Space shuttle Discovery landing on Earth. The parachute helps it to reduce its speed quickly. The directions of Discovery's velocity and acceleration are shown by the green (**v**) and gold (**a**) arrows. Note that they (**v** and **a**) point in opposite directions.

CHAPTER 2

DESCRIBING MOTION: KINEMATICS IN ONE DIMENSION

The motion of objects—baseballs, automobiles, joggers, and even the Sun and Moon—is an obvious part of everyday life. Although the ancients acquired significant insight into motion, it was not until comparatively recently, in the sixteenth and seventeenth centuries, that our modern understanding of motion was established. Many contributed to this understanding, but, as we shall soon see, two individuals stand out above the rest: Galileo Galilei (1564–1642) and Isaac Newton (1642–1727).

The study of the motion of objects, and the related concepts of force and energy, form the field called **mechanics**. Mechanics is customarily divided into two parts: **kinematics**, which is the description of how objects move, and **dynamics**, which deals with force and why objects move as they do. This chapter and the next deal with kinematics.

We start by discussing objects that move without rotating (Fig. 2–1a). Such motion is called **translational motion**. In the present chapter we will be concerned with describing an object that moves along a straight-line path, which is one-dimensional motion. In Chapter 3 we will study how to describe translational motion in two (or three) dimensions.

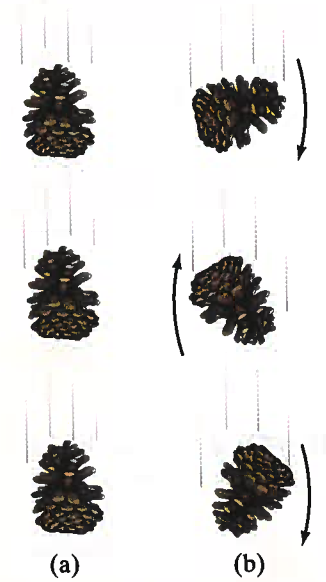


FIGURE 2–1 The pinecone in (a) undergoes pure translation as it falls, whereas in (b) it is rotating as well as translating.

2–1 Reference Frames and Displacement

All measurements are made relative to a frame of reference

Any measurement of position, distance, or speed must be made with respect to a **frame of reference**. For example, while you are on a train traveling at 80 km/h, you might notice a person who walks past you toward the front of the train at a speed of, say, 5 km/h (Fig. 2–2). Of course this is the person's speed with respect to the train as frame of reference. With respect to the ground that person is moving at a speed of 80 km/h + 5 km/h = 85 km/h. It is always important to specify the frame of reference when stating a speed. In everyday life, we usually mean "with respect to the Earth" without even thinking about it, but the reference frame should be specified whenever there might be confusion.

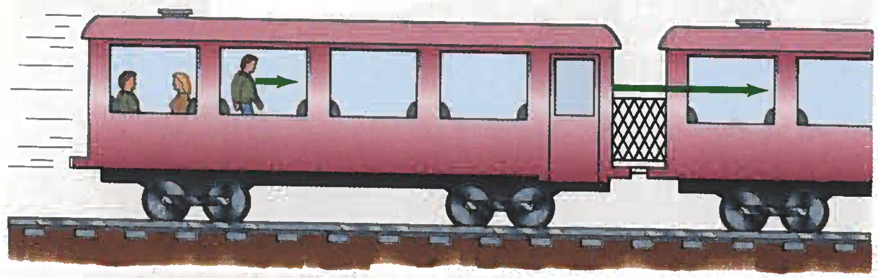


FIGURE 2–2 A person walks toward the front of a train at 5 km/h. The train is moving 80 km/h with respect to the ground, so the walking person's speed, relative to the ground, is 85 km/h.

Even distances depend on the frame of reference. For example, there is no point in telling you that Yosemite National Park is 300 km away unless I specify 300 km from where. Furthermore, when specifying the motion of an object, it is important to specify not only the speed but also the direction of motion. Often we can specify a direction by using the cardinal points, north, east, south, and west, and by "up" and "down." In physics, we often draw a set of **coordinate axes**, as shown in Fig. 2–3, to represent a frame of reference. We can always place the origin 0, and the directions of the x and y axes, as we like for convenience. Objects positioned to the right of the origin of coordinates (0) on the x axis have an x coordinate which we usually choose to be positive; then points to the left of 0 have a negative x coordinate. The position along the y axis is usually considered positive when above 0, and negative when below 0, although the reverse convention can be used if convenient. Any point on the plane can be specified by giving its x and y coordinates. In three dimensions, a z axis perpendicular to the x and y axes is also used.

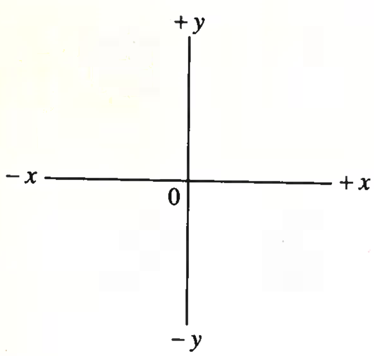


FIGURE 2–3 Standard set of xy coordinate axes.

For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. Thus the position of an object at any moment is given by its x coordinate.

Displacement

We need to make a distinction between the distance an object has traveled, and its **displacement**, which is definied as the *change in position* of the object. That is, displacement is how far the object is from its starting point. To see the distinction between total distance and displacement, imagine a person walking 70 m to the east and then turning around and walking back (west) a distance of 30 m (see Fig. 2–4). The total *distance* traveled is 100 m, but the *displacement* is only 40 m since the person is now only 40 m from the starting point.

Displacement is a quantity that has both magnitude and direction. Such quantities are called **vectors**, and are represented in diagrams by arrows. For example, in Fig. 2–4, the blue arrow represents the displacement whose magnitude is 40 m and whose direction is to the right.

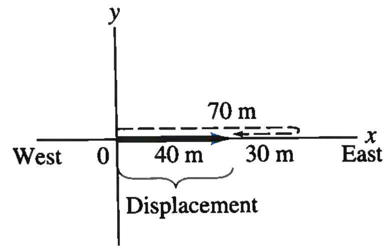


FIGURE 2–4 A person walks 70 m east, then 30 m west. The total distance traveled is 100 m (path is shown in black); but the displacement, shown as a blue arrow, is 40 m to the east.

We will deal with vectors more fully in Chapter 3. For now, we deal only with motion in one dimension, along a line, and in this case, vectors which point in one direction will have a positive sign, whereas vectors that point in the opposite direction will have a negative sign.

Let's see how this works. Consider the motion of an object over a particular time interval. Suppose that at some initial moment in time, call it t_1, the object is on the x axis at the point x_1 in the coordinate system shown in Fig. 2–5. At some later time, t_2, suppose the object is at point x_2. The displacement of our object is $x_2 - x_1$, and is represented by the arrow pointing to the right in Fig. 2–5. It is convenient to write

$$\Delta x = x_2 - x_1$$

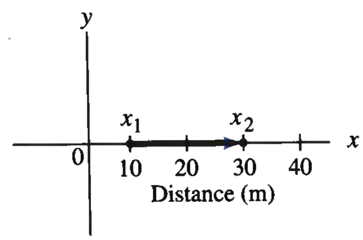


FIGURE 2–5 The arrow represents the displacement $x_2 - x_1$. Distances are in meters.

where the symbol Δ (Greek letter delta) means "change in." Then Δx means "the change in x," which is the displacement. Note that the "change in" any quantity means the final value of that quantity, minus the initial value.

To be concrete, suppose $x_1 = 10.0\text{ m}$ and $x_2 = 30.0\text{ m}$. Then

$$\Delta x = x_2 - x_1 = 30.0\text{ m} - 10.0\text{ m} = 20.0\text{ m}.$$

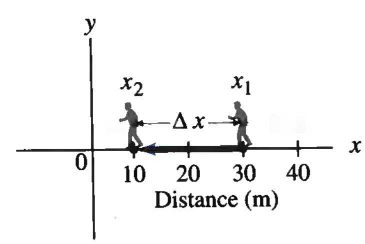
See Fig. 2–5.

Now consider a different situation, that of an object moving to the left as shown in Fig. 2–6. Here an object, say a person, starts at $x_1 = 30.0\text{ m}$ and walks to the left to the point $x_2 = 10.0\text{ m}$. In this case

$$\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m} = -20.0\text{ m}$$

and the blue arrow representing the vector displacement points to the left. This Example illustrates that when dealing with one-dimensional motion, a vector pointing to the right has a positive value, whereas one pointing to the left has a negative value.

FIGURE 2–6 For the displacement $\Delta x = x_2 - x_1 = 10.0\text{ m} - 30.0\text{ m}$, the displacement vector points to the left.



2–2 Average Velocity

The most obvious aspect of the motion of a moving object is how fast it is moving—its speed or velocity.

The term "speed" refers to how far an object travels in a given time interval. If a car travels 240 kilometers (km) in 3 hours, we say its average speed was 80 km/h. In general, the **average speed** of an object is defined as *the distance traveled along its path divided by the time it takes to travel this distance:*

Average speed

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time elapsed}}. \qquad \textbf{(2–1)}$$

Velocity

The terms velocity and speed are often used interchangeably in ordinary language. But in physics we make a distinction between the two. Speed is simply a positive number, with units. **Velocity**, on the other hand, is used to signify both the *magnitude* (numerical value) of how fast an object is moving and the *direction* in which it is moving. (Velocity is therefore a vector.) There is a second difference between speed and velocity: namely, the **average velocity** is defined in terms of *displacement*, rather than total distance traveled:

$$\text{average velocity} = \frac{\text{displacement}}{\text{time elapsed}}.$$

Average speed and average velocity often have the same magnitude, but sometimes they don't. As an example, recall the walk we described earlier, in Fig. 2–4, where a person walked 70 m east and then 30 m west. The total distance traveled was 70 m + 30 m = 100 m, but the displacement was 40 m. Suppose this walk took 70 s to complete. Then the average speed was:

$$\frac{\text{distance}}{\text{time}} = \frac{100\text{ m}}{70\text{ s}} = 1.4\text{ m/s}.$$

The magnitude of the average velocity, on the other hand, was:

$$\frac{\text{displacement}}{\text{time}} = \frac{40\text{ m}}{70\text{ s}} = 0.57\text{ m/s}.$$

This discrepancy between the speed and the magnitude of the velocity occurs in some cases, but only for the *average* values, and we rarely need be concerned with it.

To discuss one-dimensional motion of an object in general, suppose that at some moment in time, call it t_1, the object is on the x axis at point x_1 in a coordinate system, and at some later time, t_2, suppose it is at point x_2. The elapsed time is $t_2 - t_1$, and during this time interval the displacement of our object was $\Delta x = x_2 - x_1$. Then the average velocity, defined as *the displacement divided by the elapsed time*, can be written

Average velocity

$$\bar{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}, \qquad \textbf{(2–2)}$$

where v stands for velocity and the bar ($^{-}$) over the v is a standard symbol meaning "average."

PROBLEM SOLVING

+ or − sign can signify the direction for linear motion

Notice that if x_2 is less than x_1, the object is moving to the left, and then $\Delta x = x_2 - x_1$ is less than zero. The sign of the displacement, and thus of the velocity, indicates the direction: the average velocity is positive for an object moving to the right along the x axis and negative when the object moves to the left. The direction of the velocity is always the same as the direction of the displacement.

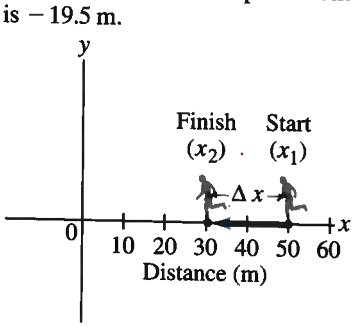
EXAMPLE 2–1 **Runner's average velocity.** The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from $x_1 = 50.0$ m to $x_2 = 30.5$ m, as shown in Fig. 2–7. What was the runner's average velocity?

SOLUTION Average velocity is the displacement divided by the elapsed time. The displacement is $\Delta x = x_2 - x_1 = 30.5\text{ m} - 50.0\text{ m} = -19.5\text{ m}$. The time interval is $\Delta t = 3.00$ s. Therefore the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-19.5\text{ m}}{3.00\text{ s}} = -6.50\text{ m/s}.$$

The displacement and average velocity are negative, which tells us (if we didn't already know it) that the runner is moving to the left along the x axis, as indicated by the arrow in Fig. 2–7. Thus we can say that the runner's average velocity is 6.50 m/s to the left.

FIGURE 2–7 Example 2–1. A person runs from $x_1 = 50.0$ m to $x_2 = 30.5$ m. The displacement is −19.5 m.



EXAMPLE 2–2 **Distance a cyclist travels.** How far can a cyclist travel in 2.5 h along a straight road if her average speed is 18 km/h?

