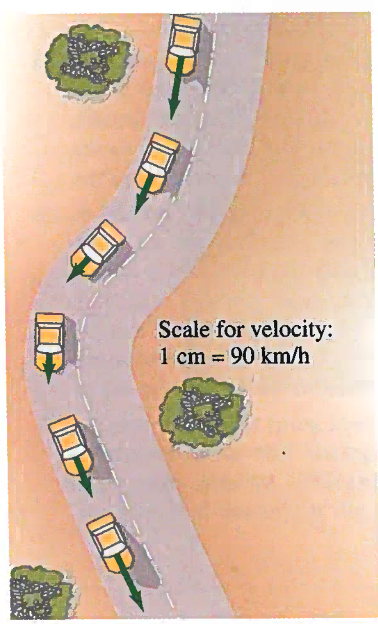
This multiflash photograph of a ping pong ball shows examples of motion in two dimensions. The arcs of the ping pong ball are parabolas that represent "projectile motion." Galileo analyzed projectile motion into its horizontal and vertical components, under the action of gravity (the gold arrow represents the downward acceleration of gravity, **g**).

CHAPTER

3 KINEMATICS IN TWO DIMENSIONS; VECTORS

FIGURE 3–1 Car traveling on a road. The green arrows represent the velocity vector at each position.



In Chapter 2 we dealt with motion along a straight line. We now consider the description of the motion of objects that move in paths in two (or three) dimensions. To do so we first need to discuss vectors and how they are added.

3–1 Vectors and Scalars

We mentioned in Chapter 2 that the term *velocity* refers not only to how fast something is moving but also to its direction. A quantity such as velocity, which has *direction* as well as *magnitude*, is a **vector** quantity. Other quantities that are also vectors are displacement, force, and momentum. However, many quantities such as mass, time, and temperature, have no direction associated with them. They are specified completely by giving a number and units. Such quantities are called **scalars**.

Drawing a diagram of a particular physical situation is always helpful in physics, and this is especially true when dealing with vectors. On a diagram, each vector is represented by an arrow. The arrow is always drawn so that it points in the direction of the vector it represents. The length of the arrow is drawn proportional to the magnitude of the vector. For example, in Fig. 3–1, arrows have been drawn representing the velocity of a car at various places as it rounds a curve. The magnitude of the velocity at each point can be read off this figure by measuring the length of the corresponding arrow and using the scale shown (1 cm = 90 km/h).

When we write the symbol for a vector, we will always use boldface type. Thus for velocity we write **v**. (In handwritten work, the symbol for a vector can be indicated by putting an arrow over it, a $\vec{v}$ for velocity.) If we are concerned only with the magnitude of the vector, we will write simply v, in italics.

3–2 Addition of Vectors—Graphical Methods

Because vectors are quantities that have direction as well as magnitude, they must be added in a special way. In this chapter, we will deal mainly with displacement vectors (for which we now use the symbol **D**) and velocity vectors (**v**). But the results will apply for other vectors we encounter later.

We use simple arithmetic for adding scalars. Simple arithmetic can also be used for adding vectors if they are in the same direction. For example, if a person walks 8 km east one day, and 6 km east the next day, the person will be 8 km + 6 km = 14 km east of the point of origin. We say that the *net* or *resultant* displacement is 14 km to the east (Fig. 3–2a). If, on the other hand, the person walks 8 km east on the first day, and 6 km west (in the reverse direction) on the second day, then the person will end up 2 km from the origin (Fig. 3–2b), so the resultant displacement is 2 km to the east. In this case, the resultant displacement is obtained by subtraction: 8 km − 6 km = 2 km.

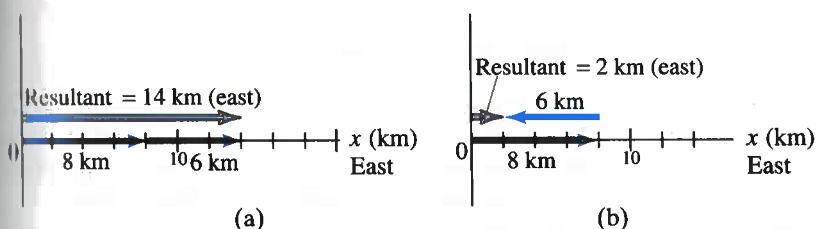


FIGURE 3–2 Combining vectors in one dimension.

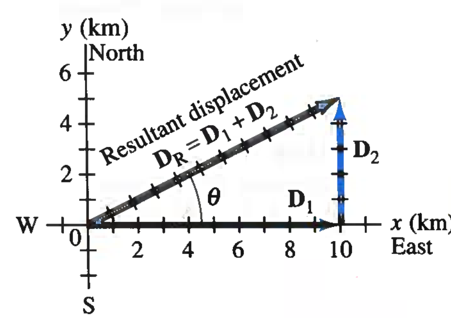
But simple arithmetic cannot be used if the two vectors are not along the same line. For example, suppose a person walks 10.0 km east and then walks 5.0 km north. These displacements can be represented on a graph in which the positive y axis points north and the positive x axis points east, Fig. 3–3. On this graph, we draw an arrow, labeled $\mathbf{D}_1$, to represent the displacement vector of the 10.0-km displacement to the east. Then we draw a second arrow, $\mathbf{D}_2$, to represent the 5.0-km displacement to the north. Both vectors are drawn to scale, as in Fig. 3–3.

After taking this walk, the person is now 10.0 km east and 5.0 km north of the point of origin. The **resultant displacement** is represented by the arrow labeled $\mathbf{D}_\mathrm{R}$ in Fig. 3–3. Using a ruler and a protractor, you can measure on this diagram that the person is 11.2 km from the origin at an angle of 27° north of east. In other words, the resultant displacement vector has a magnitude of 11.2 km and makes an angle $\theta = 27°$ with the positive x axis. The magnitude (length) of $\mathbf{D}_\mathrm{R}$ can also be obtained using the theorem of Pythagoras in this case, since D_1, D_2, and D_R form a right triangle with D_R as the hypotenuse. Thus

$$D_\mathrm{R} = \sqrt{D_1^2 + D_2^2} = \sqrt{(10.0\ \mathrm{km})^2 + (5.0\ \mathrm{km})^2} = \sqrt{125\ \mathrm{km}^2} = 11.2\ \mathrm{km}.$$

You can use the Pythagorean theorem, of course, only when the vectors are *perpendicular* to each other.

FIGURE 3–3 A person walks 10.0 km east and then 5.0 km north. These two displacements are represented by the vectors $\mathbf{D}_1$ and $\mathbf{D}_2$, which are shown as arrows. The resultant displacement vector, $\mathbf{D}_\mathrm{R}$, which is the vector sum of $\mathbf{D}_1$ and $\mathbf{D}_2$, is also shown. Measurement on the graph with ruler and protractor shows that $\mathbf{D}_\mathrm{R}$ has a magnitude of 11.2 km and points at an angle $\theta = 27°$ north of east.



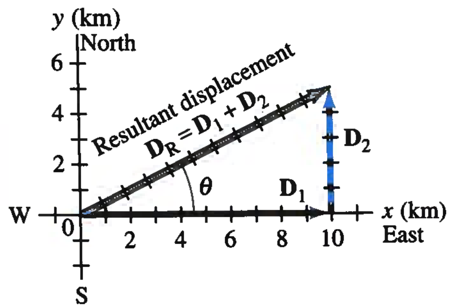


FIGURE 3–3 (Repeated from previous page.) A person walks 10.0 km east and then 5.0 km north. The resultant vector has magnitude $D_R = 11.2\,\text{km}$ at an angle $\theta = 27°$ north of east.

The resultant displacement vector, $\mathbf{D}_R$, is the sum of the vectors $\mathbf{D}_1$ and $\mathbf{D}_2$. That is,

$$\mathbf{D}_R = \mathbf{D}_1 + \mathbf{D}_2.$$

This is a *vector* equation. An important feature of adding two vectors that are not along the same line is that the magnitude of the resultant vector is not equal to the sum of the magnitudes of the two separate vectors, but is smaller than their sum:

$$D_R < D_1 + D_2. \qquad \text{[Vectors not along the same line]}$$

In our example (Fig. 3–3), $D_R = 11.2\,\text{km}$, whereas $D_1 + D_2$ equals 15 km. We generally are not interested in $D_1 + D_2$; rather we are interested in the *vector* sum of the two vectors and its magnitude, D_R. Note also that we cannot set $\mathbf{D}_R$ equal to 11.2 km, because we have a vector equation and 11.2 km is only a part of the resultant vector, its magnitude. We could write something like this, though: $\mathbf{D}_R = \mathbf{D}_1 + \mathbf{D}_2 = (11.2\,\text{km}, 27°\text{ N of E})$.

Figure 3–3 illustrates the general rules for graphically adding two vectors together, no matter what angles they make, to get their sum. The rules are as follows:

Tail-to-tip method of adding vectors

1. On a diagram, draw one of the vectors—call it $\mathbf{V}_1$—to scale.
2. Next draw the second vector, $\mathbf{V}_2$, to scale, placing its tail at the tip of the first vector and being sure its direction is correct.
3. The arrow drawn from the tail of the first vector to the tip of the second represents the *sum*, or **resultant**, of the two vectors.

Note that vectors can be translated parallel to themselves to accomplish these manipulations. The length of the resultant can be measured with a ruler and compared to the scale. Angles can be measured with a protractor. This method is known as the **tail-to-tip method of adding vectors.**

Note that it is not important in which order the vectors are added. For example, a displacement of 5.0 km north, to which is added a displacement of 10.0 km east, yields a resultant of 11.2 km and angle $\theta = 27°$ (see Fig. 3–4), the same as when they were added in reverse order (Fig. 3–3). That is,

$$\mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1.$$

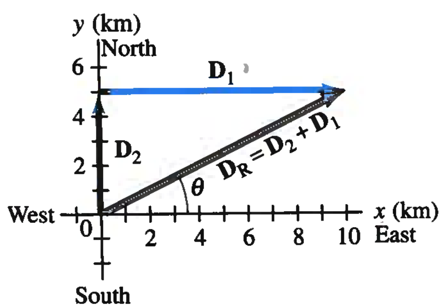
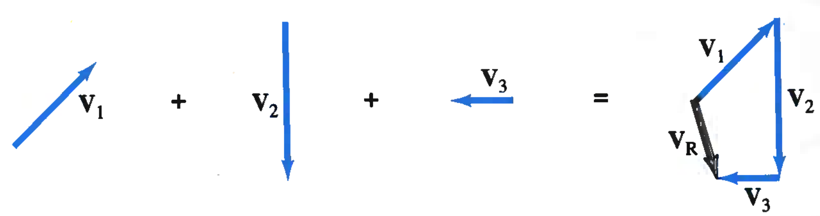


FIGURE 3–4 If the vectors are added in reverse order, the resultant is the same. (Compare Fig. 3–3.)

The tail-to-tip method of adding vectors can be extended to three or more vectors. The resultant is drawn from the tail of the first vector to the tip of the last one added. An example is shown in Fig. 3–5; the three vectors could represent displacements (northeast, south, west) or perhaps three forces. Check for yourself that you get the same resultant no matter in which order you add the three vectors.

FIGURE 3–5 The resultant of three vectors, $\mathbf{V}_R = \mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$.



A second way to add two vectors is the **parallelogram method**. It is fully equivalent to the tail-to-tip method. In this method, the two vectors are drawn starting from a common origin, and a parallelogram is constructed using these two vectors as adjacent sides as shown in Fig. 3–6b. The resultant is the diagonal drawn from the common origin. In Fig. 3–6a, the tail-to-tip method is shown, and it is clear that both methods yield the same result.

Parallelogram method of adding vectors

It is a common error to draw the sum vector as the diagonal running between the tips of the two vectors, as in Fig. 3–6c. *This is incorrect*: it does not represent the sum of the two vectors. (In fact, it represents their difference, $\mathbf{V}_2 - \mathbf{V}_1$, as we will see in the next Section.)

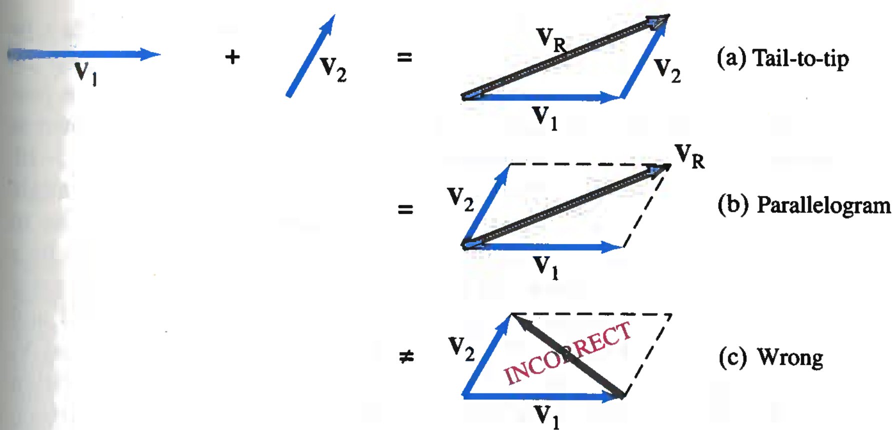


FIGURE 3–6 Vector addition by two different methods, (a) and (b). Part (c) is incorrect.

3–3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

Given a vector $\mathbf{V}$, we define the *negative* of this vector ($-\mathbf{V}$) to be a vector with the same magnitude as $\mathbf{V}$ but opposite in direction, Fig. 3–7. Note, however, that no vector is ever negative in the sense of its magnitude: the magnitude of every vector is positive. A minus sign tells us about its direction.

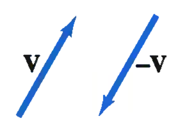


FIGURE 3–7 The negative of a vector is a vector having the same length but opposite direction.

