.0 mm apart, what is the magnetic field 10.0 cm from their midpoint, in their plane (Fig. 20–53)? Compare to the magnetic field of the Earth.

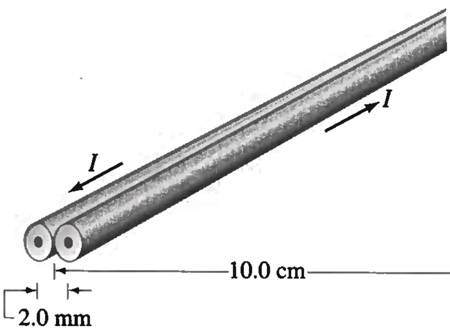


FIGURE 20–53 Problem 29.

30. (II) A compass needle points 20° E of N outdoors. However, when it is placed 8.0 cm to the east of a vertical wire inside a building, it points 55° E of N. What is the magnitude and direction of the current in the wire? The Earth's field there is 0.50×10^{-4} T and is horizontal.

31. (II) Three long parallel wires are 38.0 cm from one another. (Looking along them, they are at three corners of an equilateral triangle.) The current in each wire is 8.00 A, but that in wire A is opposite to that in wires B and C (Fig. 20–54). Determine the magnetic force per unit length on each wire due to the other two.

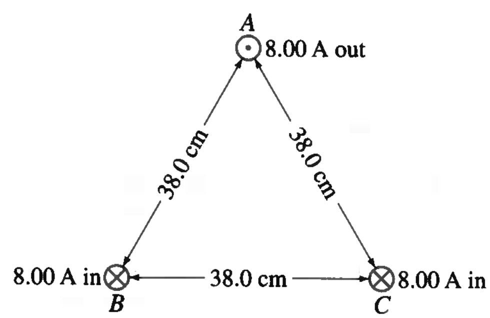


FIGURE 20–54 Problems 31 and 65.

32. (II) The magnetic field near the center of a single circular loop of radius r, carrying current I, is given by:

$$B = \frac{\mu_0 I}{2r}.$$

Assume the planetary model for the hydrogen atom, in which a single electron makes a circular orbit of radius 5.3×10^{-11} m about the nucleus. What magnitude of magnetic field would the orbiting electron produce at the nucleus?

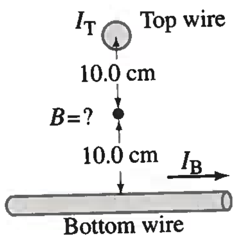


FIGURE 20–55 Problem 33.

33. (II) Two long wires are oriented so that they are perpendicular to each other, and at their closest, they are 20.0 cm apart (Fig. 20–55). What is the magnitude of the magnetic field at a point midway between them if the top one carries a current of 20.0 A and the bottom one carries 5.0 A?

34. (II) A long horizontal wire carries a current of 48 A. A second wire, made of 2.5-mm-diameter copper wire and parallel to the first but 15 cm below it, is held in suspension magnetically (Fig. 20–56). (*a*) What is the magnitude and direction of the current in the lower wire? (*b*) Is the lower wire in stable equilibrium? (*c*) Repeat parts (*a*) and (*b*) if the second wire is suspended 15 cm *above* the first due to the latter's field.

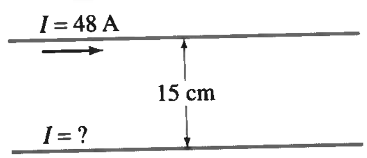


FIGURE 20–56 Problem 34.

*35. (III) Two long parallel wires 6.00 cm apart carry 16.5-A currents in the same direction. Determine the magnetic field strength at a point 12.0 cm from one wire and 13.0 cm from the other. [*Hint*: Make a drawing in a plane containing the field lines, and recall the rules for vector addition.]

SECTION 20–8

*36. (I) A 30.0-cm long solenoid 1.25 cm in diameter is to produce a field of 0.385 T at its center. How much current should the solenoid carry if it has 1000 turns of the wire?

