CHAPTER 7

1. $p = mv = (0.022 \text{ kg})(8.1 \text{ m/s}) =$ 0.18 kg ·m/s.

2. During the throwing we use momentum conservation for the one-dimensional motion:
   $0 = (m_{boat} + m_{child})v_{boat} + m_{package}v_{package}$;
   $0 = (55.0 \text{ kg} + 20.0 \text{ kg})v_{boat} + (5.40 \text{ kg})(10.0 \text{ m/s})$, which gives
   $v_{boat} =$ – 0.720 m/s (opposite to the direction of the package).

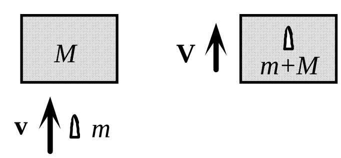
3. We find the force on the expelled gases from
   $F =$ Æ$p/$Æ$t = ($Æ$m/$Æ$t)v = (1300 \text{ kg/s})(40{,}000 \text{ m/s}) = 5.2 \times 10^7$ N.
   An equal, but opposite, force will be exerted on the rocket: 5.2×10^7 N, up.

4. For this one-dimensional motion, we take the direction of the halfback for the positive direction. For this perfectly inelastic collision, we use momentum conservation:
   $M_1v_1 + M_2v_2 = (M_1 + M_2)V$;
   $(95 \text{ kg})(4.1 \text{ m/s}) + (85 \text{ kg})(5.5 \text{ m/s}) = (95 \text{ kg} + 85 \text{ kg})V$, which gives $V = 4.8$ m/s.

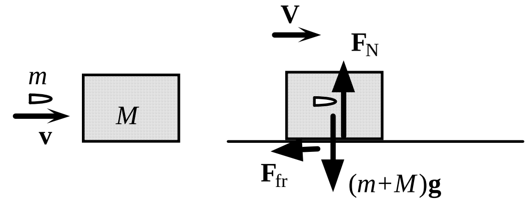
5. For the horizontal motion, we take the direction of the car for the positive direction. The load initially has no horizontal velocity. For this perfectly inelastic collision, we use momentum conservation:
   $M_1v_1 + M_2v_2 = (M_1 + M_2)V$;
   $(12{,}500 \text{ kg})(18.0 \text{ m/s}) + 0 = (12{,}500 \text{ kg} + 5750 \text{ kg})V$, which gives $V = 12.3$ m/s.

6. For the one-dimensional motion, we take the direction of the first car for the positive direction. For this perfectly inelastic collision, we use momentum conservation:
   $M_1v_1 + M_2v_2 = (M_1 + M_2)V$;
   $(9500 \text{ kg})(16 \text{ m/s}) + 0 = (9500 \text{ kg} + M_2)(6.0 \text{ m/s})$, which gives $M_2 = 1.6 \times 10^4$ kg.

7. We let V be the speed of the block and bullet immediately after the embedding and before the two start to rise.
   For this perfectly inelastic collision, we use momentum conservation:
   $mv + 0 = (M + m)V$;
   $(0.021 \text{ kg})(210 \text{ m/s}) = (0.021 \text{ kg} + 1.40 \text{ kg})V$, which gives $V = 3.10$ m/s.
   For the rising motion we use energy conservation, with the potential reference level at the ground:
   $KE_i + PE_i = KE_f + PE_f$;
   !$(M + m)V^2 + 0 = 0 + (m + M)gh$, or
   $h = V^2/2g = (3.10 \text{ m/s})^2/2(9.80 \text{ m/s}^2) =$ 0.491 m.



8. On the horizontal surface after the collision, the normal force is $F_N = (m + M)g$. We find the common speed of the block and bullet immediately after the embedding by using the work-energy principle for the sliding motion:



W_{fr} = ÆKE;

$-\mu_k(m + M)gd = 0 -$!$(M + m)V^2$;

$0.25(9.80\ m/s^2)(9.5\ m) =$!V^2, which gives $V = 6.82$ m/s.

For the collision, we use momentum conservation:

$mv + 0 = (M + m)V$;

$(0.015\ kg)v = (0.015\ kg + 1.10\ kg)(6.82\ m/s)$, which gives $v = 5.1 \times 10^2$ m/s.

9. The new nucleus and the alpha particle will recoil in opposite directions. Momentum conservation gives us

$0 = MV - m_\alpha v_\alpha$,

$0 = (57m_\alpha)V - m_\alpha(3.8 \times 10^5\ m/s)$, which gives $V =$ 6.7×10^3 m/s.

10. Because mass is conserved, the mass of the new nucleus is $M_2 = 222\ u - 4.0\ u = 218\ u$. Momentum conservation gives us

$M_1V_1 = M_2V_2 + m_\alpha v_\alpha$,

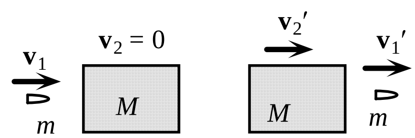
$(222\ u)(420\ m/s) = (218\ u)(350\ m/s) + (4.0\ u)v_\alpha$, which gives $v_\alpha =$ 4.2×10^3 m/s.

11. Momentum conservation gives us

$mv_1 + Mv_2 = mv_1' + Mv_2'$,

$(0.013\ kg)(230\ m/s) + 0 = (0.013\ kg)(170\ m/s) + (2.0\ kg)v_2'$,

which gives $v_2' =$ 0.39 m/s.



12. (*a*) With respect to the Earth after the explosion, one section will have a speed v_1' and the other will have a speed $v_2' = v_1' + v_{relative}$. Momentum conservation gives us

$mv =$!mv_1' + !mv_2', or

$v =$!v_1' + !$(v_1' + v_{relative}) = v_1'$ + !$v_{relative}$;

$5.80 \times 10^3\ m/s = v_1'$ + !$(2.20 \times 10^3\ m/s)$, which gives $v_1' =$ 4.70×10^3 m/s.

The other section will have

$v_2' = v_1' + v_{relative} = 4.70 \times 10^3\ m/s + 2.20 \times 10^3\ m/s =$ 6.90×10^3 m/s.

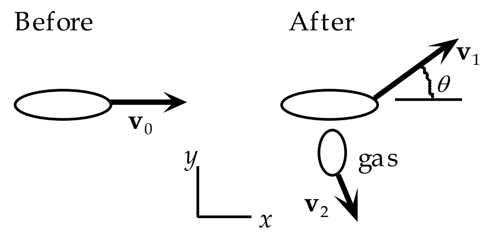
(*b*) The energy supplied by the explosion increases the kinetic energy:

E = ÆKE = [!(!$m)v_1'^2$ + !(!$m)v_2'^2$] – !mv^2

= [!(!975 kg)$(4.70 \times 10^3\ m/s)^2$ + !(!975 kg)$(6.90 \times 10^3\ m/s)^2$] – !(975 kg)$(5.80 \times 10^3\ m/s)^2$

= 5.90×10^8 J.

13. If M is the initial mass of the rocket and m_2 is the mass of the expelled gases, the final mass of the rocket is $m_1 = M - m_2$. Because the gas is expelled perpendicular to the rocket in the rocket's frame, it will still have the initial forward velocity, so the velocity of the rocket in the original direction will not change. We find the perpendicular component of the rocket's velocity after firing from



$v_{1\perp} = v_0 \tan\theta = (115\text{ m/s}) \tan 35° = 80.5\text{ m/s}.$

Using the coordinate system shown, for momentum conservation in the perpendicular direction we have

$0 + 0 = m_1 v_{1\perp} - m_2 v_{2\perp}$, or

$(M - m_2)v_{1\perp} = m_2 v_{2\perp}$;

$(3180\text{ kg} - m_2)(80.5\text{ m/s}) = m_2(1750\text{ m/s})$, which gives $m_2 =$ 140 kg.

14. We find the average force on the ball from

F = Æp/Æt = (Æmv/Æt) =[(0.0600 kg/s)(65.0 m/s) – 0]/(0.0300 s) = 130 N.

Because the weight of a 60-kg person is ≈ 600 N, this force is not large enough.

15. We find the average force on the ball from

F = Æp/Æt = m Æv/Æt =(0.145 kg)[(52.0 m/s) – (– 39.0 m/s)]/(1.00 × 10^{-3} s) = 1.32 × 10^4 N.

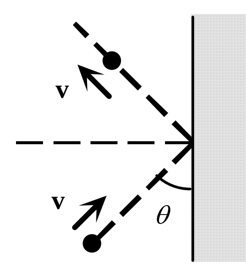
16. (*a*) We find the impulse on the ball from

Impulse = Æp = m Æv =(0.045 kg)(45 m/s – 0) = 2.0 N ·s.

(*b*) The average force is

F = Impulse/Æt = (2.0 N ·s)/(5.0 × 10^{-3} s) = 4.0 × 10^2 N.

17. The momentum parallel to the wall does not change, therefore the impulse will be perpendicular to the wall. With the positive direction toward the wall, we find the impulse on the ball from



Impulse = Æ$p_\perp$ = m Æ$v_\perp$ = $m[(-v \sin\theta) - (v \sin\theta)]$

= – $2mv \sin\theta$ = 2(0.060 kg)(25 m/s) sin 45° = – 2.1 N ·s.

The impulse on the wall is in the opposite direction: 2.1 N ·s.

18. (*a*) With the positive direction in the direction of the fullback (East), the momentum is

$p = m_{\text{fullback}} v_{\text{fullback}}$ = (115 kg)(4.0 m/s) = 4.6 × 10^2 kg ·m/s (East).