We can now define the subtraction of one vector from another: the difference between two vectors, $\mathbf{V}_2 - \mathbf{V}_1$ is defined as

$$\mathbf{V}_2 - \mathbf{V}_1 = \mathbf{V}_2 + (-\mathbf{V}_1).$$

That is, the difference between two vectors is equal to the sum of the first plus the negative of the second. Thus our rules for addition of vectors can be applied as shown in Fig. 3–8 using the tail-to-tip method.

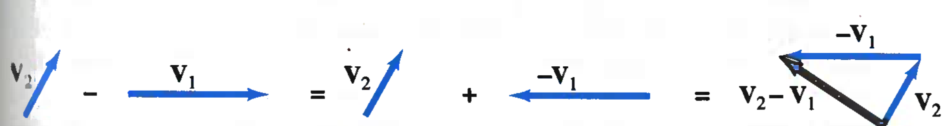


FIGURE 3–8 Subtracting two vectors: $\mathbf{V}_2 - \mathbf{V}_1$.

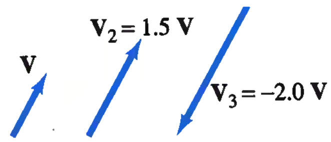


FIGURE 3–9 Multiplying a vector **V** by a scalar *c* gives a vector whose magnitude is *c* times greater and in the same direction as **V** (or opposite direction if *c* is negative).

A vector **V** can be multiplied by a scalar *c*. We define this product so that $c\mathbf{V}$ has the same direction as **V** and has magnitude cV. That is, multiplication of a vector by a positive scalar *c* changes the magnitude of the vector by a factor *c* but doesn't alter the direction. If *c* is a negative scalar, the magnitude of the product $c\mathbf{V}$ is still cV (without the minus sign), but the direction is precisely opposite to that of **V**. See Fig. 3–9.

3–4 Adding Vectors by Components

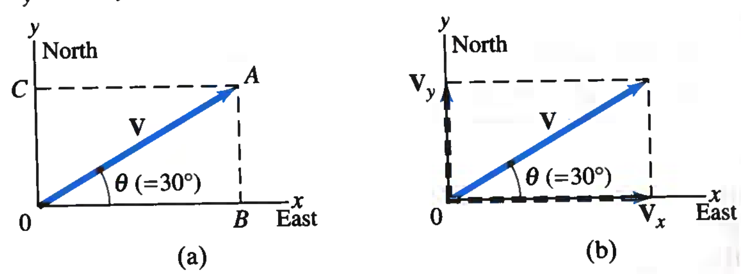
Adding vectors graphically using a ruler and protractor is often not sufficiently accurate and is not useful for vectors in three dimensions. We discuss now a more powerful and precise method for adding vectors.

Resolving a vector into components

Consider first a vector **V** that lies in a particular plane. It can be expressed as the sum of two other vectors, called the **components** of the original vector. The components are usually chosen to be along two perpendicular directions. The process of finding the components is known as **resolving the vector into its components**. An example is shown in Fig. 3–10; the vector **V** could be a displacement vector that points at an angle $\theta = 30°$ north of east, where we have chosen the positive *x* axis to be to the east and the positive *y* axis north. This vector **V** is resolved into its *x* and *y* components by drawing dashed lines from the tip (*A*) of the vector and drawing these lines perpendicular to the *x* and *y* axes (lines *AB* and *AC*). Then the lines *OB* and *OC* represent the *x* and *y* components of **V**, respectively, as shown in Fig. 3–10b. These *vector components* are written $\mathbf{V}_x$ and $\mathbf{V}_y$. We generally show vector components as arrows, like vectors, but dashed. The *scalar components*, V_x and V_y, are numbers, with units, that are given a positive or negative sign depending on whether they point along the positive or negative *x* or *y* axis. As can be seen in Fig. 3–10, $\mathbf{V}_x + \mathbf{V}_y = \mathbf{V}$ by the parallelogram method of adding vectors.

Vector components

FIGURE 3–10 Resolving a vector **V** into its components along an arbitrarily chosen set of *x* and *y* axes. Note that the components, once found, themselves represent the vector. That is, the components contain as much information as the vector itself.



Space is made up of three dimensions, and sometimes it is necessary to resolve a vector into components along three mutually perpendicular directions. In rectangular coordinates the components are $\mathbf{V}_x$, $\mathbf{V}_y$, and $\mathbf{V}_z$. Resolution of a vector in three dimensions is merely an extension of the above technique. We will mainly be concerned with situations in which the vectors are in a plane and two components are all that are necessary.

In order to add vectors using the method of components, we need to use the trigonometric functions sine, cosine, and tangent, which we now review.

Given any angle, θ, as in Fig. 3–11a, a right triangle can be constructed by drawing a line perpendicular to either of its sides, as in Fig. 3–11b. The longest side of a right triangle, opposite the right angle, is called the hypotenuse, which we label *h*. The side opposite the angle θ is labeled *o*, and the side adjacent is labeled *a*. We let *h*, *o*, and *a* represent the lengths of these sides, respectively. We now define the three trigonometric functions, sine, cosine, and

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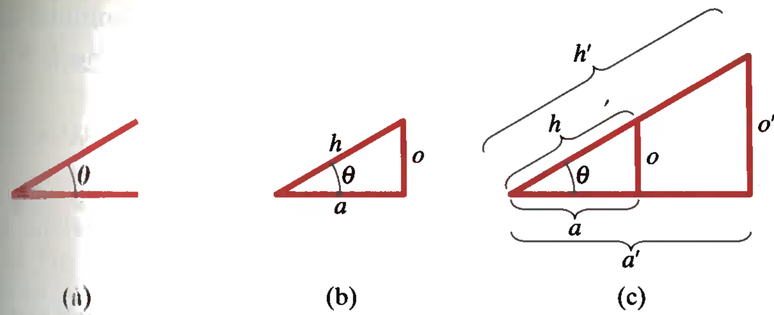


FIGURE 3–11 Starting with an angle θ as in (a), we can construct right triangles of different sizes, (b) and (c), but the ratio of the lengths of the sides does not depend on the size of the triangle.

tangent (abbreviated sin, cos, tan), in terms of the right triangle, as follows:

Trig. functions defined

$$\sin\theta = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{o}{h}$$

$$\cos\theta = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{a}{h} \qquad \textbf{(3–1)}$$

$$\tan\theta = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{o}{a}.$$

Now it is an interesting fact that if we make the triangle bigger, but keep the same angles, then the ratio of the length of one side to the other, or of one side to the hypotenuse, remains the same. That is, in Fig. 3–11c we have: $a/h = a'/h'$; $o/h = o'/h'$; and $o/a = o'/a'$. Thus the values of sine, cosine, and tangent do not depend on how big the triangle is. They depend only on the size of the angle. The values of sine, cosine, and tangent for different angles can be found using a scientific calculator, or from Tables (see inside rear cover).

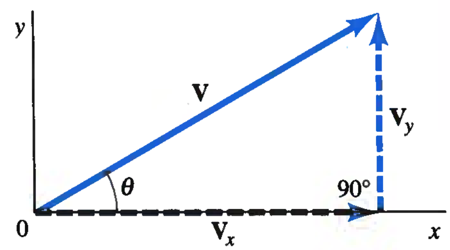
A useful trigonometric identity is

$$\sin^2\theta + \cos^2\theta = 1 \qquad \textbf{(3–2)}$$

which follows from the Pythagorean theorem ($o^2 + a^2 = h^2$ in Fig. 3–11). That is:

$$\sin^2\theta + \cos^2\theta = \frac{o^2}{h^2} + \frac{a^2}{h^2} = \frac{o^2 + a^2}{h^2} = \frac{h^2}{h^2} = 1.$$

(See also Appendix A for other details on trigonometric functions and identities.)



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

FIGURE 3–12 Finding the components of a vector using trigonometric functions.

The use of trigonometric functions for finding the components of a vector is illustrated in Fig. 3–12, where it is seen that a vector and its two components can be thought of as making up a right triangle. We then see that the sine, cosine, and tangent are as given in the figure. If we multiply the definition of $\sin\theta = V_y/V$ by V on both sides, we get

Components of a vector

$$V_y = V\sin\theta. \qquad \textbf{(3–3a)}$$

Similarly, from the definition of $\cos\theta$, we obtain

$$V_x = V\cos\theta. \qquad \textbf{(3–3b)}$$

Note that θ is chosen (by convention) to be the angle that the vector makes with the positive x axis.[†]

[†] Whichever convention is used, the vector component opposite the angle is proportional to the sine, whether we call that component x or y. Most often we use the convention that it is the y component (Eq. 3–3a).

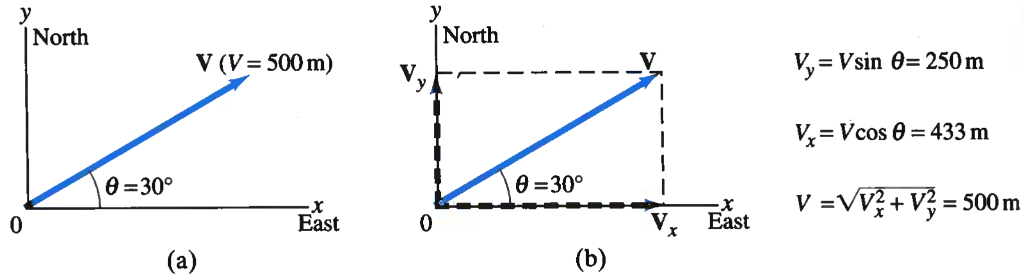


FIGURE 3–13 (a) Vector **V** represents a displacement of 500 m at a 30° angle north of east. (b) The components of **V** are $\mathbf{V}_x$ and $\mathbf{V}_y$ whose magnitudes are given on the right.

Using Eqs. 3–3, we can calculate V_x and V_y for any vector, such as that illustrated in Fig. 3–10 or Fig. 3–12. Suppose **V** represents a displacement of 500 m in a direction 30° north of east, as shown in Fig. 3–13. Then $V = 500$ m. From the trigonometric tables, sin 30° = 0.500 and cos 30° = 0.866. Then

$$V_x = V\cos\theta = (500\text{ m})(0.866) = 433\text{ m (east)},$$

$$V_y = V\sin\theta = (500\text{ m})(0.500) = 250\text{ m (north)}.$$

Note that there are two ways to specify a vector in a given coordinate system:

Two ways to specify a vector

1. We can give its components, V_x and V_y.
2. We can give its magnitude V and the angle θ it makes with the positive x axis.

We can shift from one description to the other using Eqs. 3–3, and, for the reverse, by using the theorem of Pythagoras† and the definition of tangent:

Components related to magnitude and direction

$$V = \sqrt{V_x^2 + V_y^2} \tag{3–4a}$$

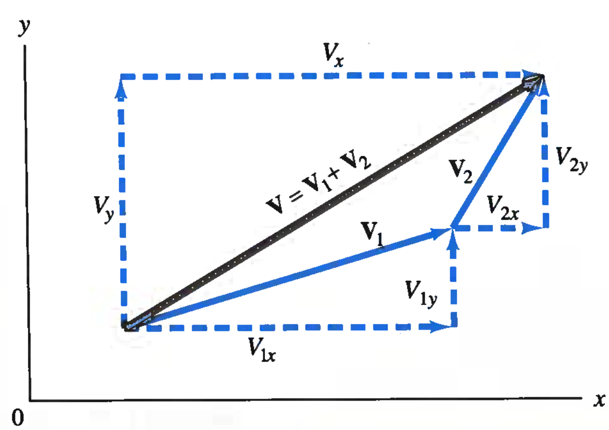
$$\tan\theta = \frac{V_y}{V_x} \tag{3–4b}$$

as can be seen in Fig. 3–12.

We can now discuss how to add vectors using components. The first step is to resolve each vector into its components. Next we can see, using Fig. 3–14

†In three dimensions, the theorem of Pythagoras becomes $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$, where V_z is the component along the third, or z, axis.

FIGURE 3–14 The components of $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ are $V_x = V_{1x} + V_{2x}$ and $V_y = V_{1y} + V_{2y}$.



that the addition of any two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ to give a resultant, $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$, implies that

$$V_x = V_{1x} + V_{2x}$$
$$V_y = V_{1y} + V_{2y}. \quad \text{(3–5)}$$

Adding vectors analytically (by components)

That is, the sum of the x components equals the x component of the resultant, and similarly for y. That this is valid can be verified by a careful examination of Fig. 3–14. But note carefully that we add all the x components together to get the x component of the resultant; and we add all the y components together to get the y component of the resultant. We do *not* add x components to y components.

If the magnitude and direction of the resultant vector are desired, they can be obtained using Eqs. 3–4.

Choice of axes can simplify effort needed

The choice of coordinate axes is, of course, always arbitrary. You can often reduce the work involved in adding vectors by a good choice of axes—for example, by choosing one of the axes to be in the same direction as one of the vectors. Then that vector will have only one nonzero component.

EXAMPLE 3–1 **Mail carrier's displacement.** A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction to the next town. She then drives in a direction 60.0° south of east for 47.0 km (Fig. 3–15a) to another town. What is her displacement from the post office?