*37. (II) You have 1.0 kg of copper and want to make a practical solenoid that produces the greatest possible magnetic field. Should you make your copper wire long and thin, short and fat, or something else? Consider other variables, such as solenoid diameter, length, and so on.

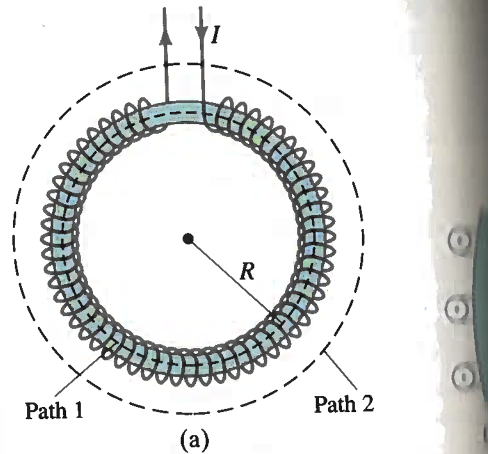


FIGURE 20–57 Problem 38. (a) A torus [illegible] (b) A section of the torus showing direction [illegible] current for three loops: ⊙ means current toward [illegible] and ⊗ means current away from viewer.

*38. (II) A torus is a solenoid in the shape [illegible] (Fig. 20–57). Use Ampère's law along the [illegible] shown dashed in Fig. 20–57a, to determine [illegible] magnetic field (*a*) inside the torus is B [illegible] where N is the total number of turns, and [illegible] the torus is $B = 0$. (*c*) Is the field inside [illegible] form like a solenoid's? If not, how does it [illegible]

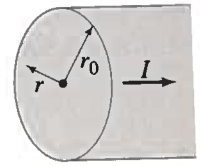
*39. (II) Use Ampère's law to show that a [illegible] netic field, such as between the pole pieces [illegible] net, Fig. 20–8, cannot drop abruptly to zero [illegible] the magnet. [*Hint*: Take as your path a rect[illegible] one vertical side inside the field and one ver[illegible] completely outside the field.]

*40. (III) A current I, flowing in a long solid [illegible] wire of radius r_0, is uniform across the [illegible] (Fig. 20–58). (*a*) Use Ampère's law to show [illegible] magnetic field inside the conductor at a di[illegible] from the center of the conductor is

$$B = \frac{\mu_0 I r}{2\pi r_0^2}.$$

Assume that the field lines are circles, just [illegible] outside the conductor. (*b*) Show that at the [illegible] the wire this agrees with the answer for the [illegible] field outside of a long wire. (*c*) Where is the [illegible] field a maximum, and what is its maximum [illegible] 1.0-mm-diameter wire carrying 15.0 A [illegible] what distance from the surface would the [illegible] 10 percent of its maximum? [*Hint*: Make a [illegible] magnetic field strength as a function of the [illegible] perpendicularly out from the central axis of [illegible]

FIGURE 20–58
Problem 40.



SECTIONS 20–9 AND 20–10

(I) A galvanometer needle deflects full scale for a [illegible]0-μA current. What current will give full-scale deflection if the magnetic field weakens to 0.860 of its original value?

(I) If the restoring spring of a galvanometer weakens by 20 percent over the years, what current will give full-scale deflection if it originally required 36 μA?

(I) If the current to a motor drops by 15 percent, by what factor does the output torque change?

(I) A single square loop of wire 22.0 cm on a side is placed with its face parallel to the magnetic field between the pole pieces of a large magnet. When [illegible]0 A flows in the coil, the torque on it is 0.325 m·N. What is the magnetic field strength?

(II) Show that the magnetic dipole moment M of an electron orbiting the proton nucleus of a hydrogen atom is related to the orbital momentum L of the electron by

$$M = \frac{e}{2m} L.$$

(II) A circular coil 18.0 cm in diameter and containing eleven loops lies flat on the ground. The Earth's magnetic field at this location has magnitude [illegible]50 $\times 10^{-5}$ T and points into the Earth at an angle of 56.0° below a line pointing due north. If a 7.70-A counterclockwise current passes through the coil, (a) determine the torque on the coil, and (b) which edge of the coil rises up, north, east, south, or west?