SOLUTION We want to find the distance traveled, so we use Eq. 2–1 letting Δx be the distance and $\bar{v}$ be the average speed, and then rewrite it as

$$\Delta x = \bar{v}\,\Delta t = (18\ \text{km/h})(2.5\ \text{h}) = 45\ \text{km}.$$

2–3 Instantaneous Velocity

If you drive a car along a straight road for 150 km in 2.0 h, the magnitude of your average velocity is 75 km/h. It is unlikely, though, that you were moving at precisely 75 km/h at every instant. To deal with this situation we need the concept of *instantaneous velocity*, which is the velocity at any instant of time. (This is the magnitude that a speedometer is supposed to indicate.) More precisely, the **instantaneous velocity** at any moment is defined as *the average velocity over an infinitesimally short time interval.* That is, starting with Eq. 2–2

$$\bar{v} = \frac{\Delta x}{\Delta t},$$

we define instantaneous velocity as the average velocity in the limit of Δt becoming extremely small, approaching zero. We can write the definition of instantaneous velocity, v, for one-dimensional motion as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}. \qquad \textbf{(2–3)}$$

Instantaneous velocity

The notation $\lim_{\Delta t \to 0}$ means the ratio $\Delta x/\Delta t$ is to be evaluated in the limit of Δt approaching zero. But we do not simply set $\Delta t = 0$ in this definition, for then Δx would also be zero, and we would have an undefined number. Rather, we are considering the *ratio* $\Delta x/\Delta t$, as a whole. As we let Δt approach zero, Δx approaches zero as well. But the ratio $\Delta x/\Delta t$ approaches some definite value, which is the instantaneous velocity at a given instant.[†]

For instantaneous velocity we use the symbol v, whereas for average velocity we use $\bar{v}$, with a bar. In the rest of this book, when we use the term "velocity" it will refer to instantaneous velocity. When we want to speak of the average velocity, we will make this clear by including the word "average." Note that the *instantaneous* speed always equals the magnitude of the instantaneous velocity. Why? Because distance and displacement become the same when they become infinitesimally small.

If an object moves at a uniform (that is, constant) velocity over a particular time interval, then its instantaneous velocity at any instant is the same as its average velocity (see Fig. 2–8a). But in many situations this is not the case. For example, a car may start from rest, speed up to 50 km/h, remain at that velocity for a time, then slow down to 20 km/h in a traffic jam, and finally stop at its destination after traveling a total of 15 km in 30 min. This trip is plotted on the graph of Fig. 2–8b. Also shown on the graph is the average velocity (dashed line), which is $\bar{v} = \Delta x/\Delta t = 15\ \text{km}/0.50\ \text{h} = 30\ \text{km/h}$.

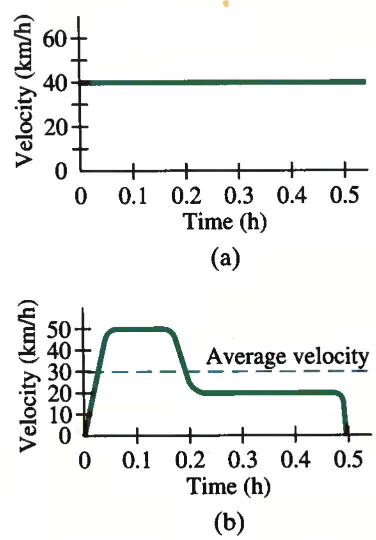


FIGURE 2–8 Velocity of a car as a function of time: (a) at constant velocity; (b) with varying velocity.

[†] More on this in Section 2–8.

2–4 Acceleration

An object whose velocity is changing is said to be accelerating. A car whose velocity increases in magnitude from zero to 80 km/h is accelerating. If one car can accomplish this change in velocity in less time than another, it is said to undergo a greater acceleration. That is, acceleration specifies how rapidly the velocity of an object is changing. **Average acceleration** is defined as the change in velocity divided by the time taken to make this change:

$$\text{average acceleration} = \frac{\text{change of velocity}}{\text{time elapsed}}.$$

In symbols, the average acceleration, $\bar{a}$, over a time interval $\Delta t = t_2 - t_1$ during which the velocity changes by $\Delta v = v_2 - v_1$, is defined as

Average acceleration

$$\bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}. \qquad (2\text{–}4)$$

Acceleration is also a vector, but for one-dimensional motion, we need only use a plus or minus sign to indicate direction relative to a chosen coordinate system.

The **instantaneous acceleration**, *a*, can be defined in analogy to instantaneous velocity, for any specific instant:

Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \qquad (2\text{–}5)$$

Here Δv represents the very small change in velocity during the very short time interval Δt.

EXAMPLE 2–3 **Average acceleration.** A car accelerates along a straight road from rest to 75 km/h in 5.0 s, Fig. 2–9. What is the magnitude of its average acceleration?

SOLUTION The car starts from rest, so $v_1 = 0$. The final velocity is $v_2 = 75\text{ km/h}$. Then from Eq. 2–4, the average acceleration is

$$\bar{a} = \frac{75\text{ km/h} - 0\text{ km/h}}{5.0\text{ s}} = 15\,\frac{\text{km/h}}{\text{s}}.$$

This is read as "fifteen kilometers per hour per second" and means that, on average, the velocity changed by 15 km/h during each second. That is, assuming the acceleration was constant, during the first second the car's velocity increased from zero to 15 km/h. During the next second its velocity increased by another 15 km/h up to 30 km/h, and so on, Fig. 2–9. (Of course, if the instantaneous acceleration was not constant, these numbers could be different.)

Careful: Do not confuse velocity with acceleration

Note carefully that *acceleration tells us how fast the velocity changes,* whereas *velocity tells us how fast the position changes.* In this last Example, the calculated acceleration contained two different time units: hours and seconds. We

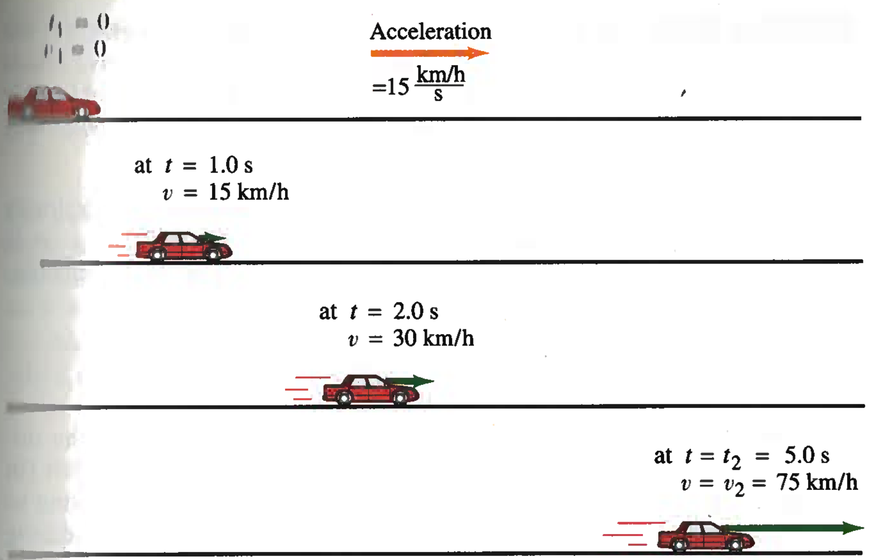


FIGURE 2–9 Example 2–3. The car is shown at the start with $v_1 = 0$ at $t_1 = 0$. It is shown three more times, at $t = 1.0$ s, $t = 2.0$ s, and $t_2 = 5.0$ s. We assume the acceleration is constant and equals 15 km/h/s. The green arrows represent the velocity vectors; the length of each represents the magnitude of the velocity at that moment. The acceleration vector is the orange arrow.

usually prefer to use only seconds. To do so we can change km/h to m/s (see Section 1–6, and Example 1–4):

$$75\text{ km/h} = \left(75\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 21\text{ m/s}.$$

Then we get

$$\bar{a} = \frac{21\text{ m/s} - 0.0\text{ m/s}}{5.0\text{ s}} = 4.2\,\frac{\text{m/s}}{\text{s}} = 4.2\,\frac{\text{m}}{\text{s}^2}.$$

We almost always write these units as m/s^2 (meters per second squared), as done here, instead of m/s/s. This is possible because:

$$\frac{\text{m/s}}{\text{s}} = \frac{\text{m}}{\text{s}\cdot\text{s}} = \frac{\text{m}}{\text{s}^2}.$$

According to the above calculation, the velocity in Example 2–3 (Fig. 2–9) changed on the average by 4.2 m/s during each second, for a total change of 21 m/s over the 5.0 s.

CONCEPTUAL EXAMPLE 2–4 **Velocity and acceleration.** (*a*) If the velocity of an object is zero, does it mean that the acceleration is zero? (*b*) If the acceleration is zero, does it mean that the velocity is zero? Think of some examples.

RESPONSE A zero velocity does not necessarily mean that the acceleration is zero, nor does a zero acceleration mean that the velocity is zero. (*a*) For example, when you put your foot on the gas pedal of your car which is at rest, the velocity starts from zero but the acceleration is not zero since the velocity of the car changes. (How else could your car start forward if its velocity weren't changing—that is, if the acceleration were zero?) (*b*) As you cruise along a straight highway at a constant velocity of 100 km/h, your acceleration is zero.

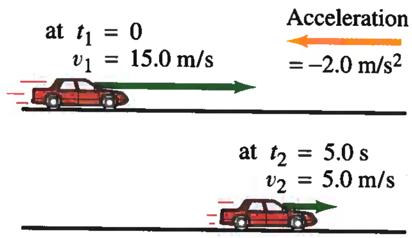
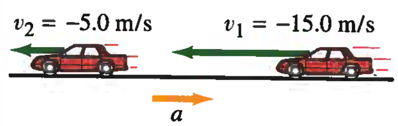


FIGURE 2–10 Example 2–5, showing the position of the car at times t_1 and t_2, as well as the car's velocity represented by the green arrows. The acceleration vector (orange) points to the left.

FIGURE 2–11 The same car as in Example 2–5, but now moving to the left and decelerating. The acceleration is

$$a = \frac{v_2 - v_1}{\Delta t} = \frac{-5.0\,\text{m/s} - (-15.0\,\text{m/s})}{5.0\,\text{s}} = \frac{-5.0\,\text{m/s} + 15.0\,\text{m/s}}{5.0\,\text{s}} = +2.0\,\text{m/s}.$$



EXAMPLE 2–5 **Car slowing down.** An automobile is moving to the right along a straight highway, which we choose to be the positive x axis (Fig. 2–10), and the driver puts on the brakes. If the initial velocity is $v_1 = 15.0\,\text{m/s}$ and it takes 5.0 s to slow down to $v_2 = 5.0\,\text{m/s}$, what was the car's average acceleration?

SOLUTION The average acceleration is equal to the change in velocity divided by the elapsed time, Eq. 2–4. Let us call the initial time $t_1 = 0$; then $t_2 = 5.0\,\text{s}$. (Note that our choice of $t_1 = 0$ doesn't affect the calculation of $\bar{a}$ because only $\Delta t = t_2 - t_1$ appears in Eq. 2–4.) Then

$$\bar{a} = \frac{5.0\,\text{m/s} - 15.0\,\text{m/s}}{5.0\,\text{s}} = -2.0\,\text{m/s}^2.$$

The negative sign appears because the final velocity is less than the initial velocity. In this case the direction of the acceleration is to the left (in the negative x direction)—even though the velocity is always pointing to the right. We say that the acceleration is $2.0\,\text{m/s}^2$ to the left, and it is shown in Fig. 2–10 as an orange arrow.

When an object is slowing down, we sometimes say it is decelerating. But be careful: deceleration does *not* mean that the acceleration is necessarily negative. For an object moving to the right along the positive x axis and slowing down (as in Fig. 2–10), the acceleration *is* negative. But the same car moving to the left (decreasing x) and slowing down has positive acceleration that points to the right, as shown in Fig. 2–11. We have a deceleration whenever the velocity and acceleration point in opposite directions.

2–5 Motion at Constant Acceleration

Let a = constant

Many practical situations occur in which the acceleration is constant or close enough that we can assume it is constant. That is, the acceleration doesn't change over time. We now treat this situation when the magnitude of the acceleration is constant and the motion is in a straight line (sometimes called **uniformly accelerated motion**). In this case, the instantaneous and average accelerations are equal.

$t_1 = 0,\ t_2 = t$
$x_1 = x_0,\ x_2 = x$
$v_1 = v_0,\ v_2 = v$

To simplify our notation, let us take the initial time in any discussion to be zero: $t_1 = 0$. We can then let $t_2 = t$ be the elapsed time. The initial position (x_1) and initial velocity (v_1) of an object will now be represented by x_0 and v_0; and at time t the position and velocity will be called x and v (rather than x_2 and v_2). The average velocity during the time t will be (from Eq. 2–2)

$$\bar{v} = \frac{x - x_0}{t - t_0} = \frac{x - x_0}{t}$$

since $t_0 = 0$. And the acceleration, which is assumed constant in time, will be (from Eq. 2–4)

$$a = \frac{v - v_0}{t}.$$

A common problem is to determine the velocity of an object after a certain time, given its acceleration. We can solve such problems by solving for v in the last equation: we multiply both sides by t and get

$$at = v - v_0$$

and then add v_0 to both sides to obtain:

$$v = v_0 + at. \qquad \text{[constant acceleration]} \qquad \textbf{(2–6)}$$

v related to a and t (a = constant)

For example, it may be known that the acceleration of a particular motorcycle is $4.0\,\mathrm{m/s^2}$, and we wish to determine how fast it will be going after, say, 6.0 s. Assuming it starts from rest ($v_0 = 0$), after 6.0 s the velocity will be $v = at = (4.0\,\mathrm{m/s^2})(6.0\,\mathrm{s}) = 24\,\mathrm{m/s}$.