SOLUTION We want to find her resultant displacement from the origin. We choose the positive x axis to be east and the positive y axis north, and resolve each displacement vector into its components (Fig. 3–15b). Since $\mathbf{D}_1$ has magnitude 22.0 km and points north, it has only a y component:

$$D_{1x} = 0, \quad D_{1y} = 22.0\ \text{km}$$

whereas $\mathbf{D}_2$ has both x and y components:

$$D_{2x} = +(47.0\ \text{km})(\cos 60°) = +(47.0\ \text{km})(0.500) = +23.5\ \text{km}$$
$$D_{2y} = -(47.0\ \text{km})(\sin 60°) = -(47.0\ \text{km})(0.866) = -40.7\ \text{km}.$$

Notice that D_{2y} is negative because this vector component points along the negative y axis. The resultant vector, $\mathbf{D}$, has components:

$$D_x = D_{1x} + D_{2x} = \quad 0\ \text{km} + \quad 23.5\ \text{km} \ = +23.5\ \text{km}$$
$$D_y = D_{1y} + D_{2y} = 22.0\ \text{km} + (-40.7\ \text{km}) = -18.7\ \text{km}$$

This specifies the resultant vector completely:

$$D_x = 23.5\ \text{km}, \qquad D_y = -18.7\ \text{km}.$$

We can also specify the resultant vector by giving its magnitude and angle using Eqs. 3–4:

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(23.5\ \text{km})^2 + (-18.7\ \text{km})^2} = 30.0\ \text{km}$$

$$\tan\theta = \frac{D_y}{D_x} = \frac{-18.7\ \text{km}}{23.5\ \text{km}} = -0.796.$$

A calculator with an INV TAN or TAN^{-1} key gives $\theta = \tan^{-1}(-0.796) = -38.5°$. The negative sign means $\theta = 38.5°$ below the x axis, Fig. 3–15c.

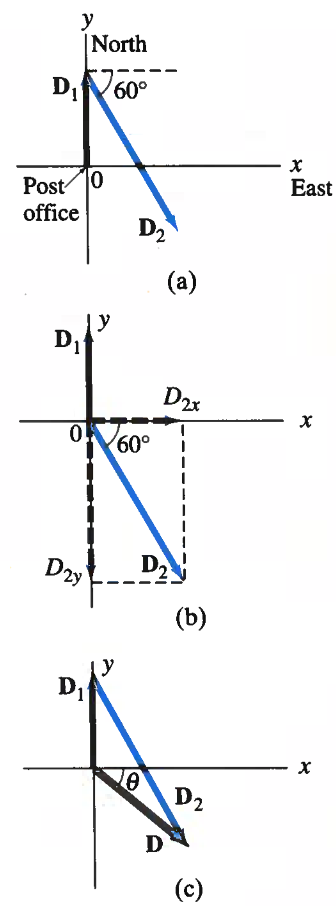


FIGURE 3–15 Example 3–1.

The signs of trigonometric functions depend on which "quadrant" the angle falls: for example, the tangent is positive in the first and third quadrants (from 0° to 90°, and 180° to 270°), but negative in the second and fourth quadrants; see Appendix A–8. The best way to keep track of angles, and to check any vector result, is always to draw a vector diagram. A vector diagram gives you something tangible to look at when analyzing a problem, and provides a check on the results.

➥ PROBLEM SOLVING Adding Vectors

Here is a brief summary of how to add two or more vectors using components:

1. Draw a diagram, adding the vectors graphically.
2. Choose x and y axes. Choose them in a way, if possible, that will make your work easier. (For example, choose one axis along the direction of one of the vectors so that vector will have only one component.)
3. Resolve each vector into its x and y components, showing each component along its appropriate (x or y) axis as a (dashed) arrow.
4. Calculate each component (when not given) using sines and cosines. If θ_1 is the angle vector $\mathbf{V}_1$ makes with the x axis, then:

$$V_{1x} = V_1 \cos\theta_1, \qquad V_{1y} = V_1 \sin\theta_1.$$

Pay careful attention to signs: any component that points along the negative x or y axis gets a negative sign.

5. Add the x components together to get the x component of the resultant. Ditto for y:

$$V_x = V_{1x} + V_{2x} + \text{any others}$$

$$V_y = V_{1y} + V_{2y} + \text{any others.}$$

This is the answer: the components of the resultant vector.

6. If you want to know the magnitude and direction of the resultant vector, use Eqs. 3–4:

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}.$$

The vector diagram you already drew helps to obtain the correct position (quadrant) of the angle θ.

EXAMPLE 3–2 **Three short trips.** An airplane trip involves three legs, with two stopovers, as shown in Fig. 3–16a. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?

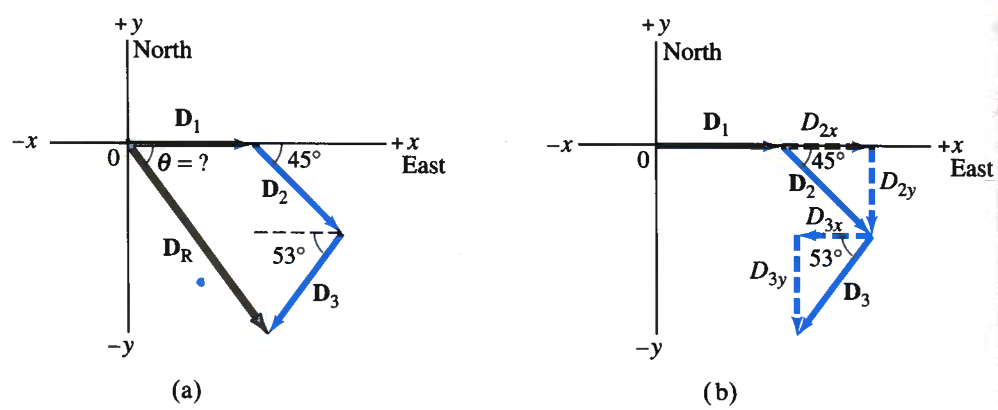


FIGURE 3–16 Example 3–2.

SOLUTION We follow the steps in the above Problem Solving box:
(1) and (2): Already shown in Fig. 3–16a, where we have taken the x axis as east (then $\mathbf{D}_1$ has only an x component).
(3): It is imperative to draw a good figure. The components are shown in Fig. 3–16b. Notice that instead of drawing all the vectors starting from a common origin, as we did in Fig. 3–15b, here we have drawn them "tail-to-tip" style, which is just as valid and may make it easier to see.
(4) Now we calculate the components:

$$\mathbf{D}_1:\; D_{1x} = +D_1 \cos 0° = D_1 = 620\text{ km}$$
$$D_{1y} = +D_1 \sin 0° = 0\text{ km}$$
$$\mathbf{D}_2:\; D_{2x} = +D_2 \cos 45° = +(440\text{ km})(0.707) = +311\text{ km}$$
$$D_{2y} = -D_2 \sin 45° = -(440\text{ km})(0.707) = -311\text{ km}$$
$$\mathbf{D}_3:\; D_{3x} = -D_3 \cos 53° = -(550\text{ km})(0.602) = -331\text{ km}$$
$$D_{3y} = -D_3 \sin 53° = -(550\text{ km})(0.799) = -439\text{ km}.$$

Note carefully that we have given a minus sign to each component that in Fig. 3–16b points in the negative x or negative y direction. We see why a good drawing is so important. We summarize the components in the table in the margin.

Vector	Components x (km)	y (km)
$\mathbf{D}_1$	620	0
$\mathbf{D}_2$	311	−311
$\mathbf{D}_3$	−331	−439
$\mathbf{D}_R$	600	−750

(5) This is easy:

$$D_x = D_{1x} + D_{2x} + D_{3x} = 620\text{ km} + 311\text{ km} - 331\text{ km} = 600\text{ km}$$
$$D_y = D_{1y} + D_{2y} + D_{3y} = 0\text{ km} - 311\text{ km} - 439\text{ km} = -750\text{ km}.$$

The x and y components are 600 km and −750 km, and point respectively to the east and south. This is one way to give the answer.
(6) We can also give the answer as

$$D_R = \sqrt{D_x^2 + D_y^2} = \sqrt{(600)^2 + (-750)^2}\text{ km} = 960\text{ km}$$
$$\tan\theta = \frac{D_y}{D_x} = \frac{-750\text{ km}}{600\text{ km}} = -1.25, \qquad \text{so } \theta = -51°,$$

where we assume only two significant figures. Thus, the total displacement has magnitude 960 km and points 51° below the x axis (south of east), as was shown in our original sketch, Fig. 3–16a.

3–5 Projectile Motion

In Chapter 2, we studied the motion of objects in one dimension in terms of displacement, velocity, and acceleration, including purely vertical motion of falling bodies undergoing acceleration due to gravity. Now we examine the more general motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, speeding bullets, and athletes doing the long jump or high

FIGURE 3–17 This strobe photograph of a soccer ball in the air shows th characteristic "parabolic" path of projectile motion.

jump. These are all examples of **projectile motion** (see Fig. 3–17), which w can describe as taking place in two dimensions. Although air resistance often important, in many cases its effect can be ignored, and we will ignore in the following analysis. We will not be concerned now with the process b which the object is thrown or projected. We consider only its motion *after* has been projected and is moving freely through the air under the action o gravity alone. Thus the acceleration of the object is that due to gravity, whi acts downward with magnitude $g = 9.80\,\mathrm{m/s^2}$, and we assume it is constant

Horizontal and vertical motion analyzed separately

Galileo first accurately described projectile motion. He showed that could be understood by analyzing the horizontal and vertical components the motion separately. This was an innovative analysis, not done in this wa

†This restricts us to objects whose distance traveled and maximum height above the E are small compared to the Earth's radius (6400 km).

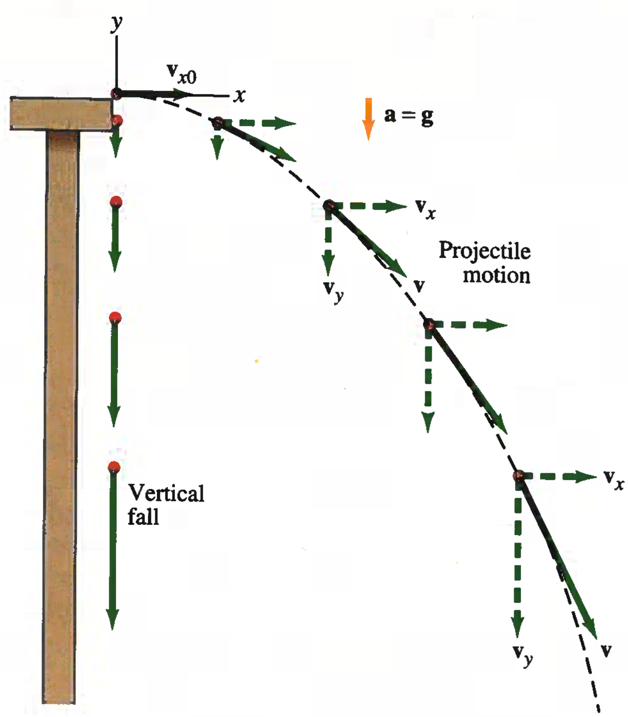
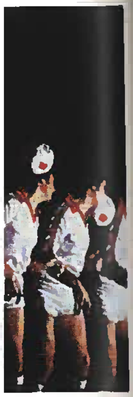


FIGURE 3–18 Projectile motion. (A vertically falling object is shown at the left for comparison.)



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by anyone prior to Galileo. (It was also idealized in that it did not take into account air resistance.) For convenience, we assume that the motion begins at time $t = 0$ at the origin of an xy coordinate system (so $x_0 = y_0 = 0$).

Let us look at a (tiny) ball rolling off the end of a table with an initial velocity v_{x0} in the horizontal (x) direction. See Fig. 3–18 (also shown is an object falling vertically, for comparison). The velocity vector **v** at each instant points in the direction of the ball's motion at that instant and is always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of the velocity, v_x and v_y, separately, and we can apply the kinematic equations (Eqs. 2–10a through 2–10d) to each.

First we examine the vertical (y) component of the motion. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration, g, the acceleration due to gravity. Thus v_y is initially zero but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upwards. Then $a_y = -g$, and from Eq. 2–10a we can write $v_y = -gt$ since the initial velocity in the vertical direction (v_{y0}) is zero. The vertical displacement, y, is given by $y = -\frac{1}{2}gt^2$, if we set $y_0 = 0$.

In the horizontal direction, on the other hand, there is no acceleration. So the horizontal component of velocity, v_x, remains constant, equal to its initial value, v_{x0}, and thus has the same magnitude at each point on the path. The two vector components, $\mathbf{v}_x$ and $\mathbf{v}_y$, can be added vectorially to obtain the velocity **v** for each point on the path, as shown in Fig. 3–18.

One result of this analysis, which Galileo himself predicted, is that *an object projected horizontally will reach the ground in the same time as an object dropped vertically*. This is because the vertical motions are the same in both cases, as shown in Fig. 3–18 where on the left a falling object is shown. Figure 3–19 is a multiple-exposure photograph of an experiment that confirms this.

If an object is projected at an upward angle, as in Fig. 3–20, the analysis is similar, except that now there is an initial vertical component of velocity, v_{y0}. Because of the downward acceleration of gravity, v_y continually decreases until the object reaches the highest point on its path in Fig. 3–20, at which point $v_y = 0$. Then v_y starts to increase in the downward direction, as shown (that is, becoming negative). As before, v_x remains constant.

FIGURE 3–19 Multiple-exposure photograph showing positions of two balls at equal time intervals. One ball was dropped from rest at the same time the other was projected horizontally outward. The vertical position of each ball is seen to be the same.

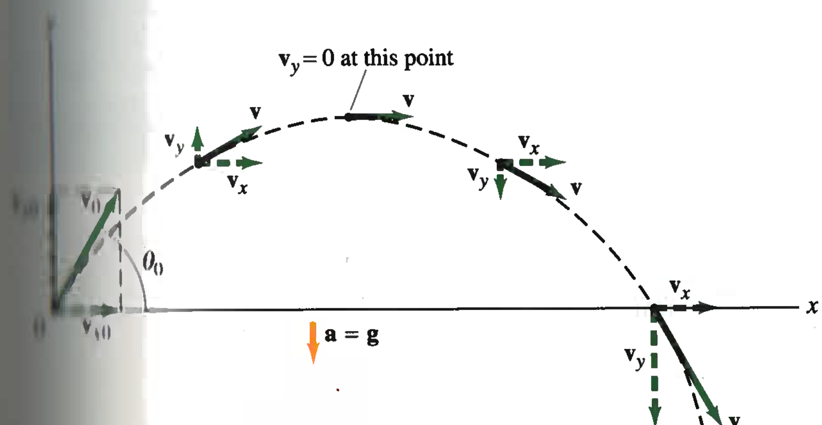


FIGURE 3–20 Path of a projectile fired with initial velocity $\mathbf{v}_0$ at angle θ to the horizontal. Path is shown in black, the velocity vectors are green arrows, and velocity components are dashed.