SECTION 20–11

(II) A rectangular sample of a metal is 3.0 cm wide and 500 μm thick. When it carries a 30-A current and is placed in a 0.80-T magnetic field it produces a [illegible]5-μV Hall emf. Determine: (a) the Hall field in the conductor; (b) the drift speed of the conduction electrons; (c) the density of free electrons in the metal.

(II) In a probe that uses the Hall effect to measure magnetic fields, a 12.0-A current passes through a [illegible]50-cm-wide 1.00-mm-thick strip of sodium metal. If the Hall emf is 2.42 μV, what is the magnitude of the magnetic field (take it perpendicular to the flat face of the strip)? Assume one free electron per atom of Na, and take its specific gravity to be 0.971.

(II) The Hall effect can be used to measure blood flow rate because the blood contains ions that constitute an electric current. (a) Does the sign of the ions influence the emf? (b) Determine the flow velocity in an artery 3.3 mm in diameter if the measured emf is 0.10 mV and B is 0.070 T. (In actual practice, an alternating magnetic field is used.)

*SECTION 20–12

*50. (I) Protons move in a circle of radius 5.10 cm in a 0.566-T magnetic field. What value of electric field could make their paths straight? In what direction must it point?

*51. (I) In a mass spectrometer, germanium atoms have radii of curvature equal to 21.0, 21.6, 21.9, 22.2, and 22.8 cm. The largest radius corresponds to an atomic mass of 76 u. What are the atomic masses of the other isotopes?

*52. (II) Suppose the electric field between the electric plates in the mass spectrometer of Fig. 20–38 is 2.48×10^4 V/m and the magnetic fields $B = B' = 0.68$ T. The source contains carbon isotopes of mass numbers 12, 13, and 14 from a long-dead piece of a tree. (To estimate atomic masses, multiply by 1.67×10^{-27} kg.) How far apart are the lines formed by the singly charged ions of each type on the photographic film? What if the ions were doubly charged?

*53. (II) (a) What value of magnetic field would make a beam of electrons, traveling to the right at a speed of 4.8×10^6 m/s, go undeflected through a region where there is a uniform electric field of 10,000 V/m pointing vertically up? (b) What is the direction of the magnetic field if it is known to be perpendicular to the electric field? (c) What is the frequency of the circular orbit of the electrons if the electric field is turned off?

*54. (II) A mass spectrometer is being used to monitor air pollutants. It is difficult, however, to separate molecules with nearly equal mass such as CO (28.0106 u) and N_2 (28.0134 u). How large a radius of curvature must a spectrometer have if these two molecules are to be separated on the film by 0.50 mm?

*55. (II) One form of mass spectrometer accelerates ions by a voltage V before they enter a magnetic field B. The ions are assumed to start from rest. Show that the mass of an ion is $m = qB^2R^2/2V$, where R is the radius of the ions' path in the magnetic field and q is their charge.

*SECTION 20–15

*56. (II) An iron-core solenoid is 36 cm long, 1.5 cm in diameter, and has 600 turns of wire. A magnetic field of 1.8 T is produced when 40 A flows in the wire. What is the permeability μ at this high field strength?

GENERAL PROBLEMS

57. Protons with momentum 4.8×10^{-16} kg·m/s are magnetically steered clockwise in a circular path 2.0 km in diameter at Fermi National Accelerator Laboratory in Illinois. What is the magnitude and direction of the field in the magnets surrounding the beam pipe?

58. A rectangular loop of wire is sitting next to a straight wire, as shown in Fig. 20–59. There is a current of 2.5 A in both wires. What is the magnitude and direction of the net force on the loop?

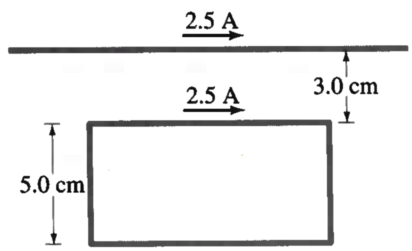


FIGURE 20–59 Problem 58.

