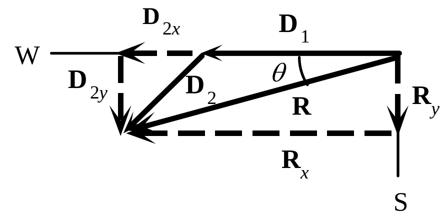
CHAPTER 3

1. We choose the west and south coordinate system shown. For the components of the resultant we have



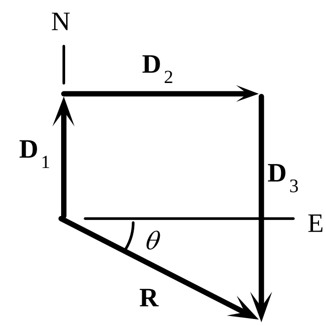
$R_W = D_1 + D_2 \cos 45°$
$= (125 \text{ km}) + (65 \text{ km}) \cos 45° = 171 \text{ km};$
$R_S = 0 + D_2$
$= 0 + (65 \text{ km}) \sin 45° = 46 \text{ km}.$

We find the resultant displacement from

$R = (R_W^2 + R_S^2)^{1/2} = [(171 \text{ km})^2 + (46 \text{ km})^2]^{1/2} =$ 177 km;

$\tan \theta = R_S/R_W = (46 \text{ km})/(171 \text{ km}) = 0.269$, which gives $\theta = 15°$ S of W.

2. We choose the north and east coordinate system shown. For the components of the resultant we have



$R_E = D_2 = 16$ blocks;
$R_N = D^1 - D_3 = 14$ blocks$- 26$ blocks $= - 12$ blocks.

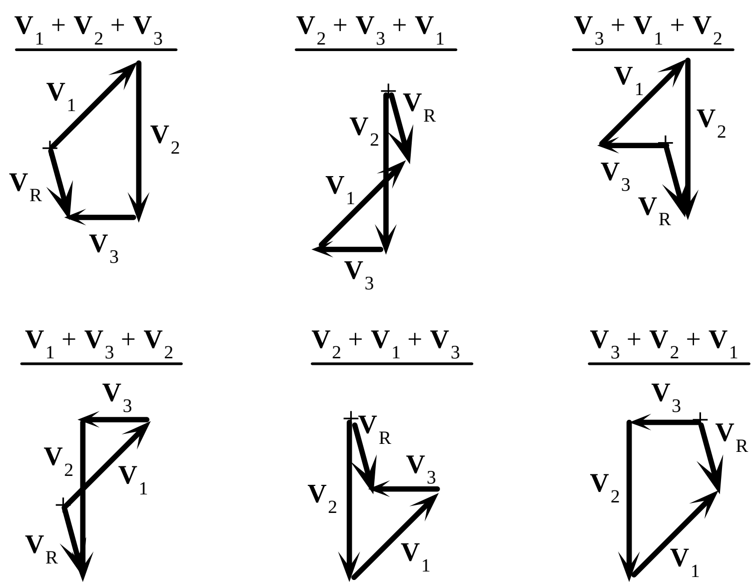
We find the resultant displacement from

$R = (R_E^2 + R_N^2)^{1/2} = [(16 \text{ blocks})^2 + (- 12 \text{ blocks})^2]^{1/2}$
$=$ 20 blocks;

$\tan \theta = R_N/R_E = (12 \text{ blocks})/(16 \text{ blocks}) = 0.75$, which gives
$\theta = 37°$ S of E.

Note that we have used the magnitude of R_N for the angle indicated on the diagram.

3. We show the six head-to-tail additions:

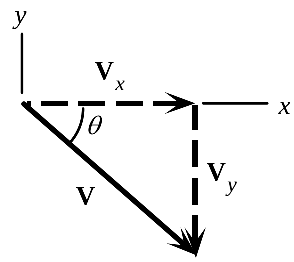


4. We find the vector from

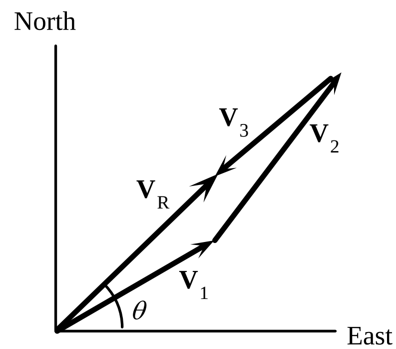
$V = (V_x^2 + V_y^2)^{1/2} = [(18.80)^2 + (-16.40)^2]^{1/2} =$ 24.95;

$\tan \theta = V_y/V_x = (-16.40)/(18.80) = -0.8723$, which gives

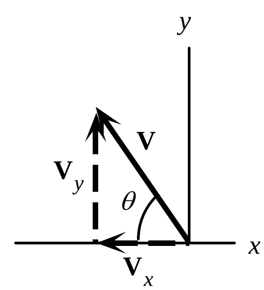
$\theta = 41.10°$ below the x-axis.



5. The resultant is 31 m, 44° N of E.



6. (*a*)



(*b*) For the components of the vector we have

$V_x = -V \cos \theta = -24.3 \cos 54.8° =$ – 14.0;

$V_y = V \sin \theta = 24.3 \sin 54.8° =$ 19.9.

(*c*) We find the vector from

$V = (V_x^2 + V_y^2)^{1/2} = [(-14.0)^2 + (19.9)^2]^{1/2}$

= 24.3;

$\tan \theta = V_y/V_x = (19.9)/(14.0) = 1.42$, which gives

$\theta = 54.8°$ above $-x$-axis.

Note that we have used the magnitude of V_x for the angle indicated on the diagram.