3–6 Solving Problems Involving Projectile Motion

We now work through several Examples of projectile motion quantitatively. We use the kinematic equations (2–10a through 2–10c) separately for the vertical and horizontal components of the motion. These equations are shown separately for the x and y components of the motion in Table 3–1, for the general case of two-dimensional motion. Note that x and y are the respective displacements, that v_x and v_y are the components of the velocity, and that a_x and a_y are the components of the acceleration. The subscript $_0$ means "at $t = 0$."

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

x Component (horizontal)		*y* Component (vertical)
$v_x = v_{x0} + a_x t$	(Eq. 2–10a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	(Eq. 2–10b)	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–10c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify these equations for the case of projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\ \text{m/s}^2$. Note that if θ is chosen relative to the $+x$ axis, as in Fig. 3–20, then $v_{x0} = v_0 \cos\theta$, and $v_{y0} = v_0 \sin\theta$.

TABLE 3–2 Kinematic Equations for Projectile Motion (y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \text{m/s}^2$)

Horizontal Motion ($a_x = 0$, $v_x =$ constant)		Vertical Motion† ($a_y = -g =$ constant)
$v_x = v_{x0}$	(Eq. 2–10a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–10b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–10c)	$v_y^2 = v_{y0}^2 - 2gy$

†If y is taken positive downward, the minus (−) signs become + signs.

PROBLEM SOLVING Projectile Motion

The approach to solving problems that we discussed in Section 2–6 also applies here. However, solving problems involving projectile motion can require a bit of creativity, and cannot be done just by simply following some rules. Certainly you must avoid just plugging numbers into equations that seem to "work."

As always, **read** carefully and **draw** a careful diagram.

1. Choose an origin and an xy coordinate system.
2. Analyze the horizontal (x) motion and the vertical (y) motion separately. If you are given the initial velocity, you may want to resolve it into its x and y components.
3. List the known and unknown quantities, choosing $a_x = 0$ and $a_y = -g$ or $+g$, where $g = 9.80\,\text{m/s}^2$, depending on whether you choose y positive up or down. Remember that v_x never changes throughout the trajectory, and that $v_y = 0$ at the highest point of any trajectory that returns downward. The velocity just before hitting the ground is generally not zero.
4. Think for a minute before jumping into the equations. A little planning goes a long way. Apply the relevant equations (Table 3–2), combining equations if necessary. You may need to combine components of a vector to get magnitude and direction (Eqs. 3–4).

EXAMPLE 3–3 **Driving off a cliff.** A movie stunt driver on a motorcycle speeds horizontally off a 50.0-m-high cliff. How fast must the motorcycle leave the cliff-top if it is to land on level ground below, 90.0 m from the base of the cliff (Fig. 3–21) where the cameras are?

SOLUTION We take the y direction to be positive upward, with the top of the cliff as $y_0 = 0$, so the bottom is at $y = -50.0$ m. First, we find how long it takes the motorcycle to reach the ground below. We use Eq. 2–10b for the vertical (y) direction (Table 3–2) with $y_0 = 0$, and $v_{y0} = 0$:

$$y = -\tfrac{1}{2}gt^2$$

We solve for t and set $y = -50.0\,\text{m}$:

$$t = \sqrt{\frac{2y}{-g}} = \sqrt{\frac{2(-50.0\,\text{m})}{-9.80\,\text{m/s}^2}} = 3.19\ \text{s}.$$

To calculate the initial velocity, v_{x0}, we again use Eq. 2–10b, but this time for the horizontal (x) direction, with $a_x = 0$ and $x_0 = 0$:

$$x = v_{x0}t$$

$$v_{x0} = \frac{x}{t} = \frac{90.0\,\text{m}}{3.19\,\text{s}} = 28.2\,\text{m/s},$$

which is 101 km/h.

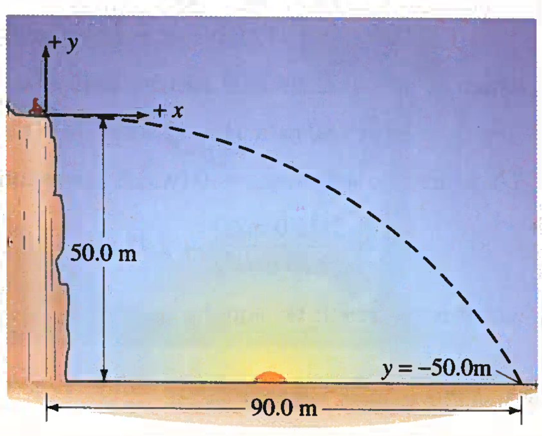


FIGURE 3–21 Example 3–3.

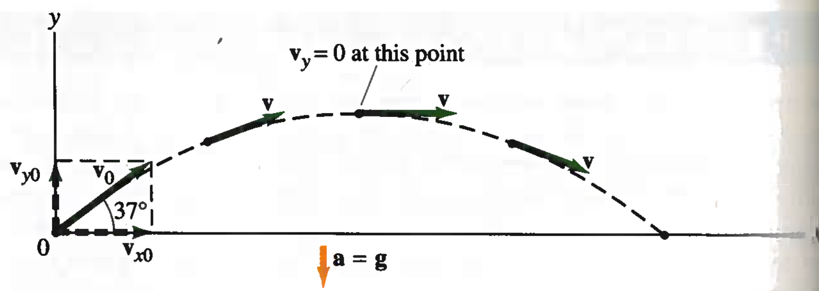


FIGURE 3–22 Example 3–4.

PHYSICS APPLIED

Sports

EXAMPLE 3–4 **A kicked football.** A football is kicked at an angle $\theta_0 = 37.0°$ with a velocity of 20.0 m/s, as shown in Fig. 3–22. Calculate (*a*) the maximum height, (*b*) the time of travel before the football hits the ground, (*c*) how far away it hits the ground, (*d*) the velocity vector at the maximum height, and (*e*) the acceleration vector at maximum height. Assume the ball leaves the foot at ground level.

SOLUTION This may seem difficult because there are so many questions. But we can deal with them one at a time. We take the y direction as positive upward. The components of the initial velocity are (Fig. 3–22)

$$v_{x0} = v_0 \cos 37.0° = (20.0\,\mathrm{m/s})(0.799) = 16.0\,\mathrm{m/s}$$

$$v_{y0} = v_0 \sin 37.0° = (20.0\,\mathrm{m/s})(0.602) = 12.0\,\mathrm{m/s}.$$

(*a*) At the maximum height, the velocity is horizontal (Fig. 3–22), so $v_y = 0$; and this occurs (see Eq. 2–10a in Table 3–2) at time

$$t = v_{y0}/g = (12.0\,\mathrm{m/s})/(9.80\,\mathrm{m/s^2}) = 1.22\,\mathrm{s}.$$

From Eq. 2–10b, with $y_0 = 0$, we have

$$\begin{aligned} y &= v_{y0}t - \tfrac{1}{2}gt^2 \\ &= (12.0\,\mathrm{m/s})(1.22\,\mathrm{s}) - \tfrac{1}{2}(9.80\,\mathrm{m/s^2})(1.22\,\mathrm{s})^2 = 7.35\,\mathrm{m}. \end{aligned}$$

Alternatively, we could have used Eq. 2–10c, solved for y, and found

$$y = \frac{v_{y0}^2 - v_y^2}{2g} = \frac{(12.0\,\mathrm{m/s})^2 - (0\,\mathrm{m/s})^2}{2(9.80\,\mathrm{m/s^2})} = 7.35\,\mathrm{m}.$$

(*b*) To find the time it takes for the ball to return to the ground, we use Eq. 2–10b with $y_0 = 0$ and also set $y = 0$ (ground level):

$$\begin{aligned} y &= y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \\ 0 &= 0 + (12.0\,\mathrm{m/s})t - \tfrac{1}{2}(9.80\,\mathrm{m/s^2})t^2 \end{aligned}$$

which is an equation that can be easily factored:

$$[\tfrac{1}{2}(9.80\,\mathrm{m/s^2})t - 12.0\,\mathrm{m/s}]t = 0.$$

There are two solutions, $t = 0$ (which corresponds to the initial point, y_0), and

$$t = \frac{2(12.0\,\mathrm{m/s})}{(9.80\,\mathrm{m/s^2})} = 2.45\,\mathrm{s},$$

which is the result we sought.

(c) The total distance traveled in the x direction is found by applying Eq. 2–10b with $x_0 = 0$, $a_x = 0$, $v_{x0} = 16.0\,\mathrm{m/s}$:

$$x = v_{x0}t = (16.0\,\mathrm{m/s})(2.45\,\mathrm{s}) = 39.2\,\mathrm{m}.$$

(d) At the highest point, there is no vertical component to the velocity. There is only the horizontal component (which remains constant throughout the flight), so $v = v_{x0} = v_0 \cos 37.0° = 16.0\,\mathrm{m/s}$.

(e) The acceleration vector is the same at the highest point as it is throughout the flight, which is $9.80\,\mathrm{m/s^2}$ downward.

CONCEPTUAL EXAMPLE 3–5 **Where does the apple land?** A child sits upright in a wagon which is moving to the right at constant speed as shown in Fig. 3–23. The child extends her hand and throws an apple straight upward (from her own point of view, Fig. 3–23a), while the wagon continues to travel forward at constant speed. If air resistance is neglected, will the apple land (*a*) behind the wagon, (*b*) in the wagon, or (*c*) in front of the wagon?

RESPONSE The child throws the apple straight up from her own point of view with initial velocity $\mathbf{v}_{0y}$ (Fig. 3–23a). But when viewed by someone on the ground, the apple also has an initial horizontal component of velocity equal to the speed of the wagon, $\mathbf{v}_{0x}$. Thus, to a person on the ground, the apple will follow the path of a projectile as shown in Fig. 3–23b. The apple experiences no horizontal acceleration, so $\mathbf{v}_{0x}$ will stay constant and equal to the speed of the wagon. As the apple follows its arc, the wagon will be directly under the apple at all times because they have the same horizontal velocity. When the apple comes down, it will drop right into the wagon, and into the outstretched hand of the child. The answer is (*b*).

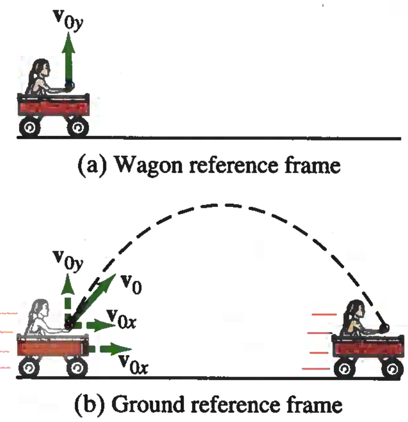


FIGURE 3–23 Conceptual Example 3–5.

CONCEPTUAL EXAMPLE 3–6 **The wrong strategy.** A boy on a small hill aims his water-balloon slingshot horizontally, straight at a second boy hanging from a tree branch a distance d away, Fig. 3–24. At the instant the water balloon is released, the second boy lets go and falls from the tree, hoping to avoid being hit. Show that he made the wrong move. (He hadn't studied physics yet.)

RESPONSE Both the water balloon and the boy in the tree start falling at the same instant, and in a time t they each fall the same vertical distance $y = \frac{1}{2}gt^2$. In the time it takes the water balloon to travel the horizontal distance d, the balloon will have the same y position as the falling boy. Splat. If the boy had stayed in the tree, he'd have saved himself the humiliation.

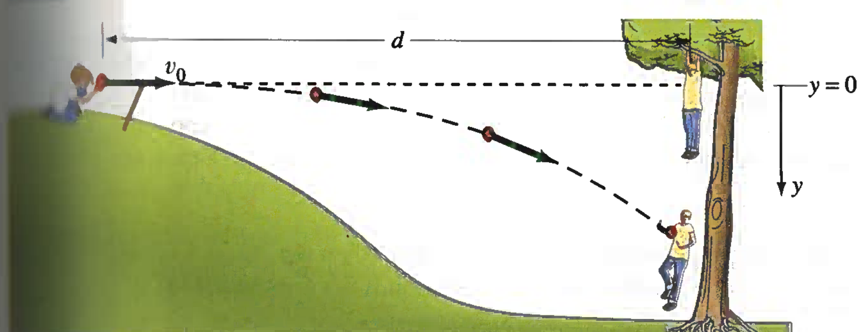


FIGURE 3–24 Conceptual Example 3–6.

PHYSICS APPLIED

Horizontal range of a projectile

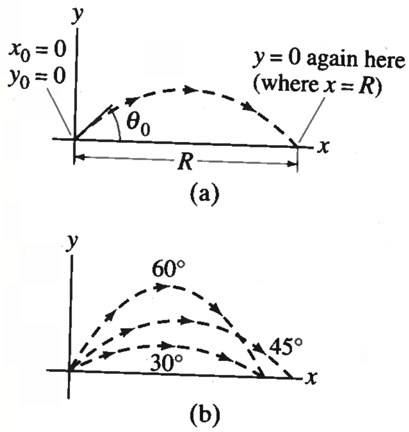


FIGURE 3–25 Example 3–7. (a) The range R of a projectile; (b) shows how generally there are two angles θ_0 that will give the same range. Can you show that if one angle is θ_{01}, the other is $\theta_{02} = 90° - \theta_{01}$?