59. A proton and an electron have the same kinetic energy upon entering a region of constant magnetic field. What is the ratio of the radii of their circular paths?

60. Near the equator, the Earth's magnetic field points almost horizontally to the north and has magnitude $B = 0.50 \times 10^{-4}$ T. What should be the magnitude and direction for the velocity of an electron if its weight is to be exactly balanced by the magnetic force?

61. Calculate the force on an airplane which has acquired a net charge of 155 C and moves with a speed of 120 m/s perpendicular to the Earth's magnetic field of 5.0×10^{-5} T.

62. The power cable for an electric trolley (Fig. 20–60) carries a horizontal current of 330 A toward the east. The Earth's magnetic field has a strength 5.0×10^{-5} T and makes an angle of dip of 22° at this location. Calculate the magnitude and direction of the magnetic force on a 10-m length of this cable.

63. A doubly charged helium atom, whose mass is 6.6×10^{-27} kg, is accelerated by a voltage of 2400 V. (*a*) What will be its radius of curvature in a uniform 0.240-T field? (*b*) What is its period of revolution?

64. A straight 1.00-mm-diameter copper wire can just "float" horizontally in air because of the force of the Earth's magnetic field **B** which is horizontal and of magnitude 5.00×10^{-5} T. What current does the wire carry?

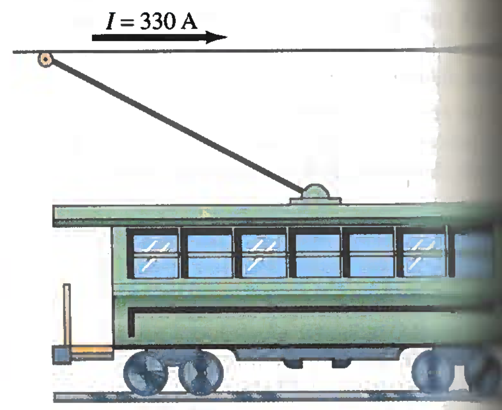


FIGURE 20–60 Problem 6[illegible]

65. In Fig. 20–54 the top wire is 2.00-mm-dia[illegible] wire and is suspended in air due to the [illegible] forces from the bottom two wires. The [illegible] through the two bottom wires is 20.0 A in [illegible] late the required current flow in the suspe[illegible]

66. Two stiff parallel wires a distance l apa[illegible] zontal plane act as rails to support a ligh[illegible] of mass m (perpendicular to each rail), [illegible] magnetic field **B**, directed vertically upwa[illegible] in diagram), acts throughout. At $t = 0$, wir[illegible] ed to the rails are connected to a cons[illegible] source and a current I begins to flow thro[illegible] tem. Determine the speed of the rod as a [illegible] time (*a*) assuming no friction between the [illegible] the rails, and (*b*) if the coefficient of fri[illegible] (*c*) In which direction does the rod mov[illegible] west, if the current through it heads north?

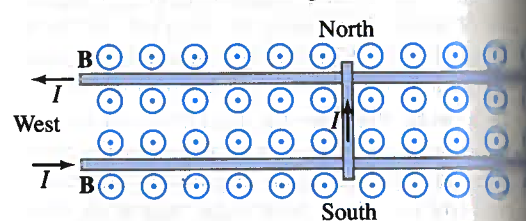


FIGURE 20–61 Looking down on a rod [illegible] rails. Problem 66.

67. Estimate the approximate maximum defl[illegible] the electron beam near the center of a TV [illegible] to the Earth's 5.0×10^{-5} T field. Assume th[illegible] is 20 cm from the electron gun where the [illegible] are accelerated (*a*) by 2.0 kV, or (*b*) by 30 [illegible] that in color TV sets, the beam must be dir[illegible] curately to within less than 1 mm in order [illegible] the correct phosphor. Because the Earth's fie[illegible] nificant here, mu-metal shields are used to re[illegible] Earth's field in the CRT. (See Section 17–10.)