(*b*) We find the impulse on the fullback from

$\text{Impulse}_{\text{fullback}}$ = Æp_{fullback}

= 0 – 4.6 × 10^2 kg ·m/s = – 4.6 × 10^2 kg ·m/s (West).

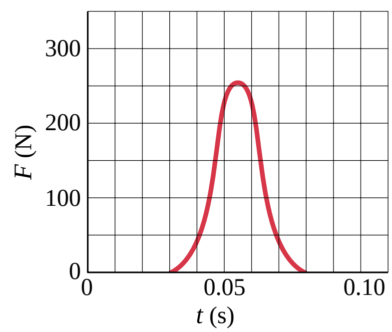
(*c*) We find the impulse on the tackler from

$\text{Impulse}_{\text{tackler}}$ = – $\text{Impulse}_{\text{fullback}}$ = + 4.6 × 10^2 kg ·m/s (East).

(*d*) We find the average force on the tackler from

F_{tackler} = $\text{Impulse}_{\text{tackler}}$/Æ$t$ = (+ 4.6 × 10^2 kg ·m/s)/(0.75 s) = 6.1 × 10^2 N (East).

19. (*a*) The impulse is the area under the F vs. t curve.
The value of each block on the graph is
$1 \text{ block} = (50 \text{ N})(0.01 \text{ s}) = 0.50 \text{ N}\cdot\text{s}$.
We estimate there are 10 blocks under the curve, so the impulse is
$\text{Impulse} = (10 \text{ blocks})(0.50 \text{ N}\cdot\text{s/block}) \approx 5.0 \text{ N}\cdot\text{s}$.



(*b*) We find the final velocity of the ball from
$\text{Impulse} = \Delta p = m\,\Delta v$;
$5.0 \text{ N}\cdot\text{s} = (0.060 \text{ kg})(v - 0)$, which gives $v = 83 \text{ m/s}$.

20. The maximum force that each leg can exert without breaking is
$(170 \times 10^6 \text{ N/m}^2)(2.5 \times 10^{-4} \text{ m}^2) = 4.25 \times 10^4 \text{ N}$,
so, if there is an even landing with both feet, the maximum force allowed on the body is 8.50×10^4 N.
We use the work-energy principle for the fall to find the landing speed:
$0 = \Delta\text{KE} + \Delta\text{PE}$;
$0 = \frac{1}{2}mv_{\text{land}}^2 - 0 + (0 - mgh_{\max})$, or $v_{\text{land}}^2 = 2gh_{\max}$.
The impulse from the maximum force changes the momentum on landing. If we take down as the positive direction and assume the landing lasts for a time t, we have
$-F_{\max}t = m\,\Delta v = m(0 - v_{\text{land}})$, or $t = mv_{\text{land}}/F_{\max}$.
We have assumed a constant force, so the acceleration will be constant. For the landing we have
$y = v_{\text{land}}t + \frac{1}{2}at^2 = v_{\text{land}}(mv_{\text{land}}/F_{\max}) + \frac{1}{2}(-F_{\max}/m)(mv_{\text{land}}/F_{\max})^2 = \frac{1}{2}mv_{\text{land}}^2/F_{\max} = mgh_{\max}/F_{\max}$;
$0.60 \text{ m} = (75 \text{ kg})(9.80 \text{ m/s}^2)h_{\max}/(8.50 \times 10^4 \text{ N})$, which gives $h_{\max} = 69 \text{ m}$.

21. For the elastic collision of the two balls, we use momentum conservation:
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;
$(0.440 \text{ kg})(3.70 \text{ m/s}) + (0.220 \text{ kg})(0) = (0.440 \text{ kg})v_1' + (0.220 \text{ kg})v_2'$.
Because the collision is elastic, the relative speed does not change:
$v_1 - v_2 = -(v_1' - v_2')$, or $3.70 \text{ m/s} - 0 = v_2' - v_1'$.
Combining these two equations, we get
$v_1' = 1.23 \text{ m/s}$, and $v_2' = 4.93 \text{ m/s}$.

22. For the elastic collision of the two pucks, we use momentum conservation:
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;
$(0.450 \text{ kg})(3.00 \text{ m/s}) + (0.900 \text{ kg})(0) = (0.450 \text{ kg})v_1' + (0.900 \text{ kg})v_2'$.
Because the collision is elastic, the relative speed does not change:
$v_1 - v_2 = -(v_1' - v_2')$, or $3.00 \text{ m/s} - 0 = v_2' - v_1'$.
Combining these two equations, we get
$v_1' = -1.00 \text{ m/s}$ (rebound), and $v_2' = 2.00 \text{ m/s}$.

23. For the elastic collision of the two billiard balls, we use momentum conservation:
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;
$m(2.00 \text{ m/s}) + m(-3.00 \text{ m/s}) = mv_1' + mv_2'$.
Because the collision is elastic, the relative speed does not change:
$v_1 - v_2 = -(v_1' - v_2')$, or $2.00 \text{ m/s} - (-3.00 \text{ m/s}) = v_2' - v_1'$.
Combining these two equations, we get
$v_1' = -3.00 \text{ m/s}$ (rebound), and $v_2' = 2.00 \text{ m/s}$.

Note that the two billiard balls exchange velocities.

24. For the elastic collision of the two balls, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$(0.060 \text{ kg})(2.50 \text{ m/s}) + (0.090 \text{ kg})(1.00 \text{ m/s}) = (0.060 \text{ kg})v_1' + (0.090 \text{ kg})v_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = - (v_1' - v_2')$, or $2.50 \text{ m/s} - 1.00 \text{ m/s} = v_2' - v_1'$.

Combining these two equations, we get

$v_1' = 0.70 \text{ m/s}$, and $v_2' = 2.20 \text{ m/s}$.

25. (*a*) For the elastic collision of the two balls, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$(0.220 \text{ kg})(5.5 \text{ m/s}) + m_2(0) = (0.220 \text{ kg})(- 3.7 \text{ m/s}) + m_2v_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = - (v_1' - v_2')$, or $5.5 \text{ m/s} - 0 = v_2' - (- 3.7 \text{ m/s})$, which gives $v_2' = 1.8 \text{ m/s}$.

(*b*) Using the result for v_2' in the momentum equation, we get $m_2 = 1.1 \text{ kg}$.

26. (*a*) For the elastic collision of the two bumper cars, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$(450 \text{ kg})(4.50 \text{ m/s}) + (550 \text{ kg})(3.70 \text{ m/s}) = (450 \text{ kg})v_1' + (550 \text{ kg})v_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = - (v_1' - v_2')$, or $4.5 \text{ m/s} - 3.7 \text{ m/s} = v_2' - v_1'$.

Combining these two equations, we get

$v_1' = 3.62 \text{ m/s}$, and $v_2' = 4.42 \text{ m/s}$.

(*b*) r the change in momentum of each we have

Æ$p_1 = m_1(v_1' - v_1) = (450 \text{ kg})(3.62 \text{ m/s} - 4.50 \text{ m/s}) =$ $- 396 \text{ kg}\cdot\text{m/s}$;

Æ$p_2 = m_2(v_2' - v_2) = (550 \text{ kg})(4.42 \text{ m/s} - 3.70 \text{ m/s}) =$ $+ 396 \text{ kg}\cdot\text{m/s}$.

As expected, the changes are equal and opposite.

27. (*a*) For the elastic collision of the two balls, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$(0.280 \text{ kg})v_1 + m_2(0) = (0.280 \text{ kg})v_1' + m_2(!v_1)$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = - (v_1' - v_2')$, or $v_1 - 0 = !v_1 - v_1'$, which gives $v_1' = - !v_1$.

Using this result in the momentum equation, we get $m_2 = 0.840 \text{ kg}$.

(*b*) The fraction transferred is

fraction = ÆKE$_2$/KE$_1$ = $!m_2(v_2'^2 - v_2^2)/!m_1v_1^2$

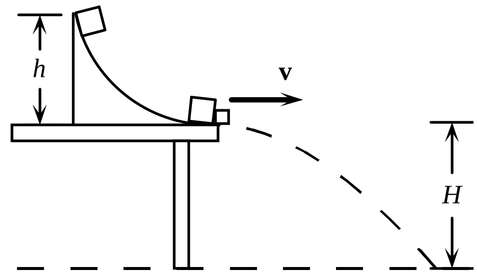
= $!m_2[(!v_1)^2 - 0]/!m_1v_1^2 = ($$m_2/m_1 = (($0.840 kg)/(0.280 kg) = 0.750.

28. We find the speed after falling a height h from energy conservation:

$!Mv^2 = Mgh$, or $v = (2gh)^{1/2}$.

The speed of the first cube after sliding down the incline and just before the collision is

$v_1 = [2(9.80 \text{ m/s}^2)(0.20 \text{ m})]^{1/2} = 1.98 \text{ m/s}$.



For the elastic collision of the two cubes, we use momentum conservation:

$Mv_1 + mv_2 = Mv_1' + mv_2'$;

$M(1.98 \text{ m/s}) + !M(0) = Mv_1' + !Mv_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_1' - v_2')$, or $1.98 \text{ m/s} - 0 = v_2' - v_1'$.

Combining these two equations, we get

$v_1' = 0.660 \text{ m/s}$, and $v_2' = 2.64 \text{ m/s}$.

Because both cubes leave the table with a horizontal velocity, they will fall to the floor in the same time, which we find from

$H = !gt^2$;

$0.90 \text{ m} = !(9.80 \text{ m/s}^2)t^2$, which gives $t = 0.429 \text{ s}$.