EXAMPLE 3–7 **Level horizontal range.** (*a*) Derive a formula for the hor-izontal range R of a projectile in terms of its initial velocity v_0 and angle θ_0. The horizontal range is defined as the horizontal distance the projectile trav-els before returning to its original height (which is typically the ground); that is, y (final) $= y_0$. See Fig. 3–25. (*b*) Suppose one of Napoleon's cannons had a muzzle velocity, v_0, of 60.0 m/s. At what angle should it have been aimed (ignore air resistance) to strike a target 320 m away?

SOLUTION (*a*) We set $x_0 = 0$ and $y_0 = 0$ at $t = 0$. After the projectile travels a horizontal distance R, it returns to the same level, $y = 0$, the final point. So to find a general expression for R, we set both $y = 0$ and $y_0 = 0$ in Eq. 2–10b for the vertical motion, and obtain

$$v_{y0}t - \tfrac{1}{2}gt^2 = 0.$$

We solve for t, which gives two solutions: $t = 0$ and $t = 2v_{y0}/g$. The first solution corresponds to the initial instant of projection and the second is the time when the projectile returns to $y = 0$. Then the range, R, will be equal to x at the moment t has this value, which we put into Eq. 2–10a for the horizontal motion ($x = v_{x0}t$, with $x_0 = 0$). Thus we have:

$$R = x = v_{x0}t = v_{x0}\left(\frac{2v_{y0}}{g}\right) = \frac{2v_{x0}v_{y0}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g} \qquad [y = y_0]$$

where we have written $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$. This is the result we sought, and it can be rewritten, using the trigonometric identity $2 \sin\theta \cos\theta = \sin 2\theta$ (Appendix A), as

Level range formula [y (*final*) $= y_0$]

$$R = \frac{v_0^2 \sin 2\theta_0}{g}. \qquad [y = y_0]$$

We see that the maximum range, for a given initial velocity, v_0, is obtained when the sine takes on its maximum value of 1.0, which occurs for $2\theta_0 = 90°$; so

$$\theta_0 = 45° \text{ for maximum range, and } R_{max} = v_0^2/g.$$

[When air resistance is important, the range is less for a given v_0, and the maximum range is obtained at an angle smaller than 45°.] Note that the maximum range increases by the square of v_0, so doubling the muzzle velocity of a cannon increases its maximum range by a factor of 4.

(*b*) From the equation we just derived, Napoleon's cannon should be aimed (assuming, unrealistically, no air resistance) at an angle θ_0 given by

$$\sin 2\theta_0 = \frac{Rg}{v_0^2} = \frac{(320\text{ m})(9.80\text{ m/s}^2)}{(60.0\text{ m/s})^2} = 0.871.$$

We want to solve for an angle θ_0 that is between 0° and 90°, which means $2\theta_0$ in this equation can be as large as 180°. Thus, $2\theta_0 = 60.6°$ is a solution, but $2\theta_0 = 180° - 60.6° = 119.4°$ is also a solution (see Appendix A–8). In general we will have two solutions, which in Napoleon's case are given by

$$\theta_0 = 30.3° \text{ or } 59.7°.$$

Either angle gives the same range. Only when $\sin 2\theta_0 = 1$ (so $\theta_0 = 45°$) is there a single solution (that is, both solutions are the same).

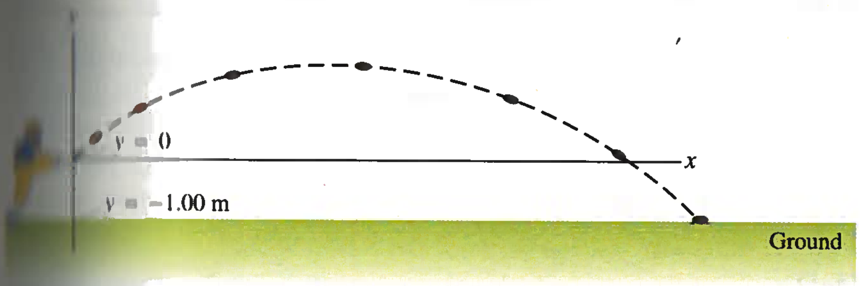


FIGURE 3–26 Example 3–8: the football leaves the punter's foot at $y = 0$, and reaches the ground where $y = -1.00$ m.

EXAMPLE 3–8 **A punt.** Suppose the football in Example 3–4 was a punt and left the punter's foot at a height of 1.00 m above the ground. How far did the football travel before hitting the ground? Set $x_0 = 0$, $y_0 = 0$.

SOLUTION We cannot use the range formula from Example 3–7 because it is valid only if y (final) $= y_0$, which is not the case here. Now we have $y_0 = 0$, and the football hits the ground where $y = -1.00$ m (see Fig. 3–26). We can get x from Eq. 2–10b, $x = v_{x0}t$, since we know that $v_{x0} = 16.0$ m/s. But first we must find t, the time at which the ball hits the ground. With $y = -1.00$ m and $v_{y0} = 12.0$ m/s (see Example 3–4), we use the equation

$$y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2,$$

and obtain

$$-1.00\text{ m} = 0 + (12.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

We rearrange this equation into standard form so we can use the quadratic formula (Appendix A–4):

$$(4.90\text{ m/s}^2)t^2 - (12.0\text{ m/s})t - (1.00\text{ m}) = 0.$$

Using the quadratic formula gives

$$t = \frac{12.0\text{ m/s} \pm \sqrt{(12.0\text{ m/s})^2 - 4(4.90\text{ m/s}^2)(-1.00\text{ m})}}{2(4.90\text{ m/s}^2)}$$

$$= 2.53\text{ s} \qquad \text{or} \qquad -0.081\text{ s}.$$

The second solution would correspond to a time previous to the kick, so it doesn't apply here. With $t = 2.53$ s for the time at which the ball touches the ground, the distance the ball traveled is (putting $v_{x0} = 16.0$ m/s, from Example 3–4):

$$x = v_{x0}t = (16.0\text{ m/s})(2.53\text{ s}) = 40.5\text{ m}.$$

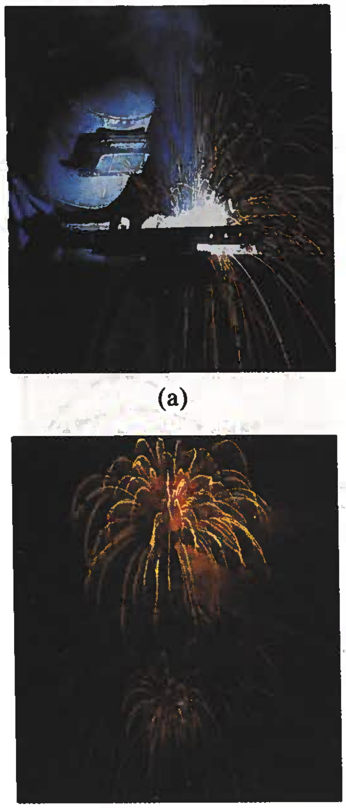
Note that our assumption in Example 3–4 that the ball leaves the foot at ground level results in an underestimate of about 1.3 m in the distance traveled.

PHYSICS APPLIED

Sports

PROBLEM SOLVING

Do not use any formula unless you are sure its range of validity fits the problem. The range formula does not apply here because $y \neq y_0$



(a)

(b)

FIGURE 3–27 Examples of projectile motion—sparks (small hot glowing pieces of metal) and fireworks. Both exhibit the parabolic path characteristic of projectile motion, although the effects of air resistance can be seen to alter the path of some trajectories.

* 3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a parabola, if we can ignore air resistance and can assume that **g** is constant. To do so, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–10b), and we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2.$$

If we write $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$, we can also write

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2.$$

In either case, we see that y as a function of x has the form

$$y = ax - bx^2,$$

where a and b are constants for any specific projectile motion. This is the well-known equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

* 3–8 Relative Velocity

We now consider how observations made in different reference frames are related to each other. For example, consider two trains approaching one another, each with a speed of 80 km/h with respect to the Earth. Observers on the Earth beside the tracks will measure 80 km/h for the speed of each of the trains. Observers on either of the trains (a different reference frame) will measure a speed of 160 km/h for the other train approaching them. Similarly, when one car traveling 90 km/h passes a second car traveling in the same direction at 75 km/h, the first car has a speed relative to the second car of 90 km/h − 75 km/h = 15 km/h.

When the velocities are along the same line, simple addition or subtraction is sufficient to obtain the relative velocity. But if they are not along the same line, we must make use of vector addition. We emphasize, as mentioned in Section 2–1, that when specifying a velocity, it is important to specify what the reference frame is.

When determining relative velocity, it is easy to make a mistake by adding or subtracting the wrong velocities. It is important, therefore, to draw a diagram and use a careful labeling process that makes things clear. Each velocity is labeled by *two subscripts: the first refers to the object, the second to the reference frame in which it has this velocity*. For example

Suppose a boat is to cross a river to the opposite side, as shown in Fig. 3–28. We let $\mathbf{v}_{BW}$ be the velocity of the **B**oat with respect to the **W**ater. (This is also what the boat's velocity would be relative to the shore if the water were still.) Similarly, $\mathbf{v}_{BS}$ is the velocity of the **B**oat with respect to the **S**hore, and $\mathbf{v}_{WS}$ is the velocity of the **W**ater with respect to the **S**hore (this is the river current). Note that $\mathbf{v}_{BW}$ is what the boat's motor produces (against the water), whereas $\mathbf{v}_{BS}$ is equal to $\mathbf{v}_{BW}$ plus the effect of the current. Therefore, the velocity of the boat relative to the shore is (see vector diagram, Fig. 3–28)

$$\mathbf{v}_{BS} = \mathbf{v}_{BW} + \mathbf{v}_{WS}. \tag{3–6}$$

By writing the subscripts via the convention above, we see that the inner subscripts (the two W's) on the right-hand side of Eq. 3–6 are the same, whereas the outer subscripts on the right of Eq. 3–6 (the B and the S) are the same as the two subscripts for the sum vector on the left, $\mathbf{v}_{BS}$. By following this convention (first subscript for the object, second for the reference frame), one can write down the correct equation relating velocities in different reference frames.† Equation 3–6 is valid in general and can be extended to three or more velocities. For example, if a fisherman on the boat walks with a velocity $\mathbf{v}_{FB}$ relative to the boat, his velocity relative to the shore is $\mathbf{v}_{FS} = \mathbf{v}_{FB} + \mathbf{v}_{BW} + \mathbf{v}_{WS}$. The equations involving relative velocity will be correct when adjacent inner subscripts are identical and when the outermost ones correspond exactly to the two on the velocity on the left of the equation. But this works only with plus signs (on the right), not minus signs.

It is often useful to remember that for any two objects or reference frames, *A* and *B*, the velocity of *A* relative to *B* has the same magnitude, but opposite direction, as the velocity of *B* relative to *A*:

$$\mathbf{v}_{BA} = -\mathbf{v}_{AB}. \tag{3–7}$$

For example, if a train is traveling 100 km/h relative to the Earth in a certain direction, objects on the Earth (such as trees) appear to an observer on the train to be traveling 100 km/h in the opposite direction.

CONCEPTUAL EXAMPLE 3–9 **Crossing a river.** A man in a rowboat is trying to cross a river that flows due west with a strong current. The man starts on the south bank and is trying to reach the north bank directly north from his starting point. He should:

1. head due north.
2. head due west.
3. head in a northwesterly direction.
4. head in a northeasterly direction.

RESPONSE The current will drag the boat westward, so to counteract that motion the boat must head in a northeasterly direction (see Fig. 3–28). The actual angle depends on the strength of the current and how fast the boat moves relative to the water. If the current is weak and the rower is strong, then the boat can head almost, but not quite, due north.

† We thus would know by inspection that (for example) the equation $\mathbf{v}_{BW} = \mathbf{v}_{BS} + \mathbf{v}_{WS}$ is wrong.

PROBLEM SOLVING

Subscripts for adding velocities: first subscript for the object; second for the reference frame

Follow the subscripts

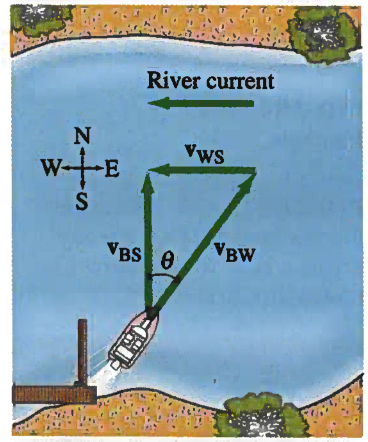


FIGURE 3–28 The boat must head upstream at an angle θ if it is to move directly across the river. Velocity vectors are shown as green arrows:
$\mathbf{v}_{BS}$ = velocity of **B**oat with respect to the **S**hore,
$\mathbf{v}_{BW}$ = velocity of **B**oat with respect to the **W**ater,
$\mathbf{v}_{WS}$ = velocity of the **W**ater with respect to the **S**hore (river current).

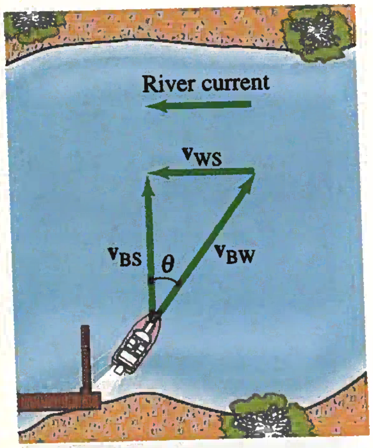
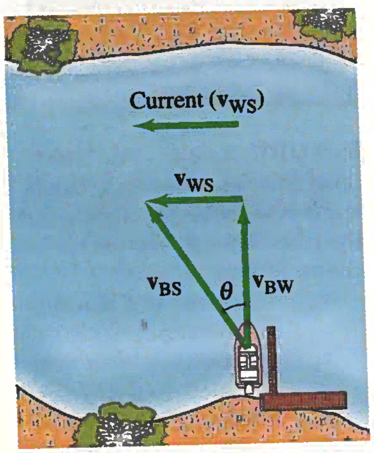


FIGURE 3–29
Example 3–10.