An electron enters a large solenoid at a 7.0° angle to the axis. If the field is a uniform 3.3×10^{-2} T, determine the radius and pitch (distance between loops) of the electron's helical path if its speed is 1.8×10^7 m/s.

The cyclotron (Fig. 20–62) is a device used to accelerate elementary particles such as protons to high speeds. Particles starting at point A with some initial velocity travel in circular orbits in the magnetic field B. The particles are accelerated to higher speeds each time they pass in the gap between the metal "dees," where there is an electric field E. (There is no electric field within the cavity of the metal dees.) The electric field changes direction each half-cycle, owing to an ac voltage $V = V_0 \sin 2\pi ft$, so that the particles are increased in speed at each passage through the gap. (*a*) Show that the frequency f of the voltage must be $f = Bq/2\pi m$, where q is the charge on the particles and m their mass. (*b*) Show that the kinetic energy of the particles increases by $2qV_0$ each revolution, assuming that the gap is small. (*c*) If the radius of the cyclotron is 2.0 m and the magnetic field strength is 0.50 T, what will be the maximum kinetic energy of accelerated protons in MeV? (*d*) How is a cyclotron like a swing?

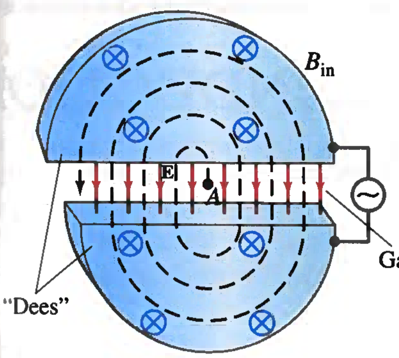


FIGURE 20–62 A cyclotron. Problem 69.

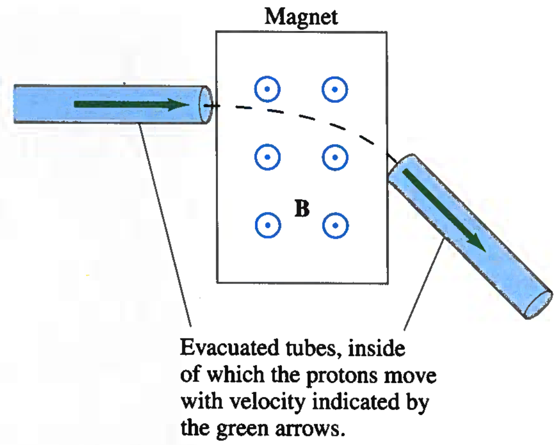


FIGURE 20–63 Problem 70.

70. Magnetic fields are very useful in particle accelerators for "beam steering"; that is, the magnetic fields can be used to change the beam's direction without altering its speed (Fig. 20–63). Show how this works with a beam of protons. What happens to protons that are not moving with the speed that the magnetic field is designed for? If the field extends over a region 5.0 cm wide and has a magnitude of 0.33 T, by approximately what angle will a beam of protons traveling at 1.0×10^7 m/s be bent?

71. A square loop of aluminum wire is 20.0 cm on a side. It is to carry 25.0 A and rotate in a 1.65-T magnetic field. (*a*) Determine the minimum diameter of the wire so that it will not fracture from tension or shear. Assume a safety factor of 10. (See Table 9–2.) (*b*) What is the resistance of a single loop of this wire?

72. The magnetic field B at the center of a circular coil of wire carrying a current I is

$$B = \frac{\mu_0 NI}{2r},$$

where N is the number of loops in the coil and r is its radius. Suppose that an electromagnet uses a coil 1.2 m in diameter made from square copper wire 1.6 mm on a side. The power supply produces 120 V at a maximum power output of 4.0 kW. (*a*) How many turns are needed to run the power supply at maximum power? (*b*) What is the magnetic field strength at the center of the coil? (*c*) If you use a greater number of turns and this same power supply (so the voltage remains at 120 V), will a greater magnetic field strength result? Explain.