Because the horizontal motion has constant velocity, we have

$x_1 = v_1't = (0.660 \text{ m/s})(0.429 \text{ s}) =$ 0.28 m;

$x_2 = v_2't = (2.64 \text{ m/s})(0.429 \text{ s}) =$ 1.1 m.

29. (*a*) For the elastic collision of the two masses, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$m_1v_1 + 0 = m_1v_1' + m_2v_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_1' - v_2')$, or $v_1 - 0 = v_2' - v_1'$.

If we multiply this equation by m_2 and subtract it from the momentum equation, we get

$(m_1 - m_2)v_1 = (m_1 + m_2)v_1'$, or $v_1' = [(m_1 - m_2)/(m_1 + m_2)]v_1$.

If we multiply the relative speed equation by m_1 and add it to the momentum equation, we get

$2m_1v_1 = (m_1 + m_2)v_2'$, or $v_2' = [2m_1/(m_1 + m_2)]v_1$.

(*b*) When $m_1 \ll m_2$ we have

$v_1' = [(m_1 - m_2)/(m_1 + m_2)]v_1 \approx [(-m_2)/(m_2)]v_1 = -v_1$;

$v_2' = [2m_1/(m_1 + m_2)]v_1 \approx [(2m_1/m_2)v_1 \approx 0$; so

$v_1' \approx -v_1, v_2' \approx 0$; the small mass rebounds with the same speed; the large mass does not move. An example is throwing a ping pong ball against a concrete block.

(*c*) When $m_1 \gg m_2$ we have

$v_1' = [(m_1 - m_2)/(m_1 + m_2)]v_1 \approx [(m_1)/(m_1)]v_1 = v_1$;

$v_2' = [2m_1/(m_1 + m_2)]v_1 \approx (2m_1/m_1)v_1 = 2v_1$; so

$v_1' \approx v_1, v_2' \approx 2v_1$; the large mass continues with the same speed; the small mass acquires a large velocity. An example is hitting a light stick with a bowling ball.

(*d*) When $m_1 = m_2$ we have

$v_1' = [(m_1 - m_2)/(m_1 + m_2)]v_1 = 0$;

$v_2' = [2m_1/(m_1 + m_2)]v_1 = (2m_1/2m_1)v_1 = v_1$; so

$v_1' \approx 0, v_2' \approx v_1$; the striking mass stops; the hit mass acquires the striking mass's velocity. An example is one billiard ball hitting an identical one.

30. We let V be the speed of the block and bullet immediately after the collision and before the pendulum swings. For this perfectly inelastic collision, we use momentum conservation:

$mv + 0 = (M + m)V;$

$(0.018 \text{ kg})(230 \text{ m/s}) = (0.018 \text{ kg} + 3.6 \text{ kg})V,$

which gives $V = 1.14$ m/s.

Because the tension does no work, we can use energy conservation for the swing:

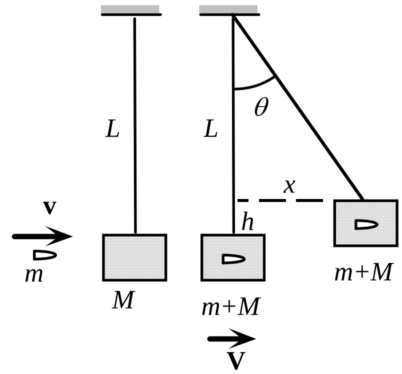
$!(M + m)V^2 = (M + m)gh$, or $V = (2gh)^{1/2}$;

$1.14 \text{ m/s} = [2(9.80 \text{ m/s}^2)h]^{1/2}$, which gives $h = 0.0666$ m.

We find the horizontal displacement from the triangle:

$L^2 = (L - h)^2 + x^2;$

$(2.8 \text{ m})^2 = (2.8 \text{ m} - 0.0666 \text{ m})^2 + x^2$, which gives $x =$ 0.61 m.



31. (*a*) The velocity of the block and projectile after the collision is

$v' = mv_1/(m + M).$

The fraction of kinetic energy lost is

fraction lost $= -$ ÆKE/KE $= -[!(m + M)v'^2 - !mv_1^2]/!mv_1^2$

$= -\{(m + M)[mv_1/(m + M)]^2 - mv_1^2\}/mv_1^2$

$= -[m/(m + M)] + 1 =$ $+ M/(m + M).$

(*b*) For the data given we have

fraction lost $= M/(m + M) = (14.0 \text{ g})/(14.0 \text{ g} + 380 \text{ g}) =$ 0.964.

32. Momentum conservation gives

$0 = m_1v_1' + m_2v_2';$

$0 = m_1v_1' + 1.5m_1v_2'$, or $v_1' = -1.5v_2'$.

The kinetic energy of each piece is

$\text{KE}_2 = !m_2v_2'^2;$

$\text{KE}_1 = !m_1v_1'^2 = !(m_2/1.5)(-1.5v_2')^2 = (1.5)!m_2v_2'^2 = 1.5\text{KE}_2$.

The energy supplied by the explosion produces the kinetic energy:

$\text{E} = \text{KE}_1 + \text{KE}_2 = 2.5\text{KE}_2;$

$7500 \text{ J} = 2.5\text{KE}_2$, which gives $\text{KE}_2 = 3000$ J.

For the other piece we have

$\text{KE}_1 = \text{E} - \text{KE}_2 = 7500 \text{ J} - 3000 \text{ J} = 4500$ J.

Thus

KE(heavier) = 3000 J; KE(lighter) = 4500 J.

33. On the horizontal surface after the collision, the normal force on the joined cars is $F_N = (m + M)g$. We find the common speed of the joined cars immediately after the collision by using the work-energy principle for the sliding motion:

$W_{fr} =$ ÆKE;

$-\mu_k(m + M)gd = 0 - !(M + m)V^2;$

$0.40(9.80 \text{ m/s}^2)(2.8 \text{ m}) = !V^2$, which gives $V = 4.68$ m/s.

For the collision, we use momentum conservation:

$mv + 0 = (m + M)V;$

$(1.0 \times 10^3 \text{ kg})v = (1.0 \times 10^3 \text{ kg} + 2.2 \times 10^3 \text{ kg})(4.68 \text{ m/s})$, which gives $v = 15$ m/s (54 km/h).

34. (*a*) For a perfectly elastic collision, we use momentum conservation:

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2', \quad \text{or} \quad m_1(v_1 - v_1') = m_2(v_2' - v_2).$$

Kinetic energy is conserved, so we have

$$\tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}m_1v_1'^2 + \tfrac{1}{2}m_2v_2'^2, \quad \text{or} \quad m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2),$$

which can be written as

$$m_1(v_1 - v_1')(v_1 + v_1') = m_2(v_2' - v_2)(v_2' + v_2).$$

When we divide this by the momentum result, we get

$$v_1 + v_1' = v_2' + v_2, \quad \text{or} \quad v_1' - v_2' = v_2 - v_1.$$

If we use this in the definition of the coefficient of restitution, we get

$$e = (v_1' - v_2')/(v_2 - v_1) = (v_2 - v_1)/(v_2 - v_1) = 1.$$

For a completely inelastic collision, the two objects move together, so we have

$v_1' = v_2'$, which gives $e = 0$.

(*b*) We find the speed after falling a height h from energy conservation:

$$\tfrac{1}{2}mv_1^2 = mgh, \quad \text{or} \quad v_1 = (2gh)^{1/2}.$$

The same expression holds for the height reached by an object moving upward:

$$v_1' = (2gh')^{1/2}.$$

Because the steel plate does not move, when we take into account the directions we have

$$e = (v_1' - v_2')/(v_2 - v_1) = [(2gh')^{1/2} - 0]/\{0 - [-(2gh)^{1/2}]\}, \text{ so } \quad e = (h'/h)^{1/2}.$$

35. Momentum conservation for the explosion gives us

$$0 = m_1v_1' + m_2v_2';$$

$$0 = m_1v_1' + 3m_1v_2', \quad \text{or} \quad v_1' = -3v_2'.$$

On the horizontal surface after the collision, the normal force on a block is $F_N = mg$.

We relate the speed of a block immediately after the collision to the distance it slides from the work-energy principle for the sliding motion:

$$W_{fr} = \Delta KE;$$

$$-\mu_k mgd = 0 - \tfrac{1}{2}mv^2, \quad \text{or} \quad d = \tfrac{1}{2}v^2/\mu_k g.$$

If we use this for each block and form the ratio, we get

$$d_1/d_2 = (v_1/v_2)^2 = (-3)^2 = 9,$$ with the lighter block traveling farther.

36. For the momentum conservation of this one-dimensional collision, we have

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$.

(*a*) If the bodies stick together, $v_1' = v_2' = V$:

(5.0 kg)(5.5 m/s) + (3.0 kg)(– 4.0 m/s) = (5.0 kg + 3.0 kg)V, which gives $V =$ $v_1' = v_2' = 1.9$ m/s.

(*b*) If the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_1' - v_2')$, or 5.5 m/s – (– 4.0 m/s) = 9.5 m/s = $v_2' - v_1'$.

The momentum equation is

(5.0 kg)(5.5 m/s) + (3.0 kg)(– 4.0 m/s) = (5.0 kg)v_1' + (3.0 kg)v_2', or

(5.0 kg)v_1' + (3.0 kg)v_2' = 15.5 kg ·m/s.

When we combine these two equations, we get $v_1' = -1.6$ m/s, $v_2' = 7.9$ m/s.

(*c*) If m_1 comes to rest, $v_1' = 0$.

(5.0 kg)(5.5 m/s) + (3.0 kg)(– 4.0 m/s) = 0 + (3.0 kg)v_2', which gives $v_1' = 0$, $v_2' = 5.2$ m/s.