Next, let us see how to calculate the position of an object after a time t when it is undergoing constant acceleration. The definition of average velocity (Eq. 2–2) is

$$\bar{v} = \frac{x - x_0}{t},$$

which we can rewrite (solving for x) as

$$x = x_0 + \bar{v}t.$$

Because the velocity increases at a uniform rate, the average velocity, $\bar{v}$, will be midway between the initial and final velocities:

$$\bar{v} = \frac{v_0 + v}{2}. \qquad \text{[constant acceleration]} \qquad \textbf{(2–7)}$$

Average velocity (when acceleration is constant)

(Careful: this is not usually valid if the acceleration is not constant.) We combine the last two equations with Eq. 2–6 and find

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v_0 + v}{2}\right)t$$

$$= x_0 + \left(\frac{v_0 + v_0 + at}{2}\right)t$$

or

$$x = x_0 + v_0t + \tfrac{1}{2}at^2. \qquad \text{[constant acceleration]} \qquad \textbf{(2–8)}$$

x related to a and t (a = constant)

Equations 2–6, 2–7, and 2–8 are three of the four most useful equations for motion at constant acceleration. We now derive the fourth equation, which is useful in situations where the time t is not known. We begin as above, with Eq. 2–7 and the equation just before it:

$$x = x_0 + \bar{v}t = x_0 + \left(\frac{v + v_0}{2}\right)t$$

Next we solve Eq. 2–6 for t, obtaining

$$t = \frac{v - v_0}{a},$$

and substituting this into the equation above we have

$$x = x_0 + \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right) = x_0 + \frac{v^2 - v_0^2}{2a}.$$

We solve this for v^2 and obtain

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \text{[constant acceleration]} \qquad \textbf{(2–9)}$$

v related to a and x (a = constant)

which is the useful equation we sought.

We now have four equations relating position, velocity, acceleration and time, when the acceleration a is constant. We collect them here in one place for further reference (the tan background screen is to emphasize their usefulness):

Kinematic equations for constant acceleration (we'll use them a lot)

$$v = v_0 + at \qquad [a = \text{constant}] \quad \text{(2–10a)}$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad [a = \text{constant}] \quad \text{(2–10b)}$$

$$v^2 = v_0^2 + 2a(x - x_0) \qquad [a = \text{constant}] \quad \text{(2–10c)}$$

$$\bar{v} = \frac{v + v_0}{2}. \qquad [a = \text{constant}] \quad \text{(2–10d)}$$

These useful equations are not valid unless a is a constant. In many cases we can set $x_0 = 0$, and this simplifies the above equations a bit. Note that x represents position, not distance, and $x - x_0$ is the displacement.

EXAMPLE 2–6 **Runway design.** You are designing an airport for small planes. One kind of airplane that might use this airfield must reach a speed before takeoff of at least 27.8 m/s (100 km/h), and can accelerate at 2.00 m/s². (*a*) If the runway is 150 m long, can this airplane reach the proper speed to take off? (*b*) If not, what minimum length must the runway have?

SOLUTION (*a*) We are given the airplane's acceleration ($a = 2.00\ \text{m/s}^2$), and we know the plane can travel a distance of 150 m. We want to find its velocity, to determine if it will be at least 27.8 m/s. We want to find v when we are given:

Known	Wanted
$x_0 = 0$	v
$v_0 = 0$	
$x = 150\ \text{m}$	
$a = 2.00\ \text{m/s}^2$	

PROBLEM SOLVING

Equations 2–10 are valid only when the acceleration is constant, which we assume in this Example

Of the above four equations, Eq. 2–10c will give us v, when we know v_0, a, x, and x_0:

$$\begin{aligned} v^2 &= v_0^2 + 2a(x - x_0) \\ &= 0 + 2(2.0\ \text{m/s}^2)(150\ \text{m}) = 600\ \text{m}^2/\text{s}^2 \\ v &= \sqrt{600\ \text{m}^2/\text{s}^2} = 24.5\ \text{m/s}. \end{aligned}$$

This runway length is *not* sufficient.

(*b*) Now we want $(x - x_0)$ given $v = 27.8\ \text{m/s}$ and $a = 2.0\ \text{m/s}^2$. So we use Eq. 2–10c, rewritten as

$$(x - x_0) = \frac{v^2 - v_0^2}{2a} = \frac{(27.8\ \text{m/s})^2 - 0}{2(2.0\ \text{m/s}^2)} = 193\ \text{m}.$$

2–6 Solving Problems

The solving of problems, such as the Examples we have already given, serves two purposes. First, solving problems is useful and practical in itself. Second, solving problems makes you think about the ideas and concepts, and applying the concepts helps you to understand them. But knowing how to do a problem—even to begin it—may not always seem easy. First, it is most important to read the problem through carefully, and more than once. Spend a moment thinking and trying to understand what physics principles, ideas, laws, and definitions might be involved. Up to this point in the book, we have been concerned mainly with the definitions of velocity and acceleration, and the "kinematic equations for constant acceleration," Eqs. 2–10, that we derived from those definitions. For now it is important to note that physics is *not* a collection of equations to be memorized. (In fact, rather than memorizing the very useful Eqs. 2–10, it is better to understand how to derive them from the definitions of velocity and acceleration as we did above.) Simply searching for an equation that might work can be disastrous and can lead you to a wrong result (and will surely not help you understand physics). A better approach is to use the following (rough) procedure, which we put in a special "box" (other such Problem Solving boxes, as an aid, will be found throughout the book):

PROBLEM SOLVING

1. **Read** *and reread* the whole problem carefully before trying to solve it.
2. **Draw** a **diagram** or picture of the situation, with coordinate axes wherever applicable. [You can choose to place the origin of coordinates and the axes wherever you like, so as to make your calculations easier. You also choose which direction is positive and which is negative. Usually we choose the x axis to the right as positive, but you could choose positive to the left.]
3. **Write down** what quantities are "known" or "given," and then what you *want* to know.
4. Think about which principles of physics apply in this problem.
5. Consider which equations (and/or definitions) relate the quantities involved. Before using them, be sure their **range of validity** includes your problem (for example, Eqs. 2–10 are valid only when the acceleration is constant). If you find an applicable equation that involves only known quantities and one desired unknown, **solve** the equation algebraically for the unknown. In many instances several sequential calculations, or a combination of equations, may be needed. It is often preferable to solve algebraically for the desired unknown before putting in numerical values.
6. Carry out the **calculation** if it is a numerical problem. Keep one or two extra digits during the calculations, but round off the final answers to the correct number of significant figures (Section 1–4).
7. Think carefully about the result you obtain: Is it **reasonable**? Does it make sense according to your own intuition and experience? A good check is to do a rough **estimate** using only powers of ten, as discussed in Section 1–7. Often it is preferable to do a rough estimate at the *start* of a numerical problem because it can help you focus your attention on finding a path toward a solution.
8. A very important aspect of doing problems is keeping track of **units**. Note that an equals sign implies the units on each side must be the same, just as the numbers must. If the units do not balance, a mistake has no doubt been made. This can serve as a **check** on your solution (but it only tells you if you're wrong, not if you're right). And: always use a consistent set of units.

PROBLEM SOLVING

Note that "starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$]

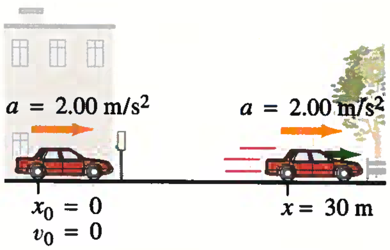


FIGURE 2–12 Example 2–7.

EXAMPLE 2–7 **Acceleration of car.** How long does it take a car to cross a 30.0-m-wide intersection after the light turns green, if it accelerates from rest at a constant $2.00\,\mathrm{m/s^2}$?

SOLUTION First we make a sketch, Fig. 2–12. Next we make a table, choosing $x_0 = 0$ and assume the car moves to the right along the positive x axis, and noting that "starting from rest" means $v = 0$ at $t = 0$ [i.e., $v_0 = 0$].

Known	Wanted
$x_0 = 0$	t
$x = 30.0\,\mathrm{m}$	
$a = 2.00\,\mathrm{m/s^2}$	
$v_0 = 0$	

Since a is constant, we can use Eqs. 2–10. Equation 2–10b is perfect since the only unknown quantity is t. Setting $v_0 = 0$ and $x_0 = 0$, we can solve Eq. 2–10b for t:

$$x = \tfrac{1}{2}at^2$$

$$t^2 = \frac{2x}{a}$$

$$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(30.0\,\mathrm{m})}{2.00\,\mathrm{m/s^2}}} = 5.48\,\mathrm{s}.$$

PROBLEM SOLVING

Check your answer

This is our answer. Note that the units come out correctly. We can check the reasonableness of the answer by calculating the final velocity $v = at = (2.00\,\mathrm{m/s^2})(5.48\,\mathrm{s}) = 10.96\,\mathrm{m/s}$, and then finding $x = x_0 + \bar{v}t = 0 + \tfrac{1}{2}(10.96\,\mathrm{m/s} + 0)(5.48\,\mathrm{s}) = 30.0\,\mathrm{m}$, which is our given distance.

PHYSICS APPLIED

Braking distances

EXAMPLE 2–8 **ESTIMATE** **Braking distances.** Estimate the minimum stopping distances for a car, which are important for traffic safety and traffic design. The problem is best dealt with in two parts: (1) the time between the decision to apply the brakes and their actual application (the "reaction time"), during which we assume $a = 0$; and (2) the actual braking period when the vehicle slows down ($a \neq 0$). The stopping distance depends on the reaction time of the driver, the initial speed of the car (the final speed is zero), and the acceleration of the car. For a dry road and good tires, good brakes can decelerate a car at a rate of about $5\,\mathrm{m/s^2}$ to $8\,\mathrm{m/s^2}$. Calculate the total stopping distance for an initial velocity of 100 km/h ($28\,\mathrm{m/s} \approx 62$ mph) and assume the acceleration of the car is

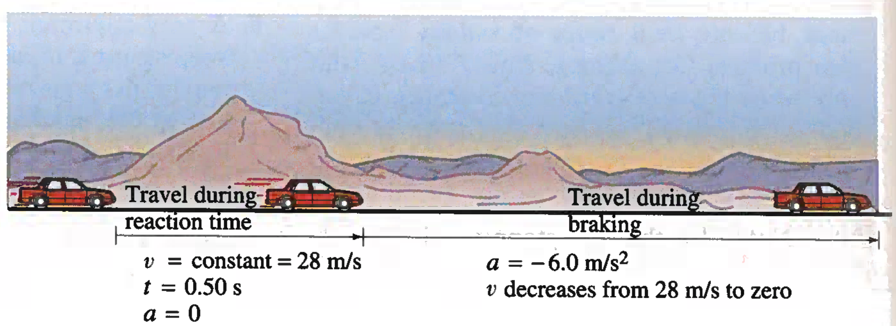


FIGURE 2–13 Example 2–8: stopping distance for a braking car.

$-6.0\,\text{m/s}^2$ (the minus sign appears because the velocity is taken to be in the positive x direction and its magnitude is decreasing). Reaction time for normal drivers varies from perhaps 0.3 s to about 1.0 s; take it to be 0.50 s.

SOLUTION The car is moving to the right in the positive x direction. We take $x_0 = 0$ for the first part of the problem, in which the car travels at a constant speed of 28 m/s during the time the driver is reacting (0.50 s). See Fig. 2–13. Thus:

Part 1: Reaction time

Part 1:

Known	Wanted
$t = 0.50\,\text{s}$	x
$v_0 = 28\,\text{m/s}$	
$v = 28\,\text{m/s}$	
$a = 0$	
$x_0 = 0$	

To find x we can use Eq. 2–10b (note that Eq. 2–10c isn't useful because x is multiplied by a, which is zero):

$$x = v_0 t + 0 = (28\,\text{m/s})(0.50\,\text{s}) = 14\,\text{m}.$$

The car travels 14 m during the driver's reaction time, until the moment the brakes are applied. Now for the second part, during which the brakes are applied and the car is brought to rest. We now take $x_0 = 14\,\text{m}$ (result of the first part):

Part 2: Braking

Part 2:

Known	Wanted
$x_0 = 14\,\text{m}$	x
$v_0 = 28\,\text{m/s}$	
$v = 0$	
$a = -6.0\,\text{m/s}^2$	

Equation 2–10a doesn't contain x; Eq. 2–10b contains x but also the unknown t. Equation 2–10c is what we want; we solve for x (after setting $x_0 = 14\text{m}$):

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$x = x_0 + \frac{v^2 - v_0^2}{2a}$$

$$= 14\,\text{m} + \frac{0 - (28\,\text{m/s})^2}{2(-6.0\,\text{m/s}^2)} = 14\,\text{m} + \frac{-784\,\text{m}^2/\text{s}^2}{-12\,\text{m/s}^2}$$

$$= 14\,\text{m} + 65\,\text{m} = 79\,\text{m}.$$

The car traveled 14 m while the driver was reacting and another 65 m during the braking period before coming to a stop. The total distance traveled was then 79 m. Under wet or icy conditions, the value of a may be only one third the value for a dry road since the brakes cannot be applied as hard without skidding, and hence stopping distances are much greater. Note also that the stopping distance during braking increases with the *square* of the speed, not just linearly with speed. If you are traveling twice as fast, it takes four times the distance to stop.