FIGURE 3–30 Example 3–11: a boat heading directly across a river whose current moves at 1.20 m/s.



EXAMPLE 3–10 **Heading upstream.** A boat's speed in still wat… v_{BW} = 1.85 m/s. If the boat is to travel directly across a river whose … rent has speed v_{WS} = 1.20 m/s, at what upstream angle must the … head? (See Fig. 3–29.)

SOLUTION Figure 3–29 has been drawn with $\mathbf{v}_{BS}$, the velocity of the Boat relative to the Shore, pointing directly across the river since this is how the boat is supposed to move. (Note that $\mathbf{v}_{BS} = \mathbf{v}_{BW} + \mathbf{v}_{WS}$.) To accomplish this, the boat needs to head upstream to offset the current pulling it downstream. Thus, $\mathbf{v}_{BW}$ points upstream at an angle θ as sh… From the diagram,

$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{1.20\text{ m/s}}{1.85\text{ m/s}} = 0.6486.$$

Thus $\theta = 40.4°$, so the boat must head upstream at a 40.4° angle.

EXAMPLE 3–11 **Heading across the stream.** The same boat (v_{BW} = 1.85 m/s) now heads directly across the stream whose current is … 1.20 m/s. (*a*) What is the velocity (magnitude and direction) of the … relative to the shore? (*b*) If the river is 110 m wide, how long will it … to cross and how far downstream will the boat be then?

SOLUTION (*a*) As shown in Fig. 3–30, the boat is pulled downst… by the current. The boat's velocity with respect to the shore, $\mathbf{v}_{BS}$, is … sum of its velocity with respect to the water, $\mathbf{v}_{BW}$, plus the velocity of … water with respect to the shore, $\mathbf{v}_{WS}$:

$$\mathbf{v}_{BS} = \mathbf{v}_{BW} + \mathbf{v}_{WS},$$

just as before. Since $\mathbf{v}_{BW}$ is perpendicular to $\mathbf{v}_{WS}$, we can get v_{BS} using … theorem of Pythagoras:

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(1.85\text{ m/s})^2 + (1.20\text{ m/s})^2} = 2.21 …$$

We can obtain the angle (note how θ is defined in diagram) from:

$$\tan\theta = v_{WS}/v_{BW} = (1.20\text{ m/s})/(1.85\text{ m/s}) = 0.6486.$$

A calculator with an INV TAN or TAN^{-1} key gives $\theta = \tan^{-1}(0.6486) = …$ Note that this angle is not equal to the angle calculated in Example 3–… (*b*) Given the river's width D = 120 m and using the definition of velocity, we solve for $t = D/v_{BW}$, where we use the velocity component in the direction of D; so t = 110 m/1.85 m/s = 60 s. The boat will have … carried downstream, in this time, a distance

$$d = v_{WS}t = (1.20\text{ m/s})(60\text{ s}) = 72\text{ m}.$$

EXAMPLE 3–12 **Airplane with crosswind.** A plane whose airspe… 200 km/h heads due north. But a 100-km/h northeast wind (that is, … ing from the northeast) suddenly begins to blow. What is the resulting … locity of the plane with respect to the ground?

SOLUTION The two velocity vectors, and their components, are sh… in Fig. 3–31a. They are drawn with a common origin for conven… $\mathbf{v}_{PA}$ represents the velocity of the plane with respect to the air; and … wind velocity is $\mathbf{v}_{AG}$, the velocity of the air with respect to the gr… The resultant velocity, $\mathbf{v}_{PG}$, the velocity of the plane with respect to …

ground, is given by:

$$\mathbf{v}_{PG} = \mathbf{v}_{PA} + \mathbf{v}_{AG}.$$

Note the use of our subscript rule, Eq. 3–6 above. Since $\mathbf{v}_{PA}$ is along the y axis, it has only a y component:

$$v_{PAx} = 0\ \text{km/h}$$

$$v_{PAy} = v_{PA} = 200\ \text{km/h}.$$

The components of v_{AG} are (note unconventional choice of 45° angle, below negative x axis):

$$v_{AGx} = -v_{AG}\cos 45° = -(100\ \text{km/h})(0.707) = -70.7\ \text{km/h}$$

$$v_{AGy} = -v_{AG}\sin 45° = -(100\ \text{km/h})(0.707) = -70.7\ \text{km/h}.$$

Both v_{AGx} and v_{AGy} are negative because their directions are, respectively, along the negative x and negative y axes. The components of the resultant velocity are

$$v_{PGx} = \quad 0\ \text{km/h} - 70.7\ \text{km/h} = -70.7\ \text{km/h},$$

$$v_{PGy} = 200\ \text{km/h} - 70.7\ \text{km/h} = +129\ \text{km/h}.$$

We find the magnitude of the resultant velocity using the Pythagorean theorem:

$$v_{PG} = \sqrt{v_{PGx}^2 + v_{PGy}^2} = 147\ \text{km/h}.$$

To find the angle θ that $\mathbf{v}_{PG}$ makes with the x axis (Fig. 3–31b), we use

$$\tan\theta = \frac{v_{PGy}}{v_{PGx}} = \frac{129\ \text{km/h}}{-70.7\ \text{km/h}} = -1.825.$$

(The negative sign results because θ is with respect to the negative x axis, which we already know from the diagram.) Then

$$\theta = \tan^{-1}(-1.825) = -61.3°.$$

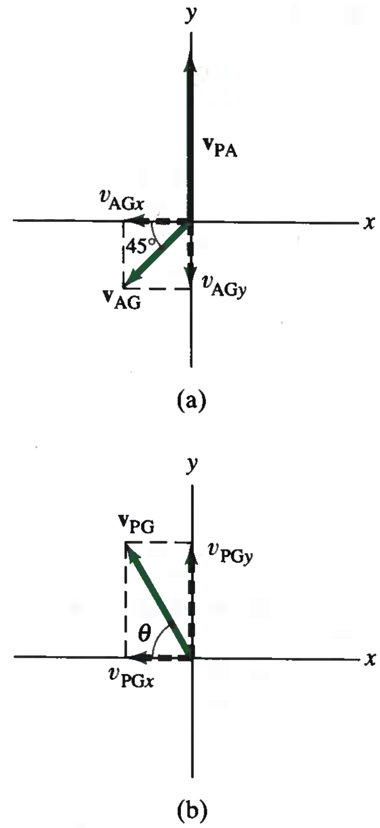


FIGURE 3–31
Example 3–12.

SUMMARY

A quantity that has both a magnitude and a direction is called a **vector**. A quantity that has only a magnitude is called a **scalar**.

Addition of vectors can be done graphically by placing the tail of each successive arrow at the tip of the previous one. The sum, or **resultant vector**, is the arrow drawn from the tail of the first to the tip of the last. Two vectors can also be added using the parallelogram method.

Vectors can be added more accurately by adding their **components** along chosen axes with the aid of trigonometric functions. A vector of magnitude V making an angle θ with the x axis has components

$$V_x = V\cos\theta \qquad V_y = V\sin\theta.$$

Given the components, we can find the magnitude and direction of a vector from

$$V = \sqrt{V_x^2 + V_y^2}, \qquad \tan\theta = \frac{V_y}{V_x}.$$

Projectile motion of an object moving in an arc near the Earth's surface can be analyzed as two separate motions if air resistance can be ignored. The horizontal component of motion is at constant velocity, whereas the vertical component is at constant acceleration, $\mathbf{g}$, just as for a body falling vertically under the action of gravity.

The velocity of an object relative to one frame of reference can be found by vector addition if its velocity relative to a second frame of reference, and the *relative velocity* of the two reference frames, are known.

QUESTIONS

1. Does the odometer of a car measure a scalar or a vector quantity? What about the speedometer?
2. Two vectors are added together, and their vector sum is zero. What can you say about the magnitude and direction of the two initial vectors?
3. Can the displacement vector for an object moving in two dimensions ever be longer than the length of path traveled by the object over the same time interval? Can it ever be less? Discuss.
4. During baseball practice, a batter hits a very high fly ball, and then runs in a straight line and catches it. Which had the greater displacement, the player or the ball?
5. If $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$, is V necessarily greater than V_1 and/or V_2? Discuss.
6. Two vectors have magnitudes $V_1 = 3.5$ km and $V_2 = 4.0$ km. What are the maximum and minimum magnitudes of their vector sum?
7. Can two vectors, of unequal magnitude, ever add up to give the zero vector? Can *three* unequal vectors? Under what conditions?
8. Can the magnitude of a vector ever (*a*) equal, or (*b*) be less than, one of its components?
9. Can a vector of magnitude zero have a nonze component?
10. One car travels due east at 50 km/h, and a secon car travels north at 50 km/h. Are their velocit equal? Explain.
11. A projectile has the least speed at what point in its pat
12. What physical factors are important for an athle doing the long jump? What about the high jump?
13. A child wishes to determine the speed a slingsh imparts to a rock. How can this be done using only meter stick, a rock, and the slingshot?
14. If you are riding on a train that speeds past anoth train moving in the same direction on an adjace track, it appears that the slower moving train is mo ing backwards. Why?
15. Two rowers, who can row at the same speed in st water, set off across a river at the same time. O heads straight across and is pulled downstream som what by the current. The other one heads upstream an angle so as to arrive at a point opposite the starti point. Which rower reaches the opposite side first?

PROBLEMS

SECTIONS 3–2 TO 3–4

1. (I) A car is driven 125 km west and then 65 km southwest. What is the displacement of the car from the point of origin (magnitude and direction)? Draw a diagram.
2. (I) A delivery truck travels 14 blocks north, 16 blocks east, and 26 blocks south. What is its final displacement from the origin? Assume the blocks are equal length.
3. (I) The three vectors in Fig. 3–32 can be added in six different orders ($\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3$, $\mathbf{V}_1 + \mathbf{V}_3 + \mathbf{V}_2$, etc.). Show on a diagram that the same resultant is obtained no matter what the order.
4. (I) If $V_x = 18.80$ units and $V_y = -16.40$ units, determine the magnitude and direction of $\mathbf{V}$.
5. (II) Graphically determine the resultant of the following three vector displacements: (1) 24 m, 30° north of east; (2) 28 m, 37° east of north; and (3) 20 m, 50° west of south.

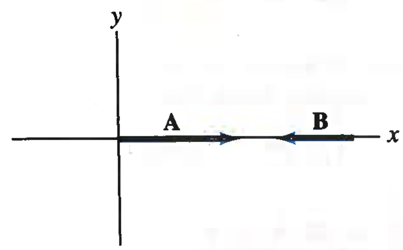
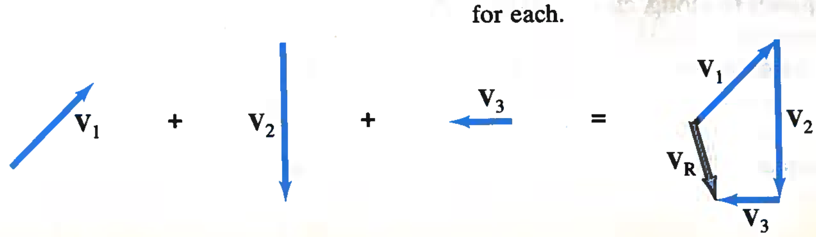


FIGURE 3–33 Problem 7.

6. (II) $\mathbf{V}$ is a vector 24.3 units in magnitude and point an angle of 54.8° above the negative x axis. (*a*) Sket this vector. (*b*) Find V_x and V_y. (*c*) Use V_x and V_y to o tain (again) the magnitude and direction of $\mathbf{V}$. [N Part (*c*) is a good way to check if you've resolved yo vector correctly.]
7. (II) Figure 3–33 shows two vectors, $\mathbf{A}$ and $\mathbf{B}$, wh magnitudes are $A = 8.31$ units and $B = 5.55$ uni Determine $\mathbf{C}$ if (*a*) $\mathbf{C} = \mathbf{A} + \mathbf{B}$, (*b*) $\mathbf{C} = \mathbf{A}$ – (*c*) $\mathbf{C} = \mathbf{B} - \mathbf{A}$. Give the magnitude and directi for each.

FIGURE 3–32 Problem 3.



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axis. (a) Sketch
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A and **B**, whose
B = 5.55 units,
b) **C** = **A** − **B**,
le and direction

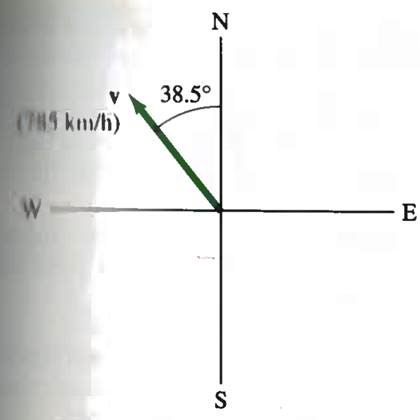

FIGURE 3–34 Problem 9.

8. (II) Vector $\mathbf{V}_1$ is 8.08 units long and points along the negative x axis. Vector $\mathbf{V}_2$ is 4.51 units long and points at +45.0° to the positive x axis. (*a*) What are the x and y components of each vector? (*b*) Determine the sum of the two vectors (magnitude and angle).

9. (I) An airplane is traveling 785 km/h in a direction 38.5° west of north (Fig. 3–34). (*a*) Find the components of the velocity vector in the northerly and westerly directions. (*b*) How far north and how far west has the plane traveled after 3.00 h?

10. (II) The components of a vector **V** are often written (V_x, V_y, V_z). What are the components and length of a vector which is the sum of the two vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, whose components are (3.0, 2.7, 0.0) and (2.9, −4.1, −1.4)?

11. (II) Three vectors are shown in Fig. 3–35. Their magnitudes are given in arbitrary units. Determine the sum of the three vectors. Give the resultant in terms of (*a*) components, (*b*) magnitude and angle with x axis.