(*d*) If m_2 comes to rest, $v_2' = 0$.

(5.0 kg)(5.5 m/s) + (3.0 kg)(– 4.0 m/s) = (5.0 kg)v_1' + 0, which gives $v_1' = 3.1$ m/s, $v_2' = 0$.

(*e*) The momentum equation is

(5.0 kg)(5.5 m/s) + (3.0 kg)(– 4.0 m/s) = (5.0 kg)(– 4.0 m/s) + (3.0 kg)v_2',

which gives $v_1' = -4.0$ m/s, $v_2' = 12$ m/s.

The result for (*c*) is reasonable. The 3.0-kg body rebounds.

The result for (*d*) is not reasonable. The 5.0-kg body would have to pass through the 3.0-kg body.

To check the result for (*e*) we find the change in kinetic energy:

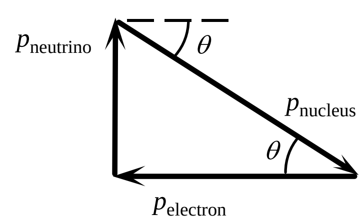
ÆKE = (!$m_1v_1'^2$ + !$m_2v_2'^2$) – (!$m_1v_1^2$ + !$m_2v_2^2$)

= ![(5.0 kg)(5.5 m/s)² + (3.0 kg)(– 4.0 m/s)²] – ![(5.0 kg)(– 4.0 m/s)² + (3.0 kg)(12 m/s)²]

= + 156 J.

Because the kinetic energy cannot increase in a simple collision, the result for (*e*) is not reasonable.

37. Because the initial momentum is zero, the momenta of the three products of the decay must add to zero. If we draw the vector diagram, we see that



$p_{nucleus} = (p_{electron}^2 + p_{neutrino}^2)^{1/2}$

$= [(9.30 \times 10^{-23}\ \text{kg·m/s})^2 + (5.40 \times 10^{-23}\ \text{kg·m/s})^2]^{1/2}$

$= 1.08 \times 10^{-22}$ kg ·m/s.

We find the angle from

$\tan \theta = p_{neutrino}/p_{electron}$

$= (5.40 \times 10^{-23}\ \text{kg·m/s})/(9.30 \times 10^{-23}\ \text{kg·m/s})$

= 0.581, so the angle is 30.1° from the direction opposite to the electron's.

38. For the collision we use momentum conservation:

x-direction: $m_1 v_1 + 0 = (m_1 + m_2)v' \cos \theta$;

$(4.3 \text{ kg})(7.8 \text{ m/s}) = (4.3 \text{ kg} + 5.6 \text{ kg})v' \cos \theta$, which gives

$v' \cos \theta = 3.39 \text{ m/s}$.

y-direction: $0 + m_2 v_2 = (m_1 + m_2)v' \sin \theta$;

$(5.6 \text{ kg})(10.2 \text{ m/s}) = (4.3 \text{ kg} + 5.6 \text{ kg})v' \sin \theta$, which gives

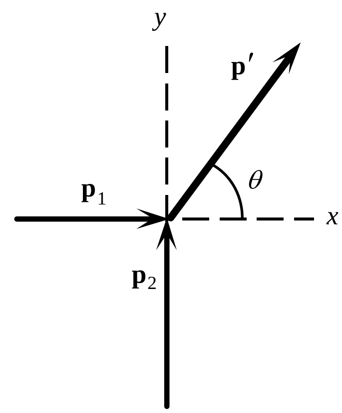
$v' \sin \theta = 5.77 \text{ m/s}$.

We find the direction by dividing the equations:

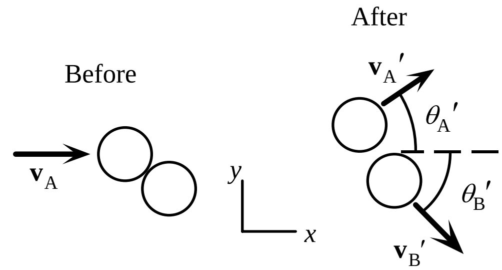
$\tan \theta = (5.77 \text{ m/s})/(3.39 \text{ m/s}) = 1.70$, so $\theta =$ 60°.

We find the magnitude by squaring and adding the equations:

$v' = [(5.77 \text{ m/s})^2 + (3.39 \text{ m/s})^2]^{1/2} =$ 6.7 m/s.



39. (*a*) Using the coordinate system shown, for momentum conservation we have



x-momentum: $m_A v_A + 0 = m_A v_A' \cos \theta_A' + m_B v_B' \cos \theta_B'$;

y-momentum: $0 + 0 = m_A v_A' \sin \theta_A' - m_B v_B' \sin \theta_B'$.

(*b*) With the given data, we have

x: $(0.400 \text{ kg})(1.80 \text{ m/s}) = (0.400 \text{ kg})(1.10 \text{ m/s}) \cos 30° + (0.500 \text{ kg})v_B' \cos \theta_B'$,

which gives $v_B' \cos \theta_B' = 0.678 \text{ m/s}$;

y: $0 = (0.400 \text{ kg})(1.10 \text{ m/s}) \sin 30° - (0.500 \text{ kg})v_B' \sin \theta_B'$,

which gives $v_B' \sin \theta_B' = 0.440 \text{ m/s}$.

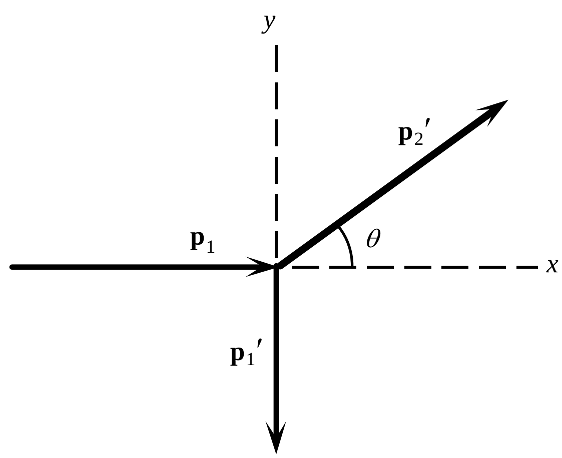
We find the magnitude by squaring and adding the equations:

$v_B' = [(0.440 \text{ m/s})^2 + (0.678 \text{ m/s})^2]^{1/2} =$ 0.808 m/s.

We find the direction by dividing the equations:

$\tan \theta_B' = (0.440 \text{ m/s})/(0.678 \text{ m/s}) = 0.649$, so $\theta_B =$ 33.0°.

40. (*a*) Using the coordinate system shown, for momentum conservation we have



x-momentum: $mv + 0 = 0 + 2mv_2' \cos \theta$, or $2v_2' \cos \theta = v$;

y-momentum: $0 + 0 = -mv_1' + 2mv_2' \sin \theta$, or $2v_2' \sin \theta = v_1'$.

If we square and add these two equations, we get

$v^2 + v_1'^2 = 4v_2'^2$.

For the conservation of kinetic energy, we have

$!mv^2 + 0 = !mv_1'^2 + !(2m)v_2'^2$, or

$v^2 - v_1'^2 = 2v_2'^2$.

When we add this to the previous result, we get

$v^2 = 3v_2'^2$.

Using this in the x-momentum equation, we get

$\cos \theta =$ Ã3/2, or $\theta =$ 30°.

(*b*) From part (*a*) we have

$v_2' = v/$Ã3.

Using the energy result, we get

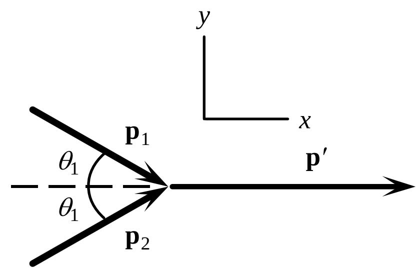
$v_1'^2 = v^2 - 2v_2'^2 = v^2 - 2v^2/3 =$ @v^2, or $v_1' = v/$Ã3.

(*c*) The fraction of the kinetic energy transferred is

fraction $= \text{KE}_2/\text{KE}_1 = !(2m)v_2'^2/!mv^2$

$= m(v^2/3)/!mv^2 =$ %.

41. Using the coordinate system shown, for momentum conservation we have



y-momentum: $-mv \sin\theta_1 + mv \sin\theta_2 = 0$, or $\theta_1 = \theta_2$.

x-momentum: $mv \cos\theta_1 + mv \cos\theta_2 = 2mv_2'$;

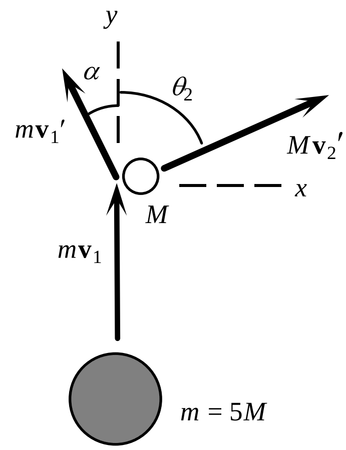
$2mv \cos\theta_1 = 2mv/3$;

$\cos\theta_1 =$ @, or $\theta_1 = 70.5° = \theta_2$.

The angle between their initial directions is

$\phi = \theta_1 + \theta_2 = 2(70.5°) =$ 141°.

42. Using the coordinate system shown, for momentum conservation we have



y-momentum: $-mv_1 + 0 = mv_1' \cos\alpha + Mv_2' \cos\theta_2$;
$5M(12.0 \text{ m/s}) = 5Mv_1' \cos\alpha + Mv_2' \cos 80°$, or
$5v_1' \cos\alpha = v_2' \cos 80° - 60.0 \text{ m/s}$.