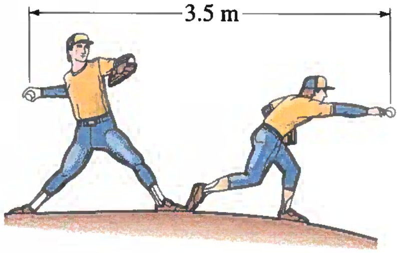


FIGURE 2–14 A baseball pitcher accelerates the ball through a displacement of about 3.5 m.

EXAMPLE 2–9 ESTIMATE **The fastball.** A baseball pitcher throws a fastball with a speed of 44 m/s. Estimate the average acceleration of the ball during the throwing motion. It is observed that in throwing the baseball, the pitcher accelerates the ball through a displacement of about 3.5 m from behind the body to the point where it is released (Fig. 2–14).

SOLUTION We want to find the acceleration a given that $x = 3.5$ m, $v_0 = 0$, and $v = 44$ m/s. We use Eq. 2–10c and solve for a:

$$a = \frac{v^2 - v_0^2}{2x}$$

$$= \frac{(44\ \text{m/s})^2 - (0\ \text{m/s})^2}{2(3.5\ \text{m})} = 280\ \text{m/s}^2.$$

This is a very large acceleration!

PHYSICS APPLIED

Car safety—air bags

EXAMPLE 2–10 ESTIMATE **Air bags.** Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed of 100 km/h (60 mph). Estimate how fast the air bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1 m. How does the use of a seat belt help the driver?

SOLUTION The car decelerates from 100 km/h to zero in a very short time and a very short distance (1 m). Noting that 100 km/h = 100×10^3 m/3600 s = 28 m/s, we can get the acceleration from Eq. 2–10c:

$$a = -\frac{v_0^2}{2x} = -\frac{(28\ \text{m/s})^2}{2.0\ \text{m}} = -390\ \text{m/s}^2.$$

This enormous acceleration takes place in a time given by (Eq. 2–10a):

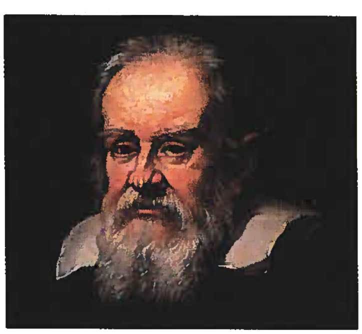
$$t = \frac{v - v_0}{a} = \frac{0 - 28\ \text{m/s}}{-390\ \text{m/s}^2} = 0.07\ \text{s}.$$

To be effective, the air bag would need to inflate faster than this.

What does the air bag do? First, it spreads the force over a larger area of the chest. This is better than being punctured by the steering column. Also, the pressure in the bag is controlled to minimize the head's maximum deceleration. The seat belt keeps the person in the correct position against the expanding air bag.

The analysis of motion we have been discussing in this chapter is basically algebraic. It is sometimes helpful to use a graphical interpretation as well, and this is discussed in the optional Section 2–8.

FIGURE 2–15 Galileo Galilei (1564–1642).



2–7 Falling Objects

One of the most common examples of uniformly accelerated motion is that of an object allowed to fall freely near the Earth's surface. That a falling object is accelerating may not be obvious at first. And beware of thinking, as was widely believed until the time of Galileo (Fig. 2–15), that heavier objects fall faster than lighter objects and that the speed of fall is proportional to how heavy the object is.

Galileo's analysis made use of his new and creative technique of imagining what would happen in idealized (simplified) cases. For free fall, he postulated that all objects would fall with the *same constant acceleration* in the absence of air or other resistance. He showed that this postulate predicts that for an object falling from rest, the distance traveled will be proportional to the square of the time (Fig. 2–16); that is, $d \propto t^2$. We can see this from Eq. 2–10b, but Galileo was the first to derive this mathematical relation. In fact, among Galileo's great contributions to science was to establish such mathematical relations, and to insist on their importance. Another great contribution of Galileo was to propose theories with specific experimental consequences that could be quantitatively checked (such as $d \propto t^2$).

To support his claim that the speed of falling objects increases as they fall, Galileo made use of a clever argument: a heavy stone dropped from a height of 2 m will drive a stake into the ground much further than will the same stone dropped from a height of only 0.2 m. Clearly, the stone must be moving faster in the former case.

As we saw, Galileo also claimed that *all* objects, light or heavy, fall with the *same* acceleration, at least in the absence of air. If you hold a piece of paper horizontally in one hand and a heavier object—say, a baseball—in the other, and release them at the same time as in Fig. 2–17a, the heavier object will reach the ground first. But if you repeat the experiment, this time crumpling the paper into a small wad (see Fig. 2–17b), you will find that the two objects reach the floor at nearly the same time.

Galileo was sure that air acts as a resistance to very light objects that have a large surface area. But in many ordinary circumstances this air resistance is negligible. In a chamber from which the air has been removed, even light objects like a feather or a horizontally held piece of paper will fall with the same acceleration as any other object (see Fig. 2–18). Such a demonstration in vacuum was of course not possible in Galileo's time, which makes Galileo's achievement all the greater. Galileo is often called the "father of modern science," not only for the content of his science (astronomical discoveries, inertia, free fall), but also for his style or approach to science (idealization and simplification, mathematization of theory, theories that have testable consequences, experiments to test theoretical predictions).

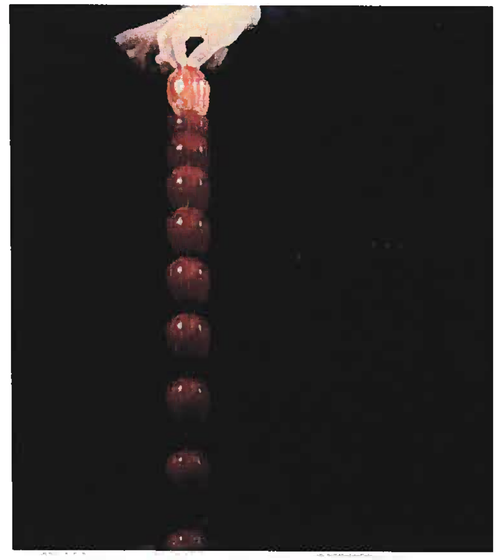
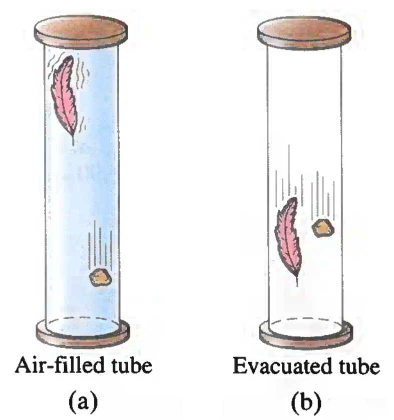


FIGURE 2–16 Multiflash photograph of a falling apple, photographed at equal time intervals. Note that the apple falls farther during each successive time interval, which means it is accelerating.

FIGURE 2–17 (a) A ball and a light piece of paper are dropped at the same time. (b) Repeated, with the paper wadded up.

FIGURE 2–18 A rock and a feather are dropped simultaneously (a) in air, (b) in a vacuum.



Galileo's specific contribution to our understanding of the motion of falling objects can be summarized as follows:

Galileo's hypothesis: free fall is at constant acceleration g

at a given location on the Earth and in the absence of air resistance, all objects fall with the same constant acceleration.

We call this acceleration the **acceleration due to gravity** on the Earth, and we give it the symbol g. Its magnitude is approximately

$$g = 9.80\ \mathrm{m/s^2}.$$

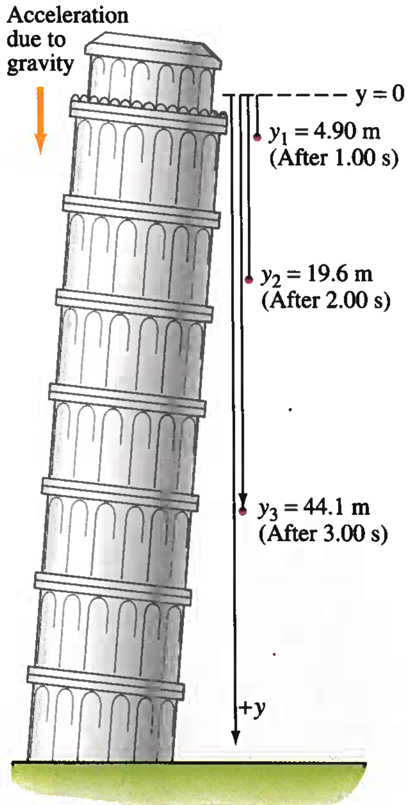
In British units g is about $32\ \mathrm{ft/s^2}$. Actually, g varies slightly according to latitude and elevation (see Table 2–1), but these variations are so small that we will ignore them for most purposes. The effects of air resistance are often small, and we will neglect them for the most part. However, air resistance will be noticeable even on a reasonably heavy object if the velocity becomes large.†

When dealing with freely falling objects we can make use of Eqs. 2–10, where for a we use the value of g given above. Also, since the motion is vertical we will substitute y in place of x, and y_0 in place of x_0. We take $y_0 = 0$ unless otherwise specified. *It is arbitrary whether we choose y to be positive in the upward direction or in the downward direction; but we must be consistent about it throughout a problem's solution.*

TABLE 2–1
Acceleration Due to Gravity at Various Locations on Earth

Location	Elevation (m)	g ($\mathrm{m/s^2}$)
New York	0	9.803
San Francisco	100	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Equator	0	9.780
North Pole (calculated)	0	9.832

FIGURE 2–19 Example 2–11. When an object is dropped from a tower, it falls with progressively greater speed and covers greater distance with each successive second. (See also Fig. 2–16.)



EXAMPLE 2–11 **Falling from a tower.** Suppose that a ball is dropped from a tower 70.0 m high. How far will it have fallen after 1.00 s, 2.00 s, and 3.00 s? Assume y is positive downward. Neglect air resistance.

SOLUTION We are given the acceleration, $a = g = +9.80\ \mathrm{m/s^2}$, which is positive because we have chosen downward as positive. Since we want to find the distance fallen given the time, t, Eq. 2–10b is the appropriate one, with $v_0 = 0$ and $y_0 = 0$. Then, after 1.00 s, the position of the ball is

$$y_1 = \tfrac{1}{2}at^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 = 4.90\ \mathrm{m},$$

so the ball has fallen a distance of 4.90 m after 1.00 s. Similarly, after 2.00 s,

$$y_2 = \tfrac{1}{2}at^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 = 19.6\ \mathrm{m},$$

and after 3.00 s,

$$y_3 = \tfrac{1}{2}at^2 = \tfrac{1}{2}(9.80\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 44.1\ \mathrm{m}.$$

See Fig. 2–19.

EXAMPLE 2–12 **Thrown down from a tower.** Suppose the ball in Example 2–11 is *thrown* downward with an initial velocity of 3.00 m/s, instead of being dropped. (*a*) What then would be its position after 1.00 s and 2.00 s? (*b*) What would its speed be after 1.00 s and 2.00 s? Compare to the speeds of a dropped ball.

†The speed of an object falling in air (or other fluid) does not increase indefinitely. If the object falls far enough, it will reach a maximum velocity called the **terminal velocity**. Acceleration due to gravity is a vector (as is any acceleration), and its direction is downward, toward the center of the Earth.

SOLUTION (*a*) We can approach this in the same way as Example 2–11, using Eq. 2–10b, but this time v_0 is not zero but is $v_0 = 3.0\,\text{m/s}$. Thus, at $t = 1.00\,\text{s}$, the position of the ball is

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(1.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(1.00\,\text{s})^2 = 7.90\,\text{m},$$

and at $t = 2.00\,\text{s}$

$$y = v_0 t + \tfrac{1}{2}at^2 = (3.00\,\text{m/s})(2.00\,\text{s}) + \tfrac{1}{2}(9.80\,\text{m/s}^2)(2.00\,\text{s})^2 = 25.6\,\text{m},$$

As expected, the ball falls farther each second than if it were dropped with $v_0 = 0$.