7. Because the vectors are parallel, the direction can be indicated by the sign.

(*a*) **C** = **A** + **B** = 8.31 + (– 5.55)

= 2.76 in the + x-direction.

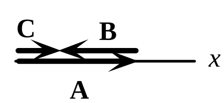
(*b*) **C** = **A** – **B** = 8.31 – (– 5.55)

= 13.86 in the + x-direction.

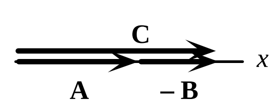
(*c*) **C** = **B** – **A** = – 5.55 – (8.31)

= – 13.86 in the + x-direction or 13.86 in the – x-direction.

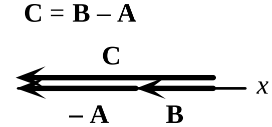
(*a*) **C** = **A** + **B**



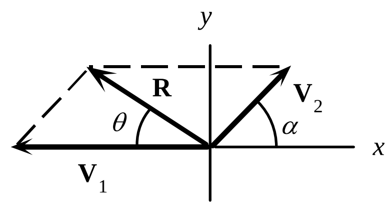
(*b*) **C** = **A** – **B**



(*c*) **C** = **B** – **A**



8. (*a*) $V_{1x} = -8.08$, $V_{1y} = 0$;
$V_{2x} = V_2 \cos 45° = 4.51 \cos 45° = 3.19$,
$V_{2y} = V_2 \sin 45° = 4.51 \sin 45° = 3.19$.

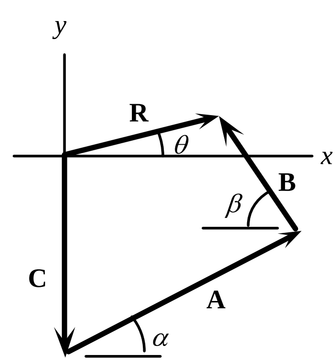


(*b*) For the components of the resultant we have
$R_x = V_{1x} + V_{2x} = -8.08 + 3.19 = -4.89$;
$R_y = V_{1y} + V_{2y} = 0 + 3.19 = 3.19$.
We find the resultant from
$R = (R_x^2 + R_y^2)^{1/2} = [(-4.89)^2 + (3.19)^2]^{1/2}$
$= 5.84$;
$\tan \theta = R_y/R_x = (3.19)/(4.89) = 0.652$, which gives
$\theta = 33.1°$ above $-x$-axis.
Note that we have used the magnitude of R_x for the angle indicated on the diagram.

9. (*a*) Using the given angle, we find the components from
$V_N = V \cos 38.5° = (785 \text{ km/h}) \cos 38.5° = 614 \text{ km/h}$;
$V_W = V \sin 38.5° = (785 \text{ km/h}) \sin 38.5° = 489 \text{ km/h}$.
(*b*) We use the velocity components to find the displacement components:
$d_N = V_N t = (614 \text{ km/h})(3.00 \text{ h}) = 1.84 \times 10^3 \text{ km}$;
$d_W = V_W t = (489 \text{ km/h})(3.00 \text{ h}) = 1.47 \times 10^3 \text{ km}$.

10. We find the components of the sum by adding the components:
$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = (3.0, 2.7, 0.0) + (2.9, -4.1, -1.4) = (5.9, -1.4, -1.4)$.
By extending the Pythagorean theorem, we find the magnitude from
$V = (V_x^2 + V_y^2 + V_z^2)^{1/2} = [(5.9)^2 + (-1.4)^2 + (-1.4)^2]^{1/2} = 6.2$.

11. (*a*) For the components we have
$R_x = A_x + B_x + C_x$
$= 66.0 \cos 28.0° - 40.0 \cos 56.0° + 0 = 35.9$;
$R_y = A_y + B_y + C_y$
$= 66.0 \sin 28.0° + 40.0 \sin 56.0° - 46.8 = 17.3$.



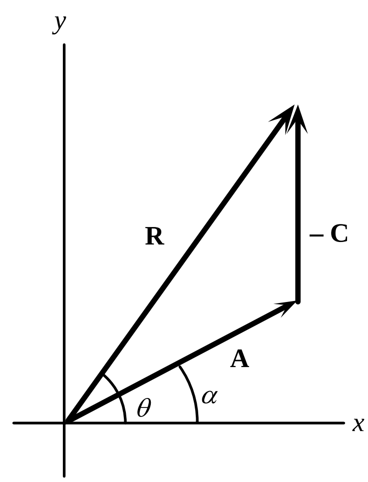
(*b*) We find the resultant from
$R = (R_x^2 + R_y^2)^{1/2} = [(35.9)^2 + (17.3)^2]^{1/2} = 39.9$;
$\tan \theta = R_y/R_x = (17.3)/(35.9) = 0.483$, which gives
$\theta = 25.8°$ above $+x$-axis.

12. For the components we have

$R_x = A_x - C_x$
$= 66.0 \cos 28.0° - 0 = 58.3;$
$R_y = A_y - C_y$
$= 66.0 \sin 28.0° - (-46.8) = 77.8.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(58.3)^2 + (77.8)^2]^{1/2} =$ 97.2;
$\tan\theta = R_y/R_x = (77.8)/(58.3) = 1.33$, which gives
$\theta = 53.1°$ above + x-axis.



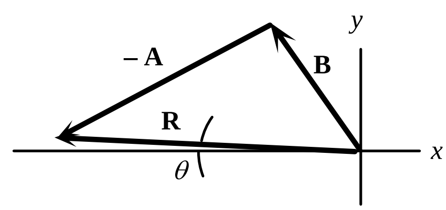
13. (*a*) For the components we have

$R_x = B_x - A_x$
$= -40.0 \cos 56.0° - 66.0 \cos 28.0° = -80.