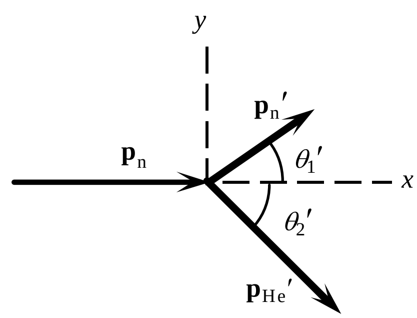
x-momentum: $0 = -mv_1' \sin\alpha + Mv_2' \sin\theta_2$;
$0 = -5Mv_1' \sin\alpha + Mv_2' \sin 80°$, or
$5v_1' \sin\alpha = v_2' \cos 80°$.

For the conservation of kinetic energy, we have
$!mv_1^2 + 0 = !mv_1'^2 + !Mv_2'^2$;
$5M(12.0 \text{ m/s})^2 = 5Mv_1'^2 + Mv_2'^2$, or
$5v_1'^2 + v_2'^2 = 720 \text{ m}^2/\text{s}^2$.

We have three equations in three unknowns: α, v_1', v_2'. We eliminate α by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

(*a*) v_2' = 3.47 m/s.
(*b*) v_1' = 11.9 m/s.
(*c*) α = 3.29°.

43. Using the coordinate system shown, for momentum conservation we have



x: $m_n v_n + 0 = m_n v_n' \cos\theta_1' + m_{He} v_{He}' \cos\theta_2'$;
$m_n(6.2 \times 10^5 \text{ m/s}) = m_n v_n' \cos\theta_1' + 4m_n v_{He}' \cos 45°$, or
$v_n' \cos\theta_1' = (6.2 \times 10^5 \text{ m/s}) - 4v_{He}' \cos 45°$.

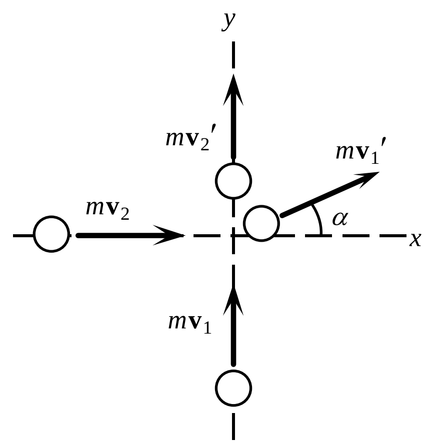
y: $0 + 0 = -m_n v_n' \sin\theta_1' + m_{He} v_{He}' \sin\theta_2'$;
$0 = -m_n v_n' \sin\theta_1' + 4m_n v_{He}' \sin 45°$, or
$v_n' \sin\theta_1' = 4v_{He}' \sin 45°$.

For the conservation of kinetic energy, we have
$!m_n v_n^2 + 0 = !m_n v_n'^2 + !m_{He} v_{He}'^2$;
$m_n(6.2 \times 10^5 \text{ m/s})^2 = m_n v_n'^2 + 4m_n v_{He}'^2$, or
$v_n'^2 + 4v_{He}'^2 = 3.84 \times 10^{11} \text{ m}^2/\text{s}^2$.

We have three equations in three unknowns: θ_1', v_n', v_{He}'. We eliminate θ_1' by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:

$\theta_1' = 76°$, $v_n' = 5.1 \times 10^5$ m/s, $v_{He}' = 1.8 \times 10^5$ m/s.

44. Using the coordinate system shown, for momentum conservation we have



x: $0 + mv_2 = mv_1' \cos \alpha + 0$;
$3.7\ \text{m/s} = v_1' \cos \alpha$;
y: $mv_1 + 0 = mv_1' \sin \alpha + mv_2'$;
$2.0\ \text{m/s} = v_1' \sin \alpha + v_2'$, or
$v_1' \sin \alpha = 2.0\ \text{m/s} - v_2'$.

For the conservation of kinetic energy, we have
$!mv_1^2 + !mv_2^2 = !mv_1'^2 + !mv_2'^2$;
$(2.0\ \text{m/s})^2 + (3.7\ \text{m/s})^2 = v_1'^2 + v_2'^2$.

We have three equations in three unknowns: α, v_1', v_2'.
We eliminate α by squaring and adding the two momentum results, and then combine this with the energy equation, with the results:
$\alpha = 0°$, $v_1' = 3.7\ \text{m/s}$, $v_2' = 2.0\ \text{m/s}$.
The two billiard balls exchange velocities.

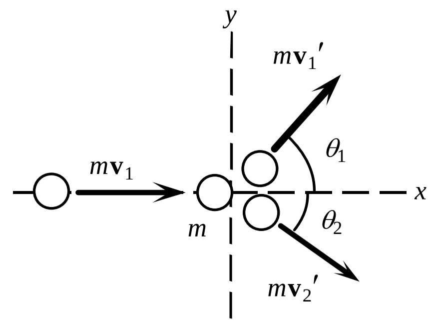
45. Using the coordinate system shown, for momentum conservation we have

x: $mv_1 + 0 = mv_1' \cos\theta_1 + mv_2' \cos\theta_2$, or

$v_1 = v_1' \cos\theta_1 + v_2' \cos\theta_2$;

y: $0 + 0 = mv_1' \sin\theta_1 - mv_2' \sin\theta_2$, or

$0 = v_1' \sin\theta_1 - v_2' \sin\theta_2$.



For the conservation of kinetic energy, we have

$!mv_1^2 + 0 = !mv_1'^2 + !mv_2'^2$;

$v_1^2 = v_1'^2 + v_2'^2$.

We square each of the momentum equations:

$v_1^2 = v_1'^2 \cos^2\theta_1 + 2v_1'v_2' \cos\theta_1 \cos\theta_2 + v_2'^2 \cos^2\theta_2$;

$0 = v_1'^2 \sin^2\theta_1 - 2v_1'v_2' \sin\theta_1 \sin\theta_2 + v_2'^2 \sin^2\theta_2$.

If we add these two equations and use $\sin^2\theta + \cos^2\theta = 1$, we get

$v_1^2 = v_1'^2 + 2v_1'v_2'(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + v_2'^2$.

If we subtract the energy equation, we get

$0 = 2v_1'v_2'(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2)$, or $\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 = 0$.

We reduce this with a trigonometric identity:

$\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 = \cos(\theta_1 + \theta_2) = 0$,

which means that $\theta_1 + \theta_2 = 90°$.

46. We choose the origin at the carbon atom. The center of mass will lie along the line joining the atoms:

$x_{CM} = (m_C x_C + m_O x_O)/(m_C + m_O)$

$= [0 + (16\ u)(1.13 \times 10^{-10}\ m)]/(12\ u + 16\ u) =$ 6.5×10^{-11} m from the carbon atom.

47. We choose the origin at the front of the car:

$x_{CM} = (m_{car}x_{car} + m_{front}x_{front} + m_{back}x_{back})/(m_{car} + m_{front} + m_{back})$

$= [(1050\ kg)(2.50\ m) + (140\ kg)(2.80\ m) + (210\ kg)(3.90\ m)]/(1050\ kg + 140\ kg + 210\ kg)$

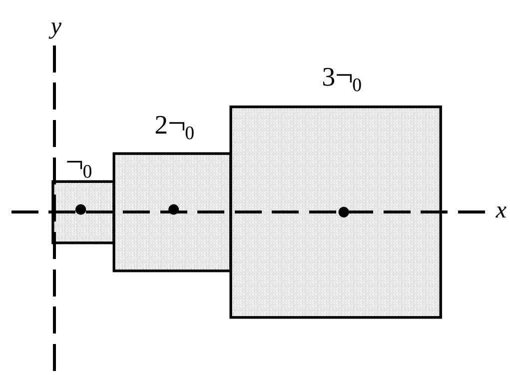
$=$ 2.74 m from the front of the car.

48. Because the cubes are made of the same material, their masses will be proportional to the volumes:

$m_1, m_2 = 2^3 m_1 = 8m_1, m_3 = 3^3 m_1 = 27m_1$.

From symmetry we see that $y_{CM} = 0$.

We choose the x-origin at the outside edge of the small cube:



$x_{CM} = (m_1x_1 + m_2x_2 + m_3x_3)/(m_1 + m_2 + m_3)$

$= \{m_1(!¬_0) + 8m_1[¬_0 + !(2¬_0)] +$

$27m_1[¬_0 + 2¬_0 + !(3¬_0)]\}/(m_1 + 8m_1 + 27m_1)$

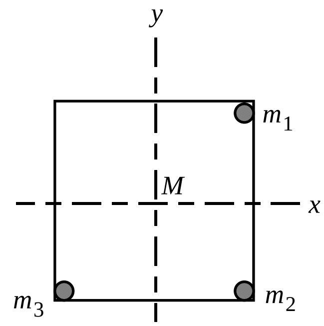
$= 138¬_0/36$

$=$ $3.83¬_0$ from the outer edge of the small cube.

49. We choose the origin at the center of the raft, which is the CM of the raft:

$x_{CM} = (Mx_{raft} + m_1x_1 + m_2x_2 + m_3x_3)/(M + m_1 + m_2 + m_3)$
$= [0 + (1200\text{ kg})(9.0\text{ m}) + (1200\text{ kg})(9.0\text{ m}) + (1200\text{ kg})(-9.0\text{ m})]/[6200\text{ kg} + 3(1200\text{ kg})]$
$=$ 1.10 m (East).