(*b*) The velocity is readily obtained from Eq. 2–10a:

$$\begin{aligned} v &= v_0 + at \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(1.00\,\text{s}) = 12.8\,\text{m/s} \quad [\text{at } t = 1.00\,\text{s}] \\ &= 3.00\,\text{m/s} + (9.80\,\text{m/s}^2)(2.00\,\text{s}) = 22.6\,\text{m/s}. \quad [\text{at } t = 2.00\,\text{s}] \end{aligned}$$

When the ball is dropped ($v_0 = 0$), the first term in the above equations is zero, so

$$\begin{aligned} v &= 0 + at \\ &= (9.80\,\text{m/s}^2)\,(1.00\,\text{s}) = 9.80\,\text{m/s} \quad [\text{at } t = 1.00\,\text{s}] \\ &= (9.80\,\text{m/s}^2)\,(2.00\,\text{s}) = 19.6\,\text{m/s}. \quad [\text{at } t = 2.00\,\text{s}] \end{aligned}$$

We see that the speed of a dropped ball increases linearly in time. (In Example 2–11 we saw that the distance fallen increases as the *square* of the time.) The downwardly thrown ball also increases linearly in speed ($\Delta v = 9.80\,\text{m/s}$ each second), but its speed at any moment is always $3.0\,\text{m/s}$ (its initial speed) higher than a falling ball.

EXAMPLE 2–13 **Ball thrown upward.** A person throws a ball *upward* into the air with an initial velocity of 15.0 m/s. Calculate (*a*) how high it goes, and (*b*) how long the ball is in the air before it comes back to his hand. We are not concerned here with the throwing action, but only with the motion of the ball *after* it leaves the thrower's hand (Fig. 2–20).

SOLUTION Let us choose y to be positive in the upward direction and negative in the downward direction. (Note: This is a different convention from that used in Examples 2–11 and 2–12.) Then the acceleration due to gravity will have a negative sign, $a = -9.80\,\text{m/s}^2$. Note that as the ball rises, its speed decreases until it reaches the highest point (B in Fig. 2–20), where its speed is zero for an instant; then it descends with increasing speed.

(*a*) To determine the maximum height, we calculate the position of the ball when its velocity equals zero ($v = 0$ at the highest point). At $t = 0$ (point A in Fig. 2–20) we have $y_0 = 0$, $v_0 = 15.0\,\text{m/s}$, and $a = -9.80\,\text{m/s}^2$. At time t (maximum height), $v = 0$, $a = -9.80\,\text{m/s}^2$, and we wish to find y. We use Eq. 2–10c (replacing x with y) and solve for y:

$$v^2 = v_0^2 + 2ay$$

$$y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (15.0\,\text{m/s})^2}{2(-9.80\,\text{m/s}^2)} = 11.5\,\text{m}.$$

The ball reaches a height of 11.5 m above the hand.

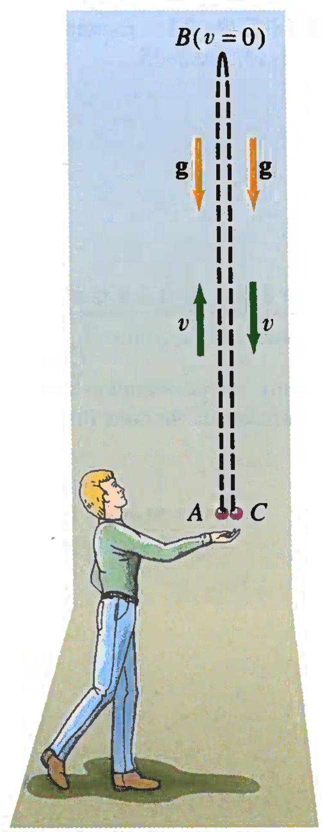


FIGURE 2–20 An object thrown into the air leaves the thrower's hand at A, reaches its maximum height at B, and returns to the original height at C. Examples 2–13, 2–14, and 2–15.

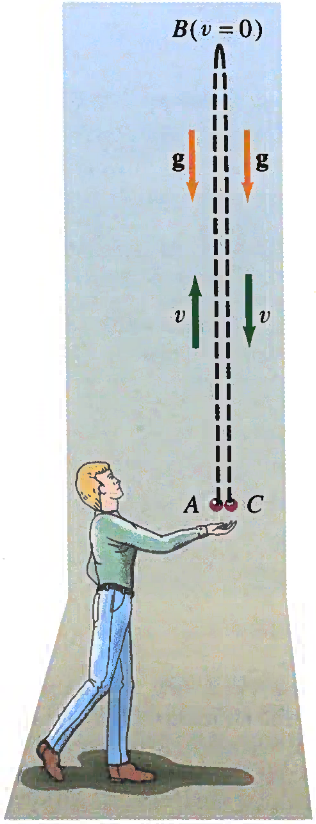


FIGURE 2–21 Examples 2–13, 2–14, and 2–15.

(*b*) Now we need to calculate how long the ball is in the air before it returns to his hand. We could do this calculation in two parts by first determining the time required for the ball to reach its highest point, and then determining the time it takes to fall back down. However, it is simpler to consider the motion from A to B to C (Fig. 2–21) in one step and use Eq. 2–10b. We can do this because y (or x) represents position or displacement, and not the total distance traveled. Thus, at both points A and C, $y = 0$. We use Eq. 2–10b with $a = -9.80\,\mathrm{m/s^2}$ and find

$$y = v_0 t + \tfrac{1}{2} a t^2$$

$$0 = (15.0\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

This equation is readily factored (we factor out one t):

$$(15.0\,\mathrm{m/s} - 4.90\,\mathrm{m/s^2}\,t)\,t = 0.$$

There are two solutions:

$$t = 0,$$

and

$$t = \frac{15.0\,\mathrm{m/s}}{4.90\,\mathrm{m/s^2}} = 3.06\,\mathrm{s}.$$

The first solution ($t = 0$) corresponds to the initial point (A) in Fig. 2–21, when the ball was first thrown and was also at $y = 0$. The second solution, $t = 3.06\,\mathrm{s}$, corresponds to point C, when the ball has returned to $y = 0$. Thus the ball is in the air 3.06 s.

Careful: *Velocity and acceleration are* not *always in the same direction*

Careful: *$a \neq 0$ even at the highest point of a trajectory*

CONCEPTUAL EXAMPLE 2–14 **Two common misconceptions.** Explain the error in these two common misconceptions: (1) that acceleration and velocity are always in the same direction, and (2) that an object thrown upward has zero acceleration at the highest point (B in Fig. 2–21).

RESPONSE Both are wrong. (1) Velocity and acceleration are *not* necessarily in the same direction. When the ball in Example 2–13 is moving upward, its velocity is positive (upward), whereas the acceleration is negative (downward). (2) At the highest point (B in Fig. 2–21), the ball has zero velocity for an instant. Is the acceleration also zero at this point? No. Gravity does not stop acting, so $a = -g = -9.80\,\mathrm{m/s^2}$ even there. Thinking that $a = 0$ at point B would lead to the conclusion that upon reaching point B, the ball would hover there. For if the acceleration (= rate of change of velocity) were zero, the velocity would remain zero, and the ball could stay up there without falling.

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EXAMPLE 2–15 **Ball thrown upward, II.** Let us consider again the ball thrown upward of Example 2–13, and make three more calculations. Calculate (*a*) how much time it takes for the ball to reach the maximum height (point B in Fig. 2–21), (*b*) the velocity of the ball when it returns to the thrower's hand (point C), and (*c*) at what time t the ball passes a point 8.00 m above the person's hand.

SOLUTION Again we take y as positive upward. (*a*) Both Eqs. 2–10a and 2–10b contain the time t with other quantities known. Let us use Eq. 2–10a with $a = -9.80\,\mathrm{m/s^2}$, $v_0 = 15.0\,\mathrm{m/s}$, and $v = 0$:

$$v = v_0 + at,$$

so

$$t = -\frac{v_0}{a} = -\frac{15.0\,\mathrm{m/s}}{-9.80\,\mathrm{m/s^2}} = 1.53\,\mathrm{s}.$$

This is just half the time it takes the ball to go up and fall back to its original position [3.06 s, calculated in part (*b*) of Example 2–13]. Thus it takes the same time to reach the maximum height as to fall back to the starting point.
(*b*) We use Eq. 2–10a with $v_0 = 15.0\,\mathrm{m/s}$ and $t = 3.06\,\mathrm{s}$ (the time calculated in Example 2–13 for the ball to come back to the hand):

$$v = v_0 + at = 15.0\,\mathrm{m/s} - (9.80\,\mathrm{m/s^2})(3.06\,\mathrm{s}) = -15.0\,\mathrm{m/s}.$$

The ball has the same magnitude of velocity when it returns to the starting point as it did initially, but in the opposite direction (this is the meaning of the negative sign). Thus, as we gathered from part (*a*), we see that the motion is symmetrical about the maximum height.

Note the symmetry: the speed at any height is the same when going up as coming down (but the direction is opposite)

(*c*) We want t, given that $y = 8.00\,\mathrm{m}$, $y_0 = 0$, $v_0 = 15.0\,\mathrm{m/s}$, and $a = -9.80\,\mathrm{m/s^2}$. We use Eq. 2–10b:

$$y = y_0 + v_0 t + \tfrac{1}{2}at^2$$

$$8.00\,\mathrm{m} = 0 + (15.0\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2.$$

To solve any quadratic equation of the form $at^2 + bt + c = 0$, where a, b, and c are constants, we can use the **quadratic formula** (see Appendix A–4):

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

PROBLEM SOLVING
Using the quadratic formula

We rewrite our equation in standard form:

$$(4.90\,\mathrm{m/s^2})t^2 - (15.0\,\mathrm{m/s})t + (8.00\,\mathrm{m}) = 0.$$

So the coefficient a is $4.90\,\mathrm{m/s^2}$, b is $-15.0\,\mathrm{m/s}$, and c is 8.00 m. Putting these into the quadratic formula, we obtain

$$t = \frac{15.0\,\mathrm{m/s} \pm \sqrt{(15.0\,\mathrm{m/s})^2 - 4(4.90\,\mathrm{m/s^2})(8.00\,\mathrm{m})}}{2(4.90\,\mathrm{m/s^2})},$$

so $t = 0.69\,\mathrm{s}$ and $t = 2.37\,\mathrm{s}$. Why are there two solutions? Are they both valid? Yes, because the ball passes $y = 8.00\,\mathrm{m}$ when it goes up ($t = 0.69\,\mathrm{s}$) and again when it comes down ($t = 2.37\,\mathrm{s}$).

Acceleration expressed in g's

The acceleration of an object, particularly rockets and fast airplanes, is often given as a multiple of $g = 9.80\,\mathrm{m/s^2}$. For example, a plane pulling out of a dive and undergoing 3.00 g's would have an acceleration of $(3.00)(9.80\,\mathrm{m/s^2}) = 29.4\,\mathrm{m/s^2}$.

* 2–8 Graphical Analysis of Linear Motion[†]

Figure 2–8 showed the graph of the velocity of a car versus time for two cases of linear motion: (*a*) constant velocity, and (*b*) a particular case in which the magnitude of the velocity varied. It is also useful to graph (or "plot") the position x as a function of time. The time t is considered the independent variable and is measured along the horizontal axis. The position, x, the dependent variable, is measured along the vertical axis.

We make a graph of x vs. t, and we make the choice that at $t = 0$, the position is $x_0 = 0$. First we consider a car moving at a constant velocity of 40 km/h, which is equivalent to 11 m/s. From Eq. 2–10b, $x = vt$, and we see that x increases by 11 m every second. Thus, the position increases linearly in time, so the graph of x vs. t is a straight line, as shown in Fig. 2–22. Each point on this straight line tells us the car's position at a particular time. For example, at $t = 3.0$ s, the position is 33 m, and at $t = 4.0$ s, $x = 44$ m, as indicated by the dashed lines. The small triangle on the graph indicates the **slope** of the straight line, which is defined as the change in the dependent variable (Δx) divided by the corresponding change in the independent variable (Δt):

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

We see, using the definition of velocity (Eq. 2–2), that the slope of the x vs. t graph is equal to the velocity. And, as can be seen from the small tri-

[†]Some sections of this book, such as this one, may be considered *optional* at the discretion of the instructor. See the Preface for more details.

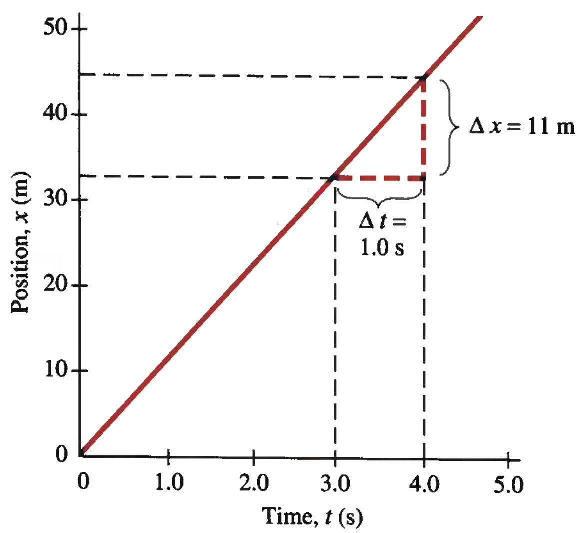

FIGURE 2–22 Graph of position vs. time for an object moving at a uniform velocity of 11 m/s.