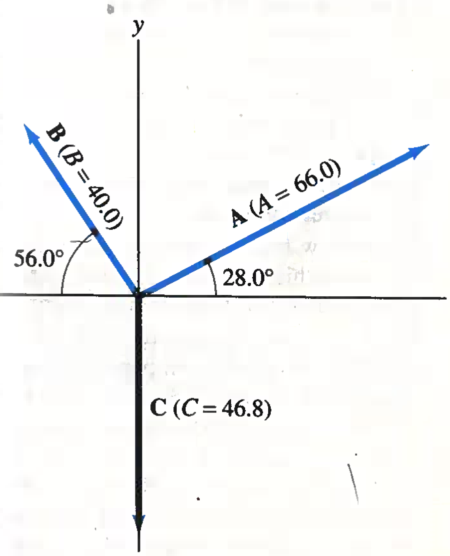

FIGURE 3–35 Problems 11, 12, 13, 14, and 15. Vector magnitudes are given in arbitrary units.

12. (II) Determine the vector **A** − **C**, given the vectors **A** and **C** in Fig. 3–35.

13. (II) (*a*) Given the vectors **A** and **B** shown in Fig. 3–35, determine **B** − **A**. (*b*) Determine **A** − **B** without using your answer in (*a*). Then compare your results and see if they are opposite.

14. (II) For the vectors given in Fig. 3–35, determine (*a*) **A** − **B** + **C**, (*b*) **A** + **B** − **C**, and (*c*) **B** − 2**A**.

15. (II) For the vectors shown in Fig. 3–35, determine (*a*) **C** − **A** − **B**, (*b*) 2**A** − 3**B** + 2**C**.

16. (II) (*a*) A skier is accelerating down a 30.0° hill at 3.80 m/s² (Fig. 3–36). What is the vertical component of her acceleration? (*b*) How long will it take her to reach the bottom of the hill, assuming she starts from rest and accelerates uniformly, if the elevation change is 335 m?

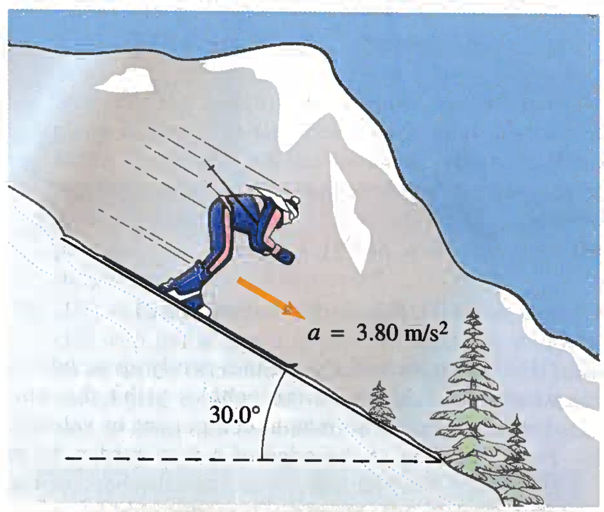

FIGURE 3–36 Problem 16.

17. (II) The summit of a mountain, 2085 m above base camp, is measured on a map to be 4580 m horizontally from the camp in a direction 32.4° west of north. What are the x, y, and z components of the displacement vector from camp to summit? What is its length? Choose the x axis east, y axis north, and z axis up.

18. (III) You are given a vector in the xy plane that has a magnitude of 90.0 units and a y component of −55.0 units. (*a*) What are the two possibilities for its x component? (*b*) Assuming the x component is known to be positive, specify the vector which, if you add it to the original one, would give a resultant vector that is 80.0 units long and points entirely in the −x direction.

SECTIONS 3–5 AND 3–6
(neglect air resistance)

19. (I) A tiger leaps horizontally from a 7.5-m-high rock with a speed of 4.5 m/s. How far from the base of the rock will she land?

20. (I) A diver running 1.6 m/s dives out horizontally from the edge of a vertical cliff and reaches the water below 3.0 s later. How high was the cliff and how far from its base did the diver hit the water?

21. (II) A fire hose held near the ground shoots water at a speed of 6.5 m/s. At what angle(s) should the nozzle point in order that the water land 2.0 m away (Fig. 3–37)? Why are there two different angles?

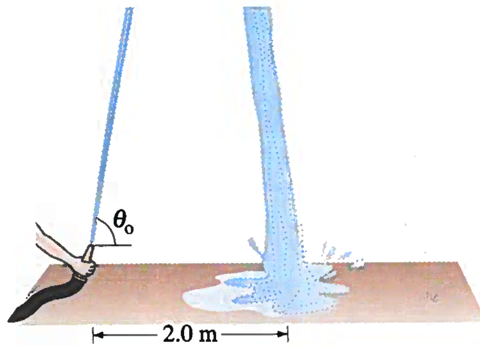

FIGURE 3–37 Problem 21.

22. (II) Romeo is chucking pebbles gently up to Juliet's window, and he wants the pebbles to hit the window with only a horizontal component of velocity. He is standing at the edge of a rose garden 8.0 m below her window and 9.0 m from the base of the wall (Fig. 3–38). How fast are the pebbles going when they hit her window?

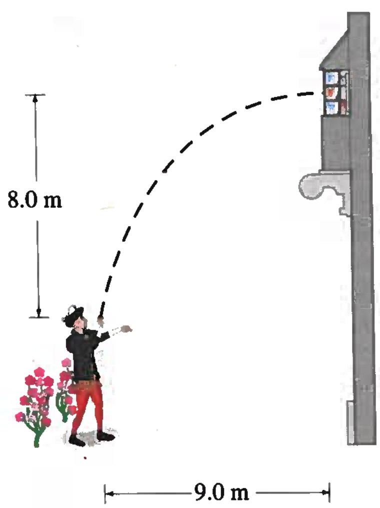

FIGURE 3–38 Problem 22.

23. (II) Suppose the kick in Example 3–4 is attempt 36.0 m from the goalposts, whose crossbar is 3.00 above the ground. If the football is directed correct ly between the goalposts, will it pass over the b and be a field goal? Show why or why not. If no from what horizontal distance must this kick made if it is to score?

24. (II) A ball is thrown horizontally from the roof of building 56 m tall and lands 45 m from the bu What was the ball's initial speed?

25. (II) Show that the speed with which a project leaves the ground is equal to its speed just before strikes the ground at the end of its journey, assumi the firing level equals the landing level.

26. (II) A football is kicked at ground level with a spe of 20.0 m/s at an angle of 37.0° to the horizont How much later does it hit the ground?

27. (II) A ball thrown horizontally at 22.2 m/s from th roof of a building lands 36.0 m from the base of th building. How high is the building?

28. (II) A shot-putter throws the shot with an initial spe of 14 m/s at a 40° angle to the horizontal. Calculate th horizontal distance traveled by the shot if it leaves th athlete's hand at a height of 2.2 m above the ground

29. (II) Determine how much farther a person can ju on the Moon as compared to the Earth if the take speed and angle are the same. The acceleration due gravity on the Moon is one-sixth what it is on Earth

30. (II) An athlete executing a long jump leaves th ground at a 30° angle and travels 7.80 m. (*a*) What w the takeoff speed? (*b*) If this speed were increased just 5.0 percent, how much longer would the jump b

31. (II) The pilot of an airplane traveling 160 km wants to drop supplies to flood victims isolated on patch of land 160 m below. The supplies should dropped how many seconds before the plane is rectly overhead?

32. (II) A hunter aims directly at a target (on the sa level) 120 m away. (*a*) If the bullet leaves the gun a speed of 250 m/s, by how much will it miss the t get? (*b*) At what angle should the gun be aimed the target will be hit?

33. (II) Show that the time required for a projectile reach its highest point is equal to the time for it return from this highest point to its original heigh

34. (II) A projectile is fired with an initial speed 40.0 m/s. Plot on graph paper its trajectory for ini projection angles of $\theta = 15°, 30°, 45°, 60°, 75°$, 90°. Plot at least 10 points for each curve.

35. (II) A projectile is fired with an initial speed 75.2 m/s at an angle of 34.5° above the horizontal a long flat firing range. Determine (*a*) the maxim height reached by the projectile, (*b*) the total tim the air, (*c*) the total horizontal distance covered (t is, the range), and (*d*) the velocity of the proje 1.50 s after firing.

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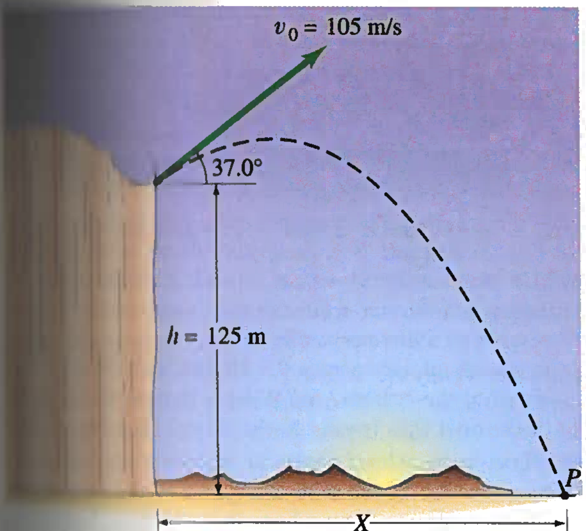


FIGURE 3–39 Problem 36.

36. (II) A projectile is shot from the edge of a cliff 125 m above ground level with an initial speed of 105 m/s at an angle of 37.0° with the horizontal, as shown in Fig. 3–39. (*a*) Determine the time taken by the projectile to hit point *P* at ground level. (*b*) Determine the range *X* of the projectile as measured from the base of the cliff. At the instant just before the projectile hits point *P*, find (*c*) the horizontal and the vertical components of its velocity, (*d*) the magnitude of the velocity, and (*e*) the angle made by the velocity vector with the horizontal.

37. (II) Revisit Conceptual Example 3–6, and assume that the boy with the slingshot is *below* the boy in the tree (Fig. 3–40), and so aims *upward*, directly at the boy in the tree. Show that again the boy in the tree makes the wrong move by letting go at the moment the water balloon is shot.

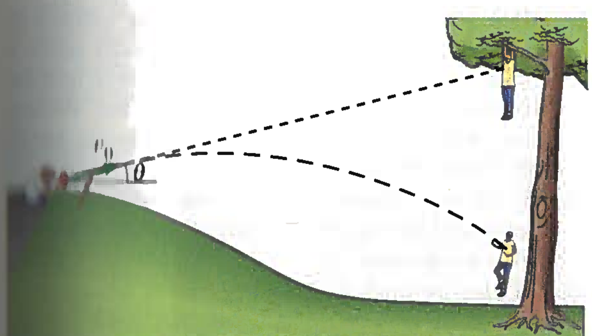


FIGURE 3–40 Problem 37.

38. (III) A rescue plane wants to drop supplies to isolated mountain climbers on a rocky ridge 235 m below. If the plane is traveling horizontally with a speed of 250 km/h (69.4 m/s), (*a*) how far in advance of the recipients (horizontal distance) must the goods be

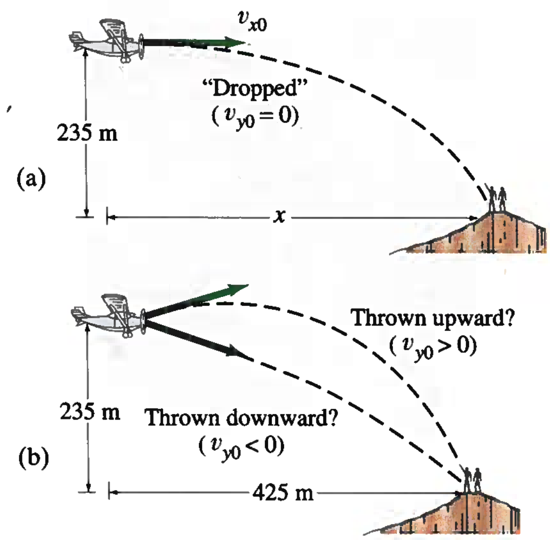


FIGURE 3–41 Problem 38.

dropped (Fig. 3–41a)? (*b*) Suppose, instead, that the plane releases the supplies a horizontal distance of 425 m in advance of the mountain climbers. What vertical velocity (up or down) should the supplies be given so that they arrive precisely at the climbers' position (Fig. 3–41b)? (*c*) With what speed do the supplies land in the latter case?

39. (III) A ball is thrown horizontally from the top of a cliff with initial speed v_0 (at $t = 0$). At any moment, its direction of motion makes an angle θ to the horizontal (Fig. 3–42). Derive a formula for θ as a function of time, *t*, as the ball follows a projectile's path.

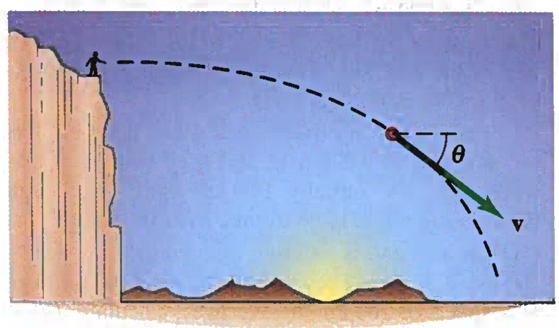


FIGURE 3–42 Problem 39.

* SECTION 3–8

* 40. (I) A person going for a morning jog on the deck of a cruise ship is running toward the bow (front) of the ship at 2.0 m/s while the ship is moving ahead at 8.5 m/s. What is the velocity of the jogger relative to the water? Later, the jogger is moving toward the stern (rear) of the ship. What is the jogger's velocity relative to the water now?