$y_{CM} = (My_{raft} + m_1y_1 + m_2y_2 + m_3y_3)/(M + m_1 + m_2 + m_3)$
$= [0 + (1200\text{ kg})(9.0\text{ m}) + (1200\text{ kg})(-9.0\text{ m}) + (1200\text{ kg})(-9.0\text{ m})]/[6200\text{ kg} + 3(1200\text{ kg})]$
$=$ – 1.10 m (South).

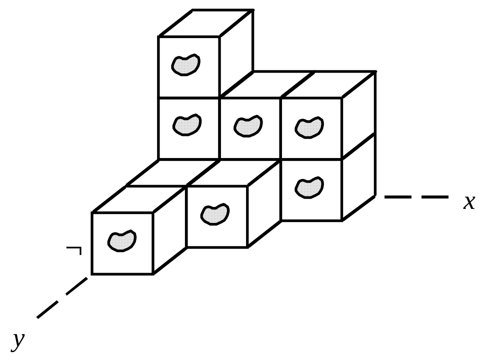


50. We choose the coordinate system shown. There are 10 cases.

$x_{CM} = (5mx_1 + 3mx_2 + 2mx_3)/(10m)$
= [5(!¬) + 3(¬ + !¬) + 2(2¬ + !¬)]/(10)
= 1.2¬.

$y_{CM} = (7my_1 + 2my_2 + my_3)/(10m)$
= [7(!¬) + 2(¬ + !¬) + (2¬ + !¬)]/(10)
= 0.9¬.

The CM is 1.2¬ from the left, and 0.9¬ from the back of the pallet.



51. We know from the symmetry that the center of mass lies on a line containing the center of the plate and the center of the hole. We choose the center of the plate as origin and x along the line joining the centers. Then $y_{CM} = 0$. A uniform circle has its center of mass at its center. We can treat the system as two circles:

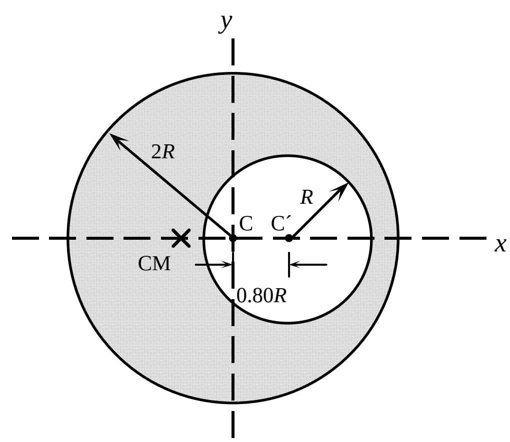
a circle of radius $2R$, density ρ and mass $\rho^1(2R)^2$ with $x_1 = 0$;

a circle of radius R, density $-\rho$ and mass $-\rho^1R^2$ with $x_2 = 0.80R$.

We find the center of mass from

$x_{CM} = (m_1x_1 + m_2x_2)/(m_1 + m_2)$
$= [4\rho^1R^2(0) - \rho^1R^2(0.80R)]/(4\rho^1R^2 - \rho^1R^2)$
$= -0.27R$.

The center of mass is along the line joining the centers $0.07R$ outside the hole.



52. If we assume a total mass of 70 kg, for one leg we have

$m_{leg} = m_{body}$!(21.5 + 9.6 + 3.4)/100 = (70 kg)!(34.5)/100 = 12 kg.

53. If we measure from the shoulder, the percentage of the height to the center of mass for each of the segments is

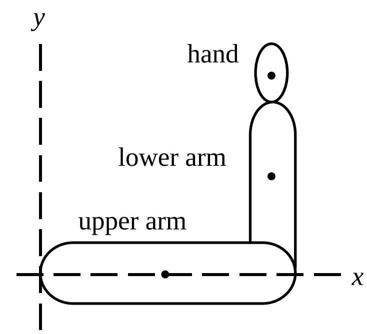
upper arm: 81.2 – 71.7 = 9.5;
lower arm: 81.2 – 55.3 = 25.9;
hand: 81.2 – 43.1 = 38.1.

Because all masses are percentages of the body mass, we can use the percentages rather than the actual mass. Thus we have

$$x_{CM} = (m_{upper}x_{upper} + m_{lower}x_{lower} + m_{hand}x_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [!(6.6)(9.5) + !(4.2)(25.9) + !(1.7)(38.1)]/[!(6.6) + !(4.2) + !(1.7)]$$
$$= 19.$$

The CM of an outstretched arm is 19% of the height.

54. We choose the shoulder as the origin. The locations of the centers of mass for each of the segments are

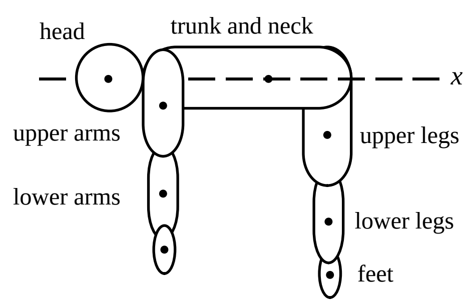


upper arm: $x_{upper} = [(81.2 – 71.7)/100] (155 \text{ cm}) = 14.7$ cm;
$y_{upper} = 0$;
lower arm: $x_{lower} = [(81.2 – 62.2)/100] (155 \text{ cm}) = 29.5$ cm;
$y_{lower} = [(62.2 – 55.3)/100] (155 \text{ cm}) = 10.7$ cm;
hand: $x_{hand} = [(81.2 – 62.2)/100] (155 \text{ cm}) = 29.5$ cm;
$y_{hand} = [(62.2 – 43.1)/100] (155 \text{ cm}) = 29.6$ cm.

Because all masses are percentages of the body mass, we can use the percentages rather than the actual mass. Thus we have

$$x_{CM} = (m_{upper}x_{upper} + m_{lower}x_{lower} + m_{hand}x_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [!(6.6)(14.7 \text{ cm}) + !(4.2)(29.5 \text{ cm}) + !(1.7)(29.5 \text{ cm})]/[!(6.6) + !(4.2) + !(1.7)]$$
$$= 22 \text{ cm}.$$
$$y_{CM} = (m_{upper}y_{upper} + m_{lower}y_{lower} + m_{hand}y_{hand})/(m_{upper} + m_{lower} + m_{hand})$$
$$= [!(6.6)(0) + !(4.2)(10.7 \text{ cm}) + !(1.7)(29.6 \text{ cm})]/[!(6.6) + !(4.2) + !(1.7)]$$
$$= 7.6 \text{ cm}.$$

55. We use the line of the torso as the origin. The vertical locations of the centers of mass for each of the segments, as a percentage of the height, are



torso and head: 0;
upper arms: $y_{ua} = –(81.2 – 71.7) = –9.5$;
lower arms: $y_{la} = –(81.2 – 55.3) = –25.9$;
hands: $y_h = –(81.2 – 43.1) = –38.1$;
upper legs: $y_{ul} = –(52.1 – 42.5) = –9.6$;
lower legs: $y_{ll} = –(52.1 – 18.1) = –33.9$;
feet: $y_f = –(52.1 – 1.8) = –50.3$.

Because all masses are percentages of the body mass, we can use the percentages rather than the actual mass. Thus we have

$$y_{CM} = (m_{ua}y_{ua} + m_{la}y_{la} + m_h y_h + m_{ul}y_{ul} + m_{ll}y_{ll} + m_f y_f)/m_{body}$$
$$= [(6.6)(-9.5) + (4.2)(-25.9) + (1.7)(-38.1) + (21.5)(-9.6) + (9.6)(-33.9) + (3.4)(-50.3)]/100$$
$$= -9.4.$$

The CM will be 9.4% of the body height below the line of the torso. For a height of 1.8 m, this is about 17 cm, so yes, this will most likely be outside the body.

56. (*a*) If we choose the origin at the center of the Earth, we have

$x_{CM} = (m_{Earth}x_{Earth} + m_{Moon}x_{Moon})/(m_{Earth} + m_{Moon})$
$= [0 + (7.35 \times 10^{22}\ \text{kg})(3.84 \times 10^{8}\ \text{m})]/(5.98 \times 10^{24}\ \text{kg} + 7.35 \times 10^{22}\ \text{kg})$
$= 4.66 \times 10^{6}\ \text{m}.$

Note that this is less than the radius of the Earth and thus is inside the Earth.

(*b*) The CM found in part (*a*) will move around the Sun on an elliptical path. The Earth and Moon will revolve about the CM. Because this is near the center of the Earth, the Earth will essentially be on the elliptical path around the Sun. The motion of the Moon about the Sun is more complicated.

57. We choose the origin of our coordinate system at the woman.

(*a*) For their CM we have

$x_{CM} = (m_{woman}x_{woman} + m_{man}x_{man})/(m_{woman} + m_{man})$
$= [0 + (90\ \text{kg})(10.0\ \text{m})]/(55\ \text{kg} + 90\ \text{kg})$
$= 6.2\ \text{m}.$

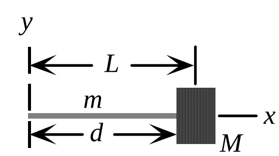
(*b*) Because the CM will not move, we find the location of the woman from

$x_{CM} = (m_{woman}x_{woman}' + m_{man}x_{man}')/(m_{woman} + m_{man})$
$6.2\ \text{m} = [(55\ \text{kg})x_{woman}' + (90\ \text{kg})(10.0\ \text{m} - 2.5\ \text{m})]/(55\ \text{kg} + 90\ \text{kg})$, which gives
$x_{woman}' = 4.1\ \text{m}.$

The separation of the two will be 7.5 m – 4.1 m = 3.4 m.