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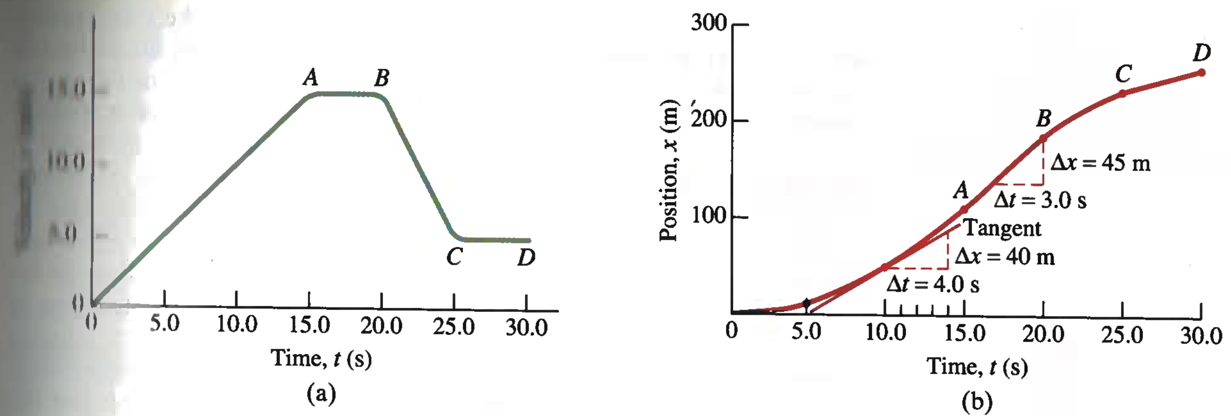

FIGURE 2–23 (a) Velocity vs. time and (b) displacement vs. time for an object with variable velocity. (See text.)

angle on the graph, $\Delta x/\Delta t = (11\text{ m})/(1.0\text{ s}) = 11\text{ m/s}$, which is the given velocity.

The slope of the x vs. t graph is everywhere the same if the velocity is constant, as in Fig. 2–22. But if the velocity changes, as in Fig. 2–23, the slope of the x vs. t graph also varies. Consider, for example, a car that accelerates uniformly from rest to 15 m/s in 15 s, after which it remains at a constant velocity of 15 m/s for the next 5.0 s, then slows down uniformly to 5.0 m/s during the following 5.0 s, and then remains at this constant velocity. The velocity as a function of time is shown in the graph of Fig. 2–23a. Now, to construct the x vs. t graph, we can use Eq. 2–10b with constant acceleration for the intervals $t = 0$ to $t = 15$ s and $t = 20$ s to $t = 25$ s, and with constant velocity for the periods $t = 15$ s to $t = 20$ s and after $t = 25$ s. The result is the x vs. t graph of Fig. 2–23b.

From the origin to point A on the plot, the x vs. t graph is not a straight line, but a curve. The **slope** of the curve at any point is defined as the *slope of the tangent to the curve at that point.* (The *tangent* is a straight line drawn so it touches the curve only at that one point, but does not pass across or through the curve.) For example, the tangent to the curve at the time $t = 10.0$ s is drawn on the graph (it is labeled "tangent"). A triangle is drawn with Δt chosen to be 4.0 s; Δx can be measured off the graph for this chosen Δt and is found to be 40 m. Thus, the slope of the curve at $t = 10.0$ s, which equals the instantaneous velocity at that instant, is $v = \Delta x/\Delta t = 40\text{ m}/4.0\text{ s} = 10\text{ m/s}$. In the region between A and B (Fig. 2–23) the x vs. t graph is a straight line and the slope can be measured using the triangle shown between $t = 17$ s and $t = 20$ s, where the increase in x is 45 m: $\Delta x/\Delta t = 45\text{ m}/3.0\text{ s} = 15\text{ m/s}$.

Slope of a curve

Suppose we were given the x vs. t graph of Fig. 2–23b. We could measure the slopes at a number of points and plot these slopes as a function of time. Since the slope equals the velocity, we could thus reconstruct the v vs. t graph! In other words, given the graph of x vs. t, we can determine the velocity as a function of time using graphical methods, instead of using equations. This technique is particularly useful when the acceleration is not constant, for then Eqs. 2–10 cannot be used.

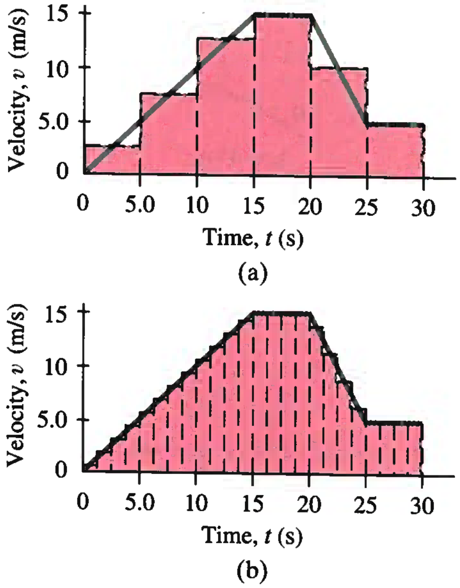

FIGURE 2–24 Determining the displacement from the graph of v vs. t is done by calculating areas.

Displacement = area under v vs. t graph

The reverse process is also possible. If we are given the v vs. t grapl we can determine the position, x, as a function of time. To do so, we us the following procedure, which we apply to the v vs. t graph of Fig. 2–24 (which is the same as Fig. 2–23a). We first divide the time axis into man subintervals (in Fig. 2–24a, only six for simplicity), which are indicated i the figure by the dashed vertical lines. In each interval, a *horizont* dashed line is drawn to indicate the average velocity during that time in terval. For example, in the first interval, the velocity increases at a constan rate from zero to 5.0 m/s, so $\bar{v}$ = 2.5 m/s; and in the fourth interval the ve locity is a constant 15 m/s, so $\bar{v}$ = 15 m/s (no horizontal dashed line shown since it coincides with the curve itself). The displacement (chang in position) during any subinterval is $\Delta x = \bar{v}\Delta t$. Thus the displacemen during each subinterval equals the product of $\bar{v}$ and Δt, and this is just th *area of the rectangle* (base × height = $\Delta t \times v$), shown shaded in rose, fo that interval. The total displacement after 25 s, say, will be the sum of th first five rectangles.

If the velocity varies a great deal, it may be difficult to estimate from the graph. To reduce this difficulty, more—but narrower—subinterval are used. That is, we make each Δt smaller, as in Fig. 2–24b. The more in tervals give a better approximation. Ideally, we could let Δt approac zero; this leads to the techniques of integral calculus, which we don't dis cuss here. The result, in any case, is that *the total displacement betwee any two times is equal to the area under the v vs. t graph between thes two times.*

EXAMPLE 2–16 **Displacement from graph.** A space probe accelerate uniformly from 50 m/s at $t = 0$ to 150 m/s at $t = 10$ s. How far did i move between $t = 2.0$ s and $t = 6.0$ s?

SOLUTION A graph of v vs. t can be drawn as shown in Fig. 2–25. W simply need to calculate the area of the shaded region shown in rose which is a trapezoid. The area will be the average of the heights (in unit of velocity) times the width (which is 4.0 s). At $t = 2.0$ s, $v = 70$ m/s; an at $t = 6.0$ s, $v = 110$ m/s. Thus the area, which equals Δx, is

$$\Delta x = \left(\frac{70\text{ m/s} + 110\text{ m/s}}{2}\right)(4.0\text{ s}) = 360\text{ m}.$$

For this case of constant acceleration, we could use Eqs. 2–10 and we woul get the same result: $a = \Delta v/\Delta t = (150\text{ m/s} - 50\text{ m/s})/10\text{ s} = 10\text{ m/s}$; a $t = 2.0$ s, $v = v_0 + at = 50\text{ m/s} + (10\text{ m/s}^2)(2.0\text{ s}) = 70\text{ m/s}$, and a $t = 6.0$ s, $v = 50\text{ m/s} + (10\text{ m/s}^2)(6.0\text{s}) = 110$ m/s; then, using Eq. 2–10c

$$\Delta x = \frac{(v^2 - v_0^2)}{2a}$$

$$= \frac{(110\text{ m/s})^2 - (70\text{ m/s})^2}{2(10\text{ m/s}^2)} = 360\text{ m}.$$

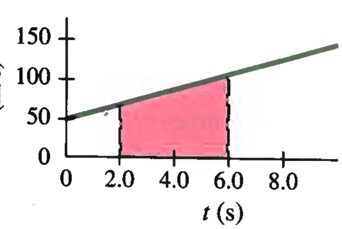

FIGURE 2–25 Example 2–16: the rose-shaded area represents the displacement during the time interval $t = 2.0$ s to $t = 6.0$ s.

In cases where the acceleration is not constant, the area can be obtaine by counting squares on graph paper.

SUMMARY

[The Summary that appears at the end of each chapter in this book gives a brief overview of the main ideas of the chapter. The Summary *cannot* serve to give an understanding of the material, which can be accomplished only by a detailed reading of the chapter.]

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame.**

The **displacement** of an object is the change in position of the object.

Average speed is the distance traveled divided by the elapsed time. An object's **average velocity** over a particular time interval Δt is the displacement Δx divided by Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

The **instantaneous velocity**, whose magnitude is the same as the *instantaneous speed*, is the average velocity taken over an infinitesimally short time interval.

Acceleration is the change of velocity per unit time. An object's **average acceleration** over a time interval Δt is

$$\bar{a} = \frac{\Delta v}{\Delta t},$$

where Δv is the change of velocity during the time interval Δt. **Instantaneous acceleration** is the average acceleration taken over an infinitesimally short time interval.

If an object moves in a straight line with constant acceleration, the velocity v and position x are related to the acceleration a, the elapsed time t, and the initial position x_0 and initial velocity v_0, by Eqs. 2–10:

$$v = v_0 + at, \qquad x = x_0 + v_0 t + \tfrac{1}{2}at^2,$$

$$v^2 = v_0^2 + 2a(x - x_0), \qquad \bar{v} = \frac{v + v_0}{2}.$$

Objects that move vertically near the surface of the Earth, either falling or having been projected vertically up or down, move with the constant downward **acceleration due to gravity** with magnitude of about $g = 9.80\ \mathrm{m/s^2}$, if air resistance can be ignored.

QUESTIONS

1. Does a car speedometer measure speed, velocity, or both?
2. Can an object have a varying velocity if its speed is constant? If yes, give examples.
3. Can an object have a varying speed if its velocity is constant? If yes, give examples.
4. When an object moves with constant velocity, does its average velocity during any time interval differ from its instantaneous velocity at any instant?
5. In drag racing, is it possible for the car with the greatest speed crossing the finish line to lose the race? Explain.
6. If one object has a greater speed than a second object, does the first necessarily have a greater acceleration? Explain, using examples.
7. Compare the acceleration of a motorcycle that accelerates from 80 km/h to 90 km/h with the acceleration of a bicycle that accelerates from rest to 10 km/h in the same time.
8. How is speed represented on a speedometer? How is acceleration represented?
9. Can an object have a northward velocity and a southward acceleration? Explain.
10. Can the velocity of an object be negative when its acceleration is positive? What about vice versa?
11. Give an example where both the velocity and acceleration are negative.
12. Is it possible for an object to have a negative acceleration while increasing in speed? If so, provide an example.
13. Two cars emerge side by side from a tunnel. Car A is traveling with a speed of 60 km/hr and has an acceleration of 40 km/hr/min. Car B has a speed of 40 km/hr and has an acceleration of 60 km/hr/min. Which car is passing the other as they come out of the tunnel? Explain your reasoning.
14. Can an object be increasing in speed as its acceleration decreases? If so, give an example. If not, explain.
15. As a freely falling object speeds up, what is happening to its acceleration due to gravity? Does it increase, decrease, or stay the same?
16. How would you estimate the maximum height you could throw a ball vertically upward? How would you estimate the maximum speed you could give it?

17. An object that is thrown vertically upward will return to its original position with the same speed as it had initially if air resistance is negligible. If air resistance is appreciable, will this result be altered, and if so, how? [*Hint*: The acceleration due to air resistance is always in a direction opposite to the motion.]

* **18.** Describe in words the motion plotted in Fig. 2–26 in terms of v, a, etc. [*Hint*: First try to duplicate the motion plotted by walking or moving your hand.]

* **19.** Describe in words the motion of the object graphed in Fig. 2–27.