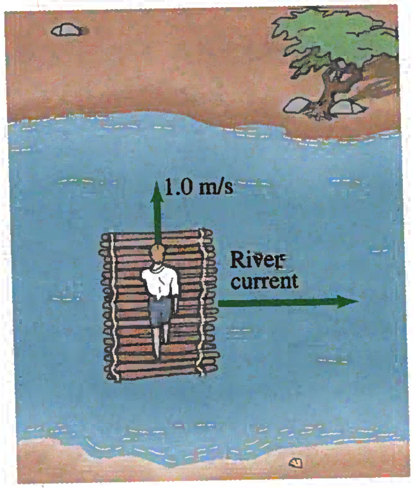


FIGURE 3–43 Problem 41.

* **41.** (II) Huck Finn walks at a speed of 1.0 m/s across his raft (that is, he walks perpendicular to the raft's motion relative to the shore). The raft is traveling down the Mississippi River at a speed of 2.7 m/s relative to the river bank (Fig. 3–43). What is the velocity (speed and direction) of Huck relative to the river bank?

* **42.** (II) You are driving south on a highway at 25 m/s (approximately 55 mph) in a snowstorm. When you last stopped, you noticed that the snow was coming down vertically, but it is passing the windows of the moving car at an angle of 30° to the horizontal. Estimate the speed of the snowflakes relative to the car and relative to the ground.

* **43.** (II) A boat can travel 2.30 m/s in still water. (*a*) If the boat points its prow directly across a stream whose current is 1.20 m/s, what is the velocity (magnitude and direction) of the boat relative to the shore? (*b*) What will be the position of the boat, relative to its point of origin, after 3.00 s? (See Fig. 3–30.)

* **44.** (II) Two planes approach each other head-on. Each has a speed of 835 km/h, and they spot each other when they are initially 10.0 km apart. How much time do the pilots have to take evasive action?

* **45.** (II) An airplane is heading due south at a speed of 500 km/h. If a wind begins blowing from the southwest at a speed of 100 km/h (average), calculate: (*a*) the velocity (magnitude and direction) of the plane relative to the ground, and (*b*) how far off course it will be after 10 min if the pilot takes no corrective action. [*Hint:* First draw a diagram.]

* **46.** (II) In what direction should the pilot aim the plane in Problem 45 so that it will fly due south?

* **47.** (II) Determine the speed of the boat with respect to the shore in Example 3–10.

* **48.** (II) A passenger on a boat moving at 1.50 m/s on a still lake walks up a flight of stairs at a speed of 0.50 m/s, Fig. 3–44. The stairs are angled at 45° pointing in the direction of motion as shown. What is the velocity of the passenger relative to the water?

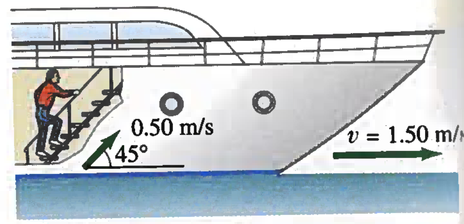


FIGURE 3–44 Problem 48.

* **49.** (II) A motorboat whose speed in still water is 3.60 m/s must aim upstream at an angle of 27.5° (with respect to a line perpendicular to the shore) in order to travel directly across the stream. (*a*) What is the speed of the current? (*b*) What is the resultant speed of the boat with respect to the shore? (See Fig. 3–29.)

* **50.** (II) A boat, whose speed in still water is 2.20 m/s, must cross a 260-m-wide river and arrive at a point 110 m upstream from where it starts (Fig. 3–45). To do so, the pilot must head the boat at a 45° upstream angle. What is the speed of the river's current?

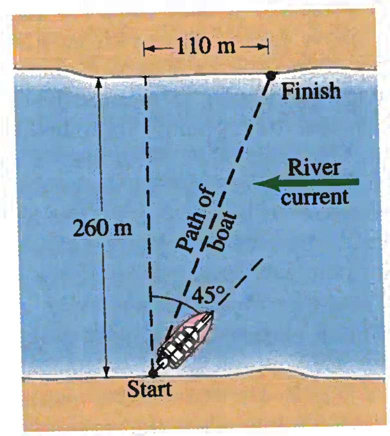


FIGURE 3–45 Problem 50.

* **51.** (II) A swimmer is capable of swimming 1.00 m/s in still water. (*a*) If she aims her body directly across a 150-m-wide river whose current is 0.80 m/s, how far downstream (from a point opposite her starting point) will she land? (*b*) How long will it take her to reach the other side?

* **52.** (II) At what upstream angle must the swimmer in Problem 51 aim, if she is to arrive at a point directly across the stream?

* **53.** (III) An airplane, whose air speed is 600 km/h, is supposed to fly in a straight path 35.0° north of east. But a steady 100 km/h wind is blowing from the north. In what direction should the plane head?

* **54.** (III) A motorcycle traveling 90.0 km/h approaches a car traveling in the same direction at 75.0 km/h. When the motorcycle is 60.0 m behind the car, the rider pushes down on the accelerator and passes the car 10.0 s later. What was the acceleration of the motorcycle?

55. (II) Two cars approach a street corner at right angles to each other (Fig. 3–46). Car 1 travels at 30 km/h and car 2 at 50 km/h. What is the relative velocity of car 1 as seen by car 2? What is the velocity of car 2 relative to car 1?

56. (III) An unmarked police car, traveling a constant 90 km/h, is passed by a speeder traveling 140 km/h. Precisely 1.00 s after the speeder passes, the policeman steps on the accelerator. If the police car's acceleration is 2.00 m/s^2, how much time elapses after the police car is passed until it overtakes the speeder (assumed moving at constant speed)?

57. (III) Assume in the previous problem that the speeder's speed is not known. If the police car accelerates uniformly as given above, and overtakes the speeder after 7.00 s, what was the speeder's speed?

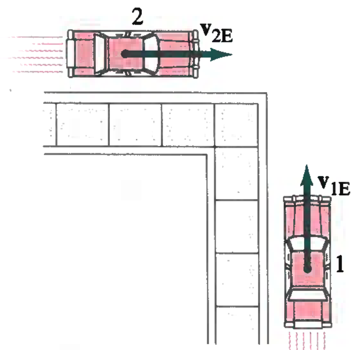


FIGURE 3–46 Problem 55.

GENERAL PROBLEMS

58. William Tell must split the apple atop his son's head from a distance of 27 m. When he aims directly at the apple, the arrow is horizontal. At what angle must he aim it to hit the apple if the arrow travels at a speed of 35 m/s?

59. Two vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, add to a resultant $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$. Describe $\mathbf{V}_1$ and $\mathbf{V}_2$ if (*a*) $V = V_1 + V_2$, (*b*) $V^2 = V_1^2 + V_2^2$, (*c*) $V_1 + V_2 = V_1 - V_2$.

60. A plumber steps out of his truck, walks 50 m east and 25 m south, and then takes an elevator 10 m into the subbasement of a building where a bad leak is occurring. What is the displacement of the plumber relative to his truck? Give your answer in components and also in magnitude and angle notation. Assume x is east, y is north, and z is up.

61. On mountainous downhill roads, escape routes are sometimes placed to the side of the road for trucks whose brakes might fail. Assuming a constant upward slope of 30°, calculate the horizontal and vertical components of the acceleration of a truck that slowed from 120 km/h to rest in 12 s. See Fig. 3–47.

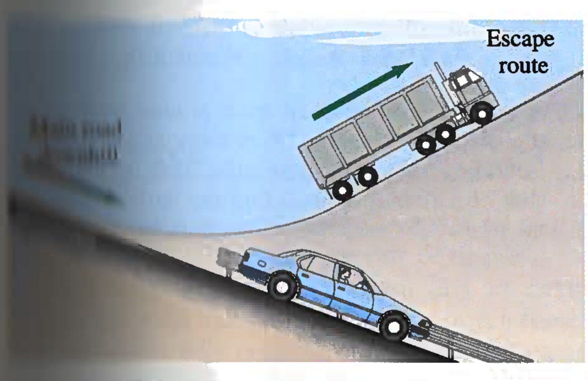


FIGURE 3–47 Problem 61.

FIGURE 3–48 Problem 63.

62. What is the y component of a vector in the xy plane whose magnitude is 88.5 and whose x component is 75.4? What is the direction of this vector (angle it makes with the x axis)?

63. Raindrops make an angle θ with the vertical when viewed through a moving train window (Fig. 3–48). If the speed of the train is v_T, what is the speed of the raindrops in the reference frame of the Earth in which they are assumed to fall vertically?

64. A light plane is headed due south with a speed relative to still air of 155 km/h. After 1.00 hour, the pilot notices that they have covered only 125 km and their direction is not south but southeast. What is the wind velocity?

65. An automobile traveling 95 km/h overtakes a 1.00-km-long train traveling in the same direction on a track parallel to the road. If the train's speed is 75 km/h, how long does it take the car to pass it and how far will the car have traveled in this time? What are the results if the car and train are traveling in opposite directions?

66. An Olympic long jumper is capable of jumping 8.0 m. Assuming his horizontal speed is 9.1 m/s as he leaves the ground, how long is he in the air and how high does he go? Assume that he lands standing upright—that is, the same way he left the ground.

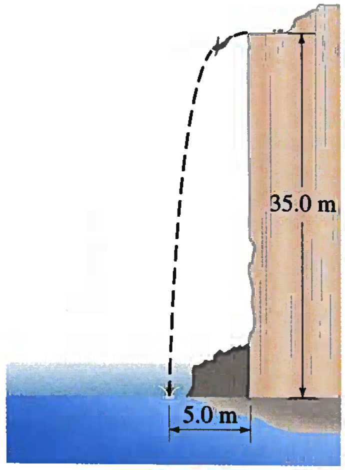


FIGURE 3–49 Problem 69.

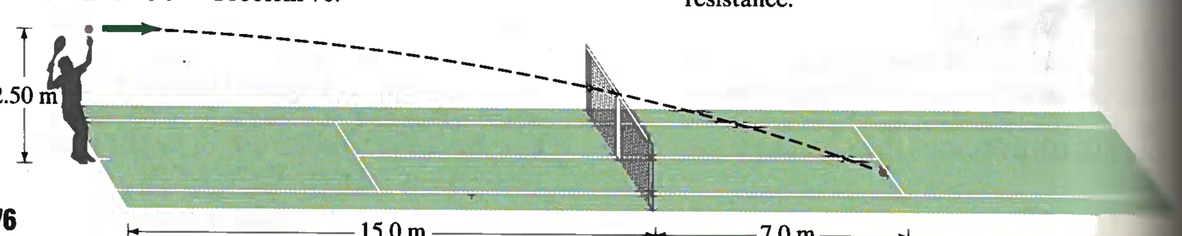
67. Apollo astronauts took a "nine iron" to the Moon and hit a golf ball about 180 m! Assuming that the swing, launch angle, and so on, were the same as on Earth where the same astronaut could hit it only 30 m, estimate the acceleration due to gravity on the surface of the Moon. (Neglect air resistance in both cases, but on the Moon there is none!)

68. When Babe Ruth hit a homer over the 12-m-high right-field fence 95 m from home plate, roughly what was the minimum speed of the ball when it left the bat? Assume the ball was hit 1.0 m above the ground and its path initially made a 40° angle with the ground.

69. The cliff divers of Acapulco push off horizontally from rock platforms about 35 m above the water, but they must clear rocky outcrops at water level that extend out into the water 5.0 m from the base of the cliff directly under their launch point. See Fig. 3–49. What minimum pushoff speed is necessary to do this? How long are they in the air?

70. At serve, a tennis player aims to hit the ball horizontally. What minimum speed is required for the ball to clear the 0.90-m-high net about 15.0 m from the server if the ball is "launched" from a height of 2.50 m? Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m of the net)? How long will it be in the air? See Fig. 3–50.

FIGURE 3–50 Problem 70.



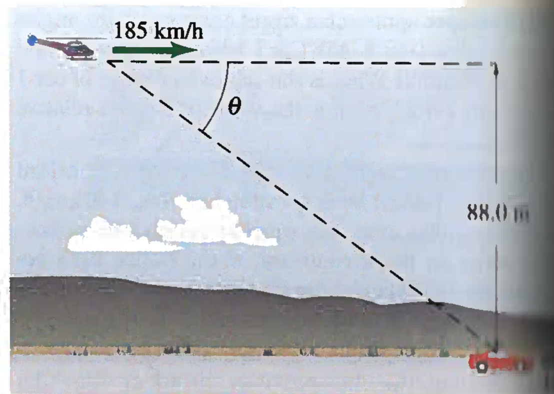


FIGURE 3–51 Problem 71.

71. Agent Tim, flying a constant 185 km/h hor[...] in a low-flying helicopter, wants to drop a sm[...] plosive onto a master criminal's automobile [...] ing 145 km/h on a level highway 88.0 m bel[...] what angle (with the horizontal) should the [...] his sights when the bomb is released (Fig. 3–51)?

72. The speed of a boat in still water is v. The bo[...] make a round-trip in a river whose current [...] speed u. Derive a formula for the time nee[...] make a round trip of total distance D if the [...] makes the round-trip by moving (*a*) upstrea[...] back downstream, (*b*) directly across the riv[...] back. We must assume $u < v$; why?

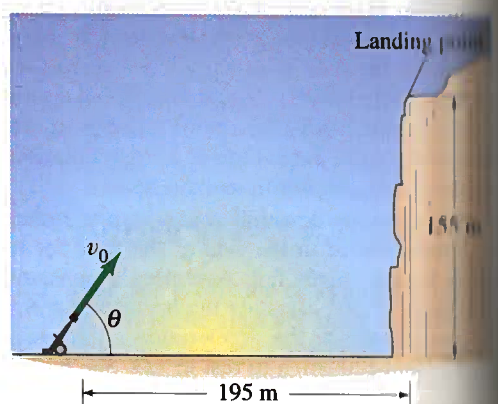


FIGURE 3–52 Problem 73.

73. A projectile is launched from ground level to [...] top of a cliff which is 195 m away and 155 m [...] (see Fig. 3–52). If the projectile lands on top of [...] cliff 7.6 s after it is fired, find the initial vel[...] the projectile (magnitude and direction). Negl[...] resistance.