(*c*) The two will meet at the CM , so he will have moved 10.0 m – 6.2 m = 3.8 m.

58. Because the two segments of the mallet are uniform, we know that the center of mass of each segment is at its midpoint.
We choose the origin at the bottom of the handle. The mallet will spin about the CM, which is the point that will follow a parabolic trajectory:



$x_{CM} = (md + ML)/(m + M)$
$= [(0.500\ \text{kg})(12.0\ \text{cm}) + (2.00\ \text{kg})(24.0\ \text{cm} + 4.00\ \text{cm})]/(0.500\ \text{kg} + 2.00\ \text{kg})$
$= 24.8\ \text{cm}.$

59. The CM will land at the same point, 2*D* from the launch site. If part I is still stopped by the explosion, it will fall straight down, as before.

(*a*) We find the location of part II from the CM:

$x_{CM} = (m_I x_I + m_{II} x_{II})/(m_I + m_{II})$
$2D = [m_I D + 3m_I x_{II}]/(m_I + 3m_I)$, which gives
$x_{II} = 7D/3$, or $2D/3$ closer to the launch site.

(*b*) For the new mass distribution, we have

$x_{CM} = (m_I x_I + m_{II} x_{II})/(m_I + m_{II})$
$2D = [3m_{II} D + m_{II} x_{II}]/(3m_{II} + m_{II})$, which gives
$x_{II} = 5D$, or $2D$ farther from the launch site.

60. The forces on the balloon, gondola, and passenger are balanced, so the CM does not move relative to the Earth. As the passenger moves down at a speed v relative to the balloon, the balloon will move up. If the speed of the balloon is v' relative to the Earth, the passenger will move down at a speed $v - v'$ relative to the Earth. We choose the location of the CM as the origin and determine the positions after a time t:

    $x_{CM} = (m_{balloon}x_{balloon} + m_{passenger}x_{passenger})/(m_{balloon} + m_{passenger})$

    $0 = [Mv't + m(v - v')t]/(M + m)$, which gives

    $v' = mv/(M + m)$ up.

    If the passenger stops, the gondola and the balloon will also stop. There will be equal and opposite impulses acting when the passenger grabs the rope to stop.

61. We find the force on the person from the magnitude of the force required to change the momentum of the air:

    F = Æp/Æt = (Æm/Æt)v

    = (40 kg/s ·m^2)(1.50 m)(0.50 m)(100 km/h)/(3.6 ks/h) = 8.3×10^2 N.

    The maximum friction force will be

    $F_{fr} = \mu mg \approx (1.0)(70\text{ kg})(9.80\text{ m/s}^2) = 6.9 \times 10^2$ N, so the forces are about the same.

62. For the system of railroad car and snow, the horizontal momentum will be constant. For the horizontal motion, we take the direction of the car for the positive direction. The snow initially has no horizontal velocity. For this perfectly inelastic collision, we use momentum conservation:

    $M_1v_1 + M_2v_2 = (M_1 + M_2)V$;

    (5800 kg)(8.60 m/s) + 0 = [5800 kg + (3.50 kg/min)(90.0 min)]V, which gives V= 8.16 m/s.

    Note that there is a vertical impulse, so the vertical momentum is not constant.

63. We find the speed after being hit from the height h using energy conservation:

    !$mv'^2 = mgh$, or $v' = (2gh)^{1/2}$ = [2(9.80 m/s^2)(55.6 m)]1/2 = 33.0 m/s.

    We see from the diagram that the magnitude of the change in momentum is

    Æ$p = m(v^2 + v'^2)^{1/2}$

    = (0.145 kg)[(35.0 m/s)^2 + (33.0 m/s)^2]$^{1/2}$ = 6.98 kg · m/s.

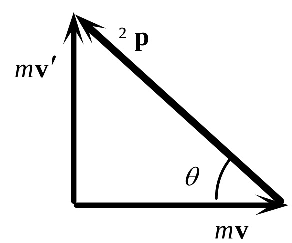
    We find the force from

    F Æt = Æp;

    $F(0.50 \times 10^{-3}$ s) = 6.98 kg · m/s, which gives F = 1.4×10^4 N.

    We find the direction of the force from

    tan $\theta = v'/v$ = (33.0 m/s)/(35.0 m/s) = 0.943, θ = 43.3°.



64. For momentum conservation we have

    x: mv_0 = %mv_x', which gives v_x' = 8v_0;

    y: 0 = @$m(2v_0)$ – %mv_y', which gives $v_y' = -v_0$.

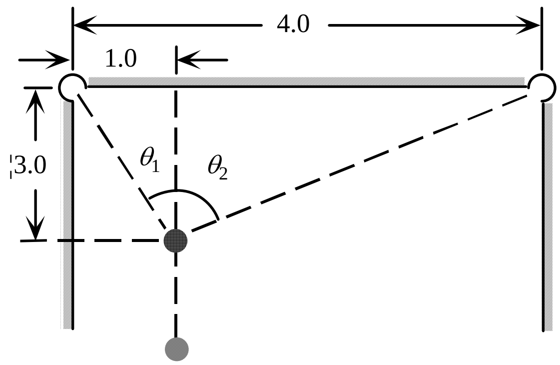
    The rocket's forward speed increases because the fuel is shot backward relative to the rocket.

65. From the result of Problem 45, the angle between the two final directions will be 90° for an elastic collision. We take the initial direction of the cue ball to be parallel to the side of the table. The angles for the two balls after the collision are

tan θ_1 = 1.0/Ã3.0, which gives θ_1 = 30°;

tan θ_2 = (4.0 – 1.0)/Ã3.0, which gives θ_2 = 60°.

Because their sum is 90°,
this will be a "scratch shot".



66. In the reference frame of the capsule before the push, we take the positive direction in the direction the capsule will move.

(*a*) Momentum conservation gives us

$mv_{\text{astronaut}} + Mv_{\text{satellite}} = mv_{\text{astronaut}}' + Mv_{\text{satellite}}'$,

0 + 0 = (140 kg)(– 2.50 m/s) + (1800 kg)$v_{\text{satellite}}'$, which gives $v_{\text{satellite}}'$ = 0.194 m/s.

(*b*) We find the force on the satellite from

$F_{\text{satellite}}$ = Æ$p_{\text{satellite}}$/Æt = $m_{\text{satellite}}$ Æ$v_{\text{satellite}}$/Æt

=(1800 kg)(0.194 m/s – 0)/(0.500 s) = 700 N.

There will be an equal but opposite force on the astronaut.

67. For each of the elastic collisions with a step, conservation of kinetic energy means that the velocity reverses direction but has the same magnitude. Thus the golf ball always rebounds to the height from which it started. Thus, after five bounces, the bounce height will be 4.00 m.

68. For the elastic collision of the two balls, we use momentum conservation:

$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$;

$mv_1 + 0 = m(-v_1/4) + m_2v_2'$, or $m_2v_2' = 5mv_1/4$.

Because the collision is elastic, the relative speed does not change:

$v_1 - 0 = -(v_1' - v_2')$; $v_1 = v_2' - (-v_1/4)$, or $v_2' = 3v_1/4$.

Combining these two equations, we get

$m_2 = 5m/3$.

69. On the horizontal surface, the normal force on a car is $F_N = mg$. We find the speed of a car immediately after the collision by using the work-energy principle for the succeeding sliding motion:

W_{fr} = ÆKE;

– $\mu_k mgd$ = 0 – !mv^2.

We use this to find the speeds of the cars after the collision:

0.60(9.80 m/s^2)(15 m) = !$v_A'^2$, which gives v_A' = 13.3 m/s;

0.60(9.80 m/s^2)(30 m) = !$v_B'^2$, which gives v_B' = 18.8 m/s.

For the collision, we use momentum conservation:

$m_Av_A + m_Bv_B = m_Av_A' + m_Bv_B'$;

(2000 kg)v_A + 0 = (2000 kg)(13.3 m/s) + (1000 kg)(18.8 m/s), which gives v_A = 22.7 m/s.

We find the speed of car A before the brakes were applied by using the work-energy principle for the preceding sliding motion:

W_{fr} = ÆKE;

$-\mu_k m_A g d = !m_A v_A^2 - !m_A v_{A0}^2$;

$-0.60(9.80\ \mathrm{m/s^2})(15\ \mathrm{m}) = ![(22.7\ \mathrm{m/s})^2 - v_{A0}^2]$, which gives $v_{A0} = 26.3\ \mathrm{m/s} = 94.7\ \mathrm{km/h} =$ 59 mi/h.

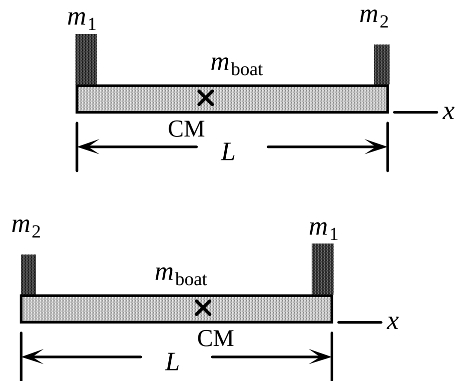
70. We choose the origin of our coordinate system at the initial position of the 75-kg person. For the location of the center of mass of the system we have

$x_{CM} = (m_1x_1 + m_2x_2 + m_{boat}x_{boat})/(m_1 + m_2 + m_{boat})$

$= [(75\text{ kg})(0) + (60\text{ kg})(2.0\text{ m}) + (80\text{ kg})(1.0\text{ m})]/(75\text{ kg} + 60\text{ kg} + 80\text{ kg}) = 0.93\text{ m}.$

Thus the CM will be 2.0 m – 0.93 m = 1.07 m from the 60-kg person. When the two people exchange seats, the CM will not move. The end where the 75-kg person started, which was 0.93 m from the CM, is now 1.07 m from the CM, that is, the boat must have moved 1.07 m – 0.93 m =

0.14 m toward the initial position of the 75-kg person.