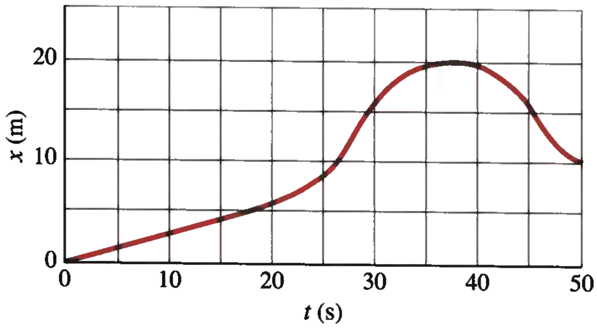

FIGURE 2–26 Question 18, Problems 51, 52, and 57.

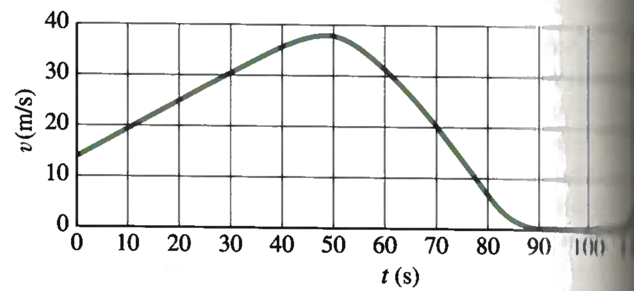

FIGURE 2–27 Question 19, Problems 53, 56, and

PROBLEMS

[The problems at the end of each chapter are ranked I, II, or III according to estimated difficulty, with I problems being easiest. The problems are arranged by Section, meaning that the reader should have read up to and including that Section, but not only that Section—problems often depend on earlier material. Finally, there is a set of unranked "General Problems" not arranged by Section number.]

SECTIONS 2–1 TO 2–3

1. (I) What must be your average speed in order to travel 230 km in 3.25 h?
2. (I) A bird can fly 25 km/h. How long does it take to fly 15 km?
3. (I) If you are driving 110 km/h along a straight road and you look to the side for 2.0 s, how far do you travel during this inattentive period?
4. (I) 65 mph is how many (*a*) km/h, (*b*) m/s, and (*c*) ft/s?
5. (II) You are driving home from school steadily at 65 mph for 130 miles. It then begins to rain and you slow to 55 mph. You arrive home after driving 3 hours and 20 minutes. (*a*) How far is your hometown from school? (*b*) What was your average speed?
6. (II) According to a rule-of-thumb, every five seconds between a lightning flash and the following thunder gives the distance of the storm in miles. Assuming that the flash of light arrives in essentially no time at all, estimate the speed of sound in m/s from this rule.
7. (II) A person jogs eight complete laps around a quarter-mile track in a total time of 12.5 min. Calculate (*a*) the average speed and (*b*) the average velocity, in m/s.
8. (II) A horse canters away from its trainer in a straight line, moving 130 m away in 14.0 s. It then turns abruptly and gallops halfway back in 4.8 s. Calculate (*a*) its average speed and (*b*) its average velocity for the entire trip, using "away from the trainer" as the positive direction.
9. (II) Two locomotives approach each other on parallel tracks. Each has a speed of 95 km/h with respect to the ground. If they are initially 8.5 km apart, how long will it be before they reach each other? (See Fig. 2–28.)

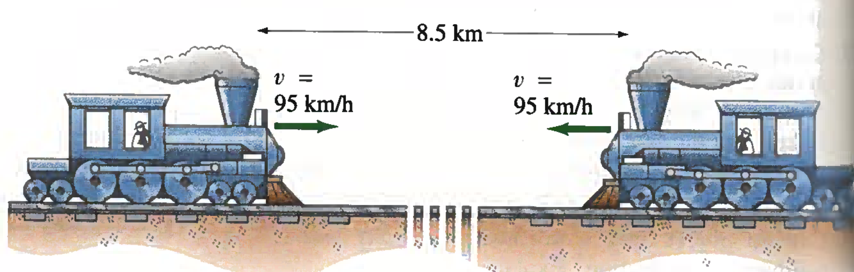

FIGURE 2–28 Problem 9.

10. (II) An airplane travels 2100 km at a speed of 800 km/h, and then encounters a tailwind that boosts its speed to 1000 km/h for the next 1800 km. What was the total time for the trip? What was the average speed of the plane for this trip? [*Hint*: Think carefully before using Eq. 2–10d.]

11. (II) Calculate the average speed and average velocity of a complete round-trip in which the outgoing 200 km is covered at 90 km/h, followed by a one-hour lunch break, and the return 200 km is covered at 50 km/h.

12. (III) A bowling ball traveling with constant speed hits the pins at the end of a bowling lane 16.5 m long. The bowler hears the sound of the ball hitting the pins 2.50 s after the ball is released from his hands. What is the speed of the ball? The speed of sound is 340 m/s.

SECTION 2–4

13. (I) A sports car accelerates from rest to 95 km/h in 6.2 s. What is its average acceleration in m/s^2?

14. (I) At highway speeds, a particular automobile is capable of an acceleration of about $1.6\ m/s^2$. At this rate, how long does it take to accelerate from 80 km/h to 110 km/h?

15. (I) A sprinter accelerates from rest to 10.0 m/s in 1.35 s. What is her acceleration (*a*) in m/s^2, and (*b*) in km/h^2?

16. (II) A sports car is advertised to be able to stop in a distance of 50 m from a speed of 90 km/h. What is its acceleration in m/s^2? How many *g*'s is this ($g = 9.80\ m/s^2$)?

17. (III) The position of a racing car, which starts from rest at $t = 0$ and moves in a straight line, has been measured as a function of time, as given in the following table. Estimate (*a*) its velocity and (*b*) its acceleration as a function of time. Display each in a table and on a graph.

t(s)	0	0.25	0.50	0.75	1.00	1.50	2.00	2.50
x(m)	0	0.11	0.46	1.06	1.94	4.62	8.55	13.79

t(s)	3.00	3.50	4.00	4.50	5.00	5.50	6.00
x(m)	20.36	28.31	37.65	48.37	60.30	73.26	87.16

SECTIONS 2–5 AND 2–6

18. (I) The principal kinematic equations, Eqs. 2–10a through 2–10d, become particularly simple if the initial speed is zero. Write down the equations for this special case. (Also put $x_0 = 0$.)

19. (I) A car accelerates from 12 m/s to 25 m/s in 6.0 s. What was its acceleration? How far did it travel in this time? Assume constant acceleration.

20. (I) A car slows down from 20 m/s to rest in a distance of 85 m. What was its acceleration, assumed constant?

21. (I) A light plane must reach a speed of 30 m/s for takeoff. How long a runway is needed if the (constant) acceleration is $3.0\ m/s^2$?

22. (II) A world-class sprinter can burst out of the blocks to essentially top speed (of about 11.5 m/s) in the first 15.0 m of the race. What is the average acceleration of this sprinter and how long does it take her to reach that speed?

23. (II) A car slows down from a speed of 25.0 m/s to rest in 5.00 s. How far did it travel in that time?

24. (II) In coming to a stop, a car leaves skid marks 80 m long on the highway. Assuming a deceleration of $7.00\ m/s^2$, estimate the speed of the car just before braking.

25. (II) A car traveling 45 km/h slows down at a constant $0.50\ m/s^2$ just by "letting up on the gas." Calculate (*a*) the distance the car coasts before it stops, (*b*) the time it takes to stop, and (*c*) the distance it travels during the first and fifth seconds.

26. (II) A car traveling at 90 km/h strikes a tree. The front end of the car compresses and the driver comes to rest after traveling 0.80 m. What was the average acceleration of the driver during the collision? Express the answer in terms of "*g*'s," where $1.00\ g = 9.80\ m/s^2$.

27. (II) Determine the stopping distances for an automobile with an initial speed of 90 km/h and human reaction time of 1.0 s: (*a*) for an acceleration $a = -4.0\ m/s^2$; (*b*) for $a = -8.0\ m/s^2$.

28. (III) Show that the equation for the stopping distance of a car is $d_S = v_0 t_R - v_0^2/(2a)$, where v_0 is the initial speed of the car, t_R is the driver's reaction time, and a is the constant acceleration (and is negative).

29. (III) A speeding motorist traveling 120 km/h passes a stationary police officer. The officer immediately begins pursuit at a constant acceleration of 10.0 km/h/s (note the mixed units). How much time will it take for the police officer to reach the speeder, assuming that the speeder maintains a constant speed? How fast will the police officer be traveling at this time?

30. (III) A person driving her car at 50 km/h approaches an intersection just as the traffic light turns yellow. She knows that the yellow light lasts only 2.0 s before turning to red, and she is 30 m away from the near side of the intersection (Fig. 2–29). Should she try to stop, or should she make a run for it? The intersection is 15 m wide. Her car's maximum deceleration is $-6.0\ m/s^2$, whereas it can accelerate from 50 km/h to 70 km/h in 6.0 s. Ignore the length of her car and her reaction time.

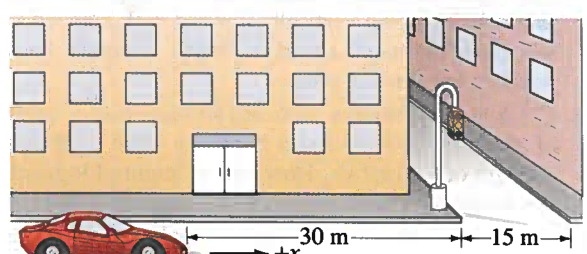

FIGURE 2–29 Problem 30.

31. (III) A runner hopes to complete the 10,000-m run in less than 30.0 min. After exactly 27.0 min, there are still 1100 m to go. The runner must then accelerate at $0.20\,\mathrm{m/s^2}$ for how many seconds in order to achieve the desired time?

SECTION 2–7 [neglect air resistance]

32. (I) Calculate the acceleration of the baseball in Example 2–9 in "g's."

33. (I) If a car rolls gently ($v_0 = 0$) off a vertical cliff, how long does it take it to reach 90 km/h?

34. (I) A stone is dropped from the top of a cliff. It is seen to hit the ground below after 3.50 s. How high is the cliff?

35. (I) Calculate (*a*) how long it took King Kong to fall straight down from the top of the Empire State Building (380 m high), and (*b*) his velocity just before "landing"?

36. (II) A foul ball is hit straight up into the air with a speed of about 25 m/s. (*a*) How high does it go? (*b*) How long is it in the air?

37. (II) A kangaroo jumps to a vertical height of 2.7 m. How long was it in the air before returning to Earth?

38. (II) A ballplayer catches a ball 3.3 s after throwing it vertically upward. With what speed did he throw it, and what height did it reach?

39. (II) Draw graphs of (*a*) the speed and (*b*) the distance fallen, as a function of time, for an object falling under the influence of gravity from $t = 0$ to $t = 5.00$ s. Ignore air resistance and assume $v_0 = 0$.

40. (II) The best rebounders in basketball have a vertical leap (that is, the vertical movement of a fixed point on their body) of about 120 cm. (*a*) What is their initial "launch" speed off the ground? (*b*) How long are they in the air?

41. (II) A helicopter is ascending vertically with a speed of 5.50 m/s. At a height of 105 m above the Earth, a package is dropped from a window. How much time does it take for the package to reach the ground?

42. (II) For an object falling freely from rest, show that the distance traveled during each successive second increases in the ratio of successive odd integers (1, 3, 5, etc.). (This was first shown by Galileo.) See Figs. 2–16 and 2–19.

43. (II) If air resistance is neglected, show (algebraically) that a ball thrown vertically upward with a speed v_0 will have the same speed, v_0, when it comes back down to the starting point.

44. (II) A stone is thrown vertically upward with a speed of 20.0 m/s. (*a*) How fast is it moving when it reaches a height of 12.0 m? (*b*) How long is required to reach this height? (*c*) Why are there two answers to (*b*)?

45. (II) Estimate the time between each photoflash of the apple in Fig. 2–16 (or number of photoflashes per second). Assume the apple is about 10 cm in diameter.

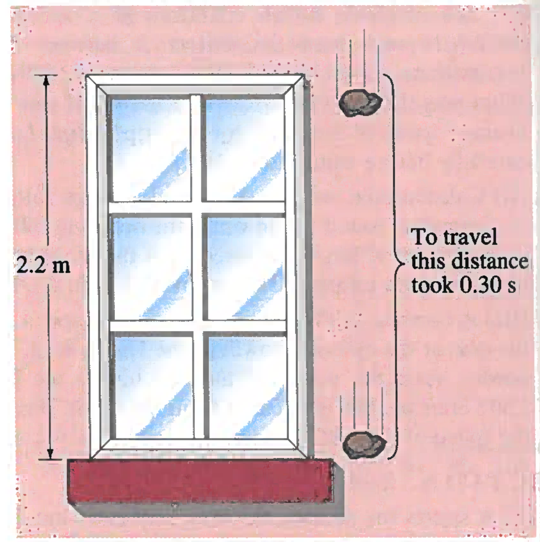

FIGURE 2–30 Problem 46.

46. (III) A falling stone takes 0.30 s to travel past a wi dow 2.2 m tall (Fig. 2–30). From what height abo the top of the window did the stone fall?

47. (III) A rock is dropped from a sea cliff and th sound of it striking the ocean is heard 3.4 s later. the speed of sound is 340 m/s, how high is the cliff

48. (III) Suppose you adjust your garden hose nozzle f a hard stream of water. You point the nozzle vert cally upward at a height of 1.5 m above the grou (Fig. 2–31). When you quickly move the nozzle aw from the vertical, you hear the water striking th ground next to you for another 2.0 s. What is th water speed as it leaves the nozzle?

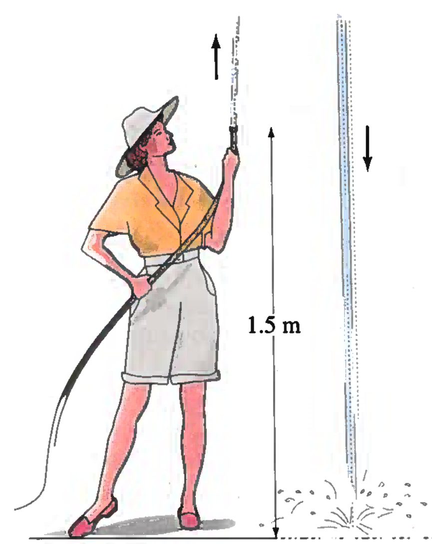

FIGURE 2–31 Problem 48.

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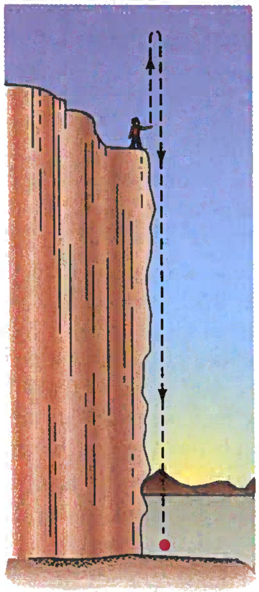


FIGURE 2-32 Problem 49.

49. (III) A stone is thrown vertically upward with a speed of 12.0 m/s from the edge of a cliff 75.0 m high (Fig. 2-32). (a) How much later does it reach the bottom of the cliff? (b) What is its speed just before hitting? (c) What total distance did it travel?

50. (III) A baseball is seen to pass upward by a window 25 m above the street with a vertical speed of 12 m/s. If the ball was thrown from the street, (a) what was its initial speed, (b) what altitude does it reach, (c) when was it thrown, and (d) when does it reach the street again?

SECTION 2-8

51. (I) The position of a rabbit along a straight tunnel as a function of time is plotted in Fig. 2-26. What is its instantaneous velocity (a) at $t = 10.0$ s and (b) at $t = 30.0$ s? What is its average velocity (c) between $t = 0$ and $t = 5.0$ s, (d) between $t = 25.0$ s and $t = 30.0$ s, and (e) between $t = 40.0$ s and $t = 50.0$ s?

52. (I) In Fig. 2-26, (a) during what time periods, if any, is the object's velocity constant? (b) At what time is its velocity the greatest? (c) At what time, if any, is the velocity zero? (d) Does the object run in one direction or in both along its tunnel during the time shown?

53. (I) Figure 2-27 shows the velocity of a train as a function of time. (a) At what time was its velocity greatest? (b) During what periods, if any, was the velocity constant? (c) During what periods, if any, was the acceleration constant? (d) When was the magnitude of the acceleration greatest?

54. (II) A high-performance automobile can accelerate approximately as shown in the velocity-time graph of Fig. 2-33. (The short flat spots in the curve represent shifting of the gears.) (a) Estimate the average acceleration of the car in second gear and in fourth gear. (b) Estimate how far the car traveled while in fourth gear.

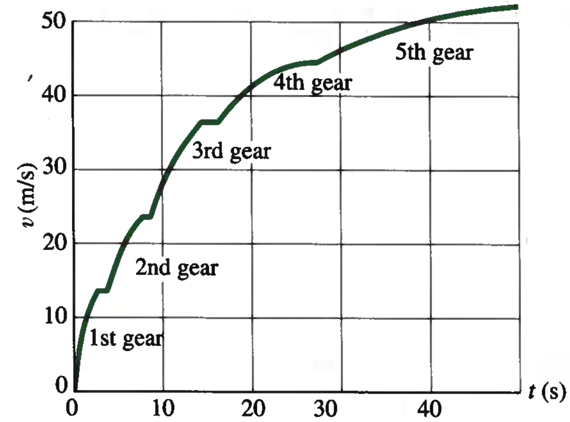


FIGURE 2-33 The velocity of a high-performance automobile as a function of time, starting from a dead stop. The jumps in the curve represent gear shifts. (Problems 54 and 55.)

* 55. (II) Estimate the average acceleration of the car in the previous problem (Fig. 2-33) when it is in (a) first, (b) third, and (c) fifth gear. (d) What is its average acceleration through the first four gears?

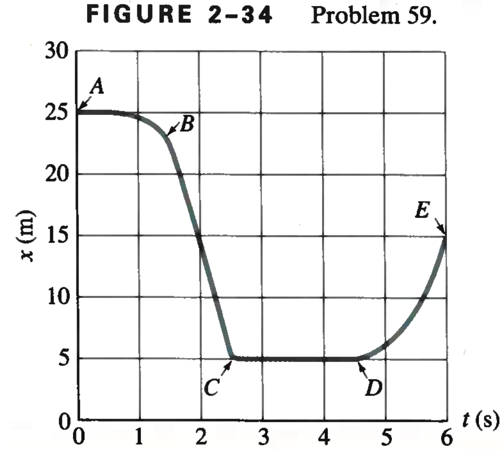
* 56. (II) In Fig. 2-27, estimate the distance the object traveled during (a) the first minute and (b) the second minute.

* 57. (II) Construct the v vs. t graph for the object whose displacement as a function of time is given by Fig. 2-26.

* 58. (II) Construct an x vs. t graph for the object whose velocity as a function of time is given by Fig. 2-27.

* 59. (II) Figure 2-34 is a position versus time graph for the motion of an object along the x axis. As the object moves from A to B: (a) Is the object moving in the positive or negative direction? (b) Is the object speeding up or slowing down? (c) Is the acceleration of the object positive or negative? Next, for the time interval from D to E: (d) Is the object moving in the positive or negative direction? (e) Is the object speeding up or slowing down? (f) Is the acceleration of the object positive or negative? (g) Finally, answer these same three questions for the time interval from C to D.

FIGURE 2-34 Problem 59.



GENERAL PROBLEMS

60. A person jumps from a fourth-story window 15.0 m above a firefighter's safety net. The survivor stretches the net 1.0 m before coming to rest, Fig. 2–35. (*a*) What was the average deceleration experienced by the survivor when slowed to rest by the net? (*b*) What would you do to make it "safer" (that is, generate a smaller deceleration): would you stiffen or loosen the net? Explain.

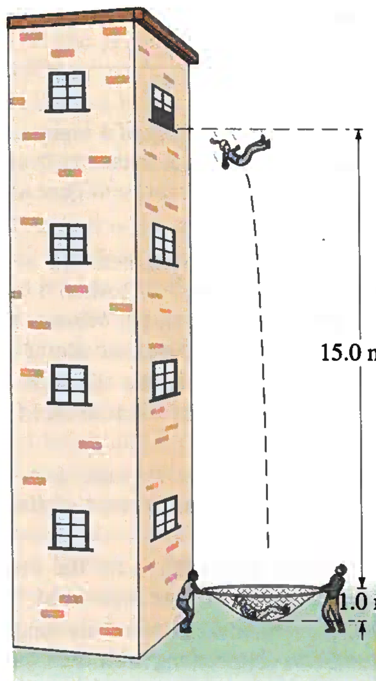


FIGURE 2–35 Problem 60.

61. The acceleration due to gravity on the Moon is about one sixth what it is on Earth. If an object is thrown vertically upward on the Moon, how many times higher will it go than it would on Earth, assuming the same initial velocity?

62. A person who is properly constrained by an over-the-shoulder seat belt has a good chance of surviving a car collision if the deceleration does not exceed 30 "g's" ($1.00\ g = 9.80\ \mathrm{m/s^2}$). Assuming uniform deceleration of this value, calculate the distance over which the front end of the car must be designed to collapse if a crash brings the car to rest from 100 km/h.

63. A race car driver must average 200.0 km/h over the course of a time trial lasting ten laps. If the first nine laps were done at 199.0 km/h, what average speed must be maintained for the last lap?

64. A car manufacturer tests its cars for front-end coll… by hauling them up on a crane and dropping them f… a certain height. (*a*) Show that the speed just before… car hits the ground, after falling from rest a vertical dis-tance H, is given by $\sqrt{2gH}$. What height corresponds… a collision at (*b*) 50 km/h? (*c*) 100 km/h?

65. A first stone is dropped from the roof of a buil… 2.00 s after that, a second stone is thrown straight… with an initial speed of 30.0 m/s, and it is observed… the two stones land at the same time. (*a*) How long… it take the first stone to reach the ground? (*b*) … high is the building? (*c*) What are the speeds of the… stones just before they hit the ground?

66. A 90-m-long train begins uniform accelera… from rest. The front of the train has a speed… 20 m/s when it passes a railway worker wh… standing 180 m from where the front of the… started. What will be the speed of the last car… passes the worker? (See Fig. 2–36.)

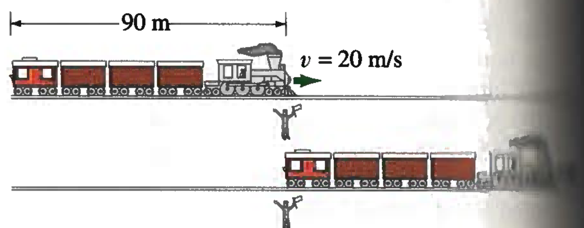


FIGURE 2–36 Problem 66.

67. A police car at rest, passed by a speeder travel… a constant 110 km/h, takes off in hot pursuit. The po-lice officer catches up to the speeder in 700 m, main-taining a constant acceleration. (*a*) Qualitatively… the position versus time graph for both cars from… police car's start to the catch-up point. (*b*) Calc… how long it took the police officer to overtake… speeder, (*c*) calculate the required police car acceler-ation, and (*d*) calculate the speed of the police car… the overtaking point.

68. In the design of a rapid transit system, it is nec… to balance out the average speed of a train… the distance between stops. The more stops the… the slower the train's average speed. To get an ide… this problem, calculate the time it takes a train… make a 36-km trip in two situations: (*a*) the stati… at which the trains must stop are 0.80 km apart… (*b*) the stations are 3.0 km apart. Assume that at… station the train accelerates at a rate of $1.1\ \mathrm{m/s^2}$… it reaches 90 km/h, then stays at this speed until… brakes are applied for arrival at the next station… which time it decelerates at $-2.0\ \mathrm{m/s^2}$. Assume… stops at each intermediate station for 20 s.

69. Pelicans tuck their wings and free fall straight down when diving for fish. Suppose a pelican starts its dive from a height of 16.0 m and cannot change its path once committed. If it takes a fish 0.20 s to perform evasive action, at what minimum height must it spot the pelican to escape? Assume the fish is at the surface of the water.

70. In putting, the force with which a golfer strikes a ball is planned so that the ball will stop within some small distance of the cup, say 1.0 m long or short, in case the putt is missed. Accomplishing this from an uphill lie (that is, putting downhill, see Fig. 2–37) is more difficult than from a downhill lie. To see why, assume that on a particular green the ball decelerates constantly at $2.0\ \mathrm{m/s^2}$ going downhill, and constantly at $3.0\ \mathrm{m/s^2}$ going uphill. Suppose we have an uphill lie 7.0 m from the cup. Calculate the allowable range of initial velocities we may impart to the ball so that it stops in the range 1.0 m short to 1.0 m long of the cup. Do the same for a downhill lie 7.0 m from the cup. What in your results suggests that the downhill putt is more difficult?

71. A car is behind a truck going 25 m/s on the highway. The driver looks for an opportunity to pass, guessing that his car can accelerate at $1.0\ \mathrm{m/s^2}$, and he gauges that he has to cover the 20-m length of the truck, plus 10 m clear room at the rear of the truck and 10 m more at the front of it. In the oncoming lane, he sees a car approaching, probably also traveling at 25 m/s. He estimates that the car is about 400 m away. Should he attempt the pass? Give details.

72. A stone is dropped from the roof of a high building. A second stone is dropped 1.50 s later. How far apart are the stones when the second one has reached a speed of 12.0 m/s?

73. Bond is standing on a bridge, 10 m above the road below, and his pursuers are getting too close for comfort. He spots a flatbed truck loaded with mattresses approaching at 30 m/s, which he measures by knowing that the telephone poles the truck is passing are 20 m apart in this country. The bed of the truck is 1.5 m above the road, and Bond quickly calculates how many poles away the truck should be when he jumps down from the bridge onto the truck, making his getaway. How many poles is it?

FIGURE 2–37 Problem 70. Golf on Wednesday morning.