71. (*a*) We take the direction of the meteor for the positive direction. For this perfectly inelastic collision, we use momentum conservation:

$M_{meteor}v_{meteor} + M_{Earth}v_{Earth} = (M_{meteor} + M_{Earth})V;$

$(10^8\text{ kg})(15 \times 10^3\text{ m/s}) + 0 = (10^8\text{ kg} + 6.0 \times 10^{24}\text{ kg})V$, which gives $V = 2.5 \times 10^{-13}\text{ m/s}$.

(*b*) The fraction transformed was

fraction $= \Delta KE_{Earth}/KE_{meteor} = \frac{1}{2}m_{Earth}V^2/\frac{1}{2}m_{meteor}v_{meteor}^2$

$= (6.0 \times 10^{24}\text{ kg})(2.5 \times 10^{-13}\text{ m/s})^2/(10^8\text{ kg})(15 \times 10^3\text{ m/s})^2 =$ 1.7×10^{-17}.

(*c*) The change in the Earth's kinetic energy was

$\Delta KE_{Earth} = \frac{1}{2}m_{Earth}V^2$

$= \frac{1}{2}(6.0 \times 10^{24}\text{ kg})(2.5 \times 10^{-13}\text{ m/s})^2 =$ 0.19 J.

72. Momentum conservation gives

$0 = m_1v_1' + m_2v_2'$, or $v_2'/v_1' = -m_1/m_2$.

The ratio of kinetic energies is

$KE_2/KE_1 = \frac{1}{2}m_2v_2'^2/\frac{1}{2}m_1v_1'^2 = (m_2/m_1)(v_2'/v_1')^2 = 2.$

When we use the result from momentum, we get

$(m_2/m_1)(-m_1/m_2)^2 = 2$, which gives $m_1/m_2 =$ 2.

73. (*b*) The force would become zero at

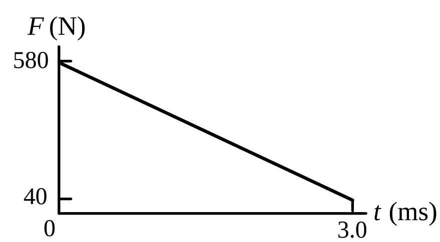
$t = 580/(1.8 \times 10^5) = 3.22 \times 10^{-3}$ s.

At $t = 3.0 \times 10^{-3}$ s the force is

$580 - (1.8 \times 10^5)(3.0 \times 10^{-3}\text{ s}) = +40$ N.

The impulse is the area under the F vs. t curve, and consists of a triangle and a rectangle:

Impulse $= \frac{1}{2}(580\text{ N} - 40\text{ N})(3.0 \times 10^{-3}\text{ s}) + (40\text{ N})(3.0 \times 10^{-3}\text{ s}) =$ 0.93 N·s.



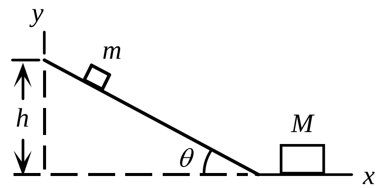
(*c*) We find the mass of the bullet from

Impulse $= \Delta p = m\,\Delta v;$

$0.93\text{ N}\cdot\text{s} = m(220\text{ m/s} - 0)$, which gives $m = 4.23 \times 10^{-3}$ kg = 4.2 g.

74. We find the speed for falling or rising through a height h from energy conservation:

$\frac{1}{2}mv^2 = mgh$, or $v^2 = 2gh$.



(*a*) The speed of the first block after sliding down the incline and just before the collision is

$v_1 = [2(9.80 \text{ m/s}^2)(3.60 \text{ m})]^{1/2} = 8.40 \text{ m/s}$.

For the elastic collision of the two blocks, we use momentum conservation:

$mv_1 + Mv_2 = mv_1' + Mv_2'$;

$(2.20 \text{ kg})(8.40 \text{ m/s}) + (7.00 \text{ kg})(0) = (2.20 \text{ kg})v_1' + (7.00 \text{ kg})v_2'$.

Because the collision is elastic, the relative speed does not change:

$v_1 - v_2 = -(v_1' - v_2')$, or $8.40 \text{ m/s} - 0 = v_2' - v_1'$.

Combining these two equations, we get

$v_1' = -4.38 \text{ m/s}$, $v_2' = 4.02 \text{ m/s}$.

(*b*) We find the height of the rebound from

$v_1'^2 = 2gh'$;

$(-4.38 \text{ m/s})^2 = 2(9.80 \text{ m/s}^2)h'$, which gives $h' = 0.979$ m.

The distance along the incline is

$d = h'/\sin\theta = (0.979 \text{ m})/\sin 30° =$ 1.96 m.

75. Because energy is conserved for the motion up and down the incline, mass m will return to the level with the speed $-v_1'$. For a second collision to occur, mass m must be moving faster than mass M: $-v_1' \geq v_2'$.

In the first collision, the relative speed does not change:

$v_1 - 0 = -(v_1' - v_2')$, or $-v_1' = v_1 - v_2'$,

so the condition becomes $v_1 - v_2' \geq v_2'$, or $v_1 \geq 2v_2'$.

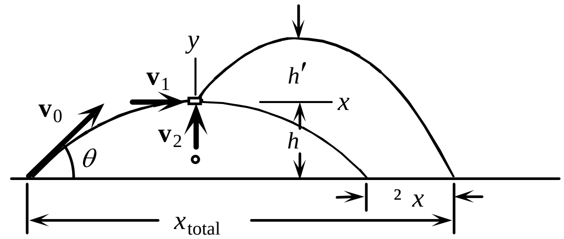
For the first collision, we use momentum conservation:

$mv_1 + 0 = mv_1' + Mv_2'$, or $v_1 - v_1' = (M/m)v_2'$.

When we use the two versions of the condition, we get $v_1 - v_1' \geq 3v_2'$, so we need

$(M/m) \geq 3$, or $m \leq M/3$.

76. (*a*) If the skeet were not hit by the pellet, the horizontal distance it would travel can be found from the range expression for projectile motion:



$R = (v_0^2/g) \sin 2\theta$

$= [(30 \text{ m/s})^2/(9.80 \text{ m/s}^2)] \sin 2(30°) = 79.5$ m.

At the collision the skeet will have the x-component of the initial velocity:

$v_1 = v_0 \cos \theta = (30 \text{ m/s}) \cos 30° = 26.0$ m/s.

We use energy conservation to find the height attained by the skeet when the collision occurs:

$!Mv_0^2 = !Mv_1^2 + Mgh;$

$!(30 \text{ m/s})^2 = !(26.0 \text{ m/s})^2 + (9.80 \text{ m/s}^2)h$, which gives $h = 11.5$ m.

Using the coordinate system shown, for momentum conservation of the collision we have

x: $Mv_1 + 0 = (M + m)V_x;$

(250 g)(26.0 m/s) = (250 g + 15 g)V_x, which gives $V_x = 24.5$ m/s;

y: $0 + mv_2 = (M + m)V_y;$

(15 g)(200 m/s) = (250 g + 15 g)V_y, which gives $V_y = 11.3$ m/s.

We use energy conservation to find the additional height attained by the skeet after the collision:

$!(M + m)(V_x^2 + V_y^2) = !(M + m)V_x^2 + (M + m)gh';$

$![(24.5 \text{ m/s})^2 + (11.3 \text{ m/s})^2] = !(24.5 \text{ m/s})^2 + (9.80 \text{ m/s}^2)h'$, which gives $h' = 6.54$ m.

(*b*) We find the time for the skeet to reach the ground from the vertical motion:

$y = y_0 + V_y t + !(-g)t^2;$

$-11.5 \text{ m} = 0 + (11.3 \text{ m/s})t - !(9.80 \text{ m/s}^2)t^2.$

The positive solution to this quadratic equation is $t = 3.07$ s.

The horizontal distance from the collision is

$x = V_x t = (24.5 \text{ m/s})(3.07 \text{ s}) = 75$ m.

The total horizontal distance covered is

$x_{\text{total}} = !R + x = !(79.5 \text{ m}) + 75 \text{ m} = 115$ m.

Because of the collision, the skeet will have traveled an additional distance of

$Æx = x_{\text{total}} - R = 115 \text{ m} - 79.5 \text{ m} =$ 35 m.

77. Obviously the spacecraft will have negligible effect on the motion of Saturn. In the reference frame of Saturn, we can treat this as the equivalent of a small mass "bouncing off" a massive object. The relative velocity of the spacecraft in this reference frame will be reversed.

The initial relative velocity of the spacecraft is

$v_{\text{SpS}} = v_{\text{Sp}} - v_{\text{S}} = 10.4 \text{ km/s} - (-9.6 \text{ km/s}) = 20.0$ km/s.

so the final relative velocity is $v_{\text{SpS}}' = -20.0$ km/s. Therefore, we find the final velocity of the spacecraft from

$v_{\text{SpS}}' = v_{\text{Sp}}' - v_{\text{S}};$

$-20.0 \text{ km/s} = v_{\text{Sp}}' - (-9.6 \text{ km/s})$, which gives $v_{\text{Sp}}' = -29.6$ km/s,

so the final speed of the spacecraft is 29.6 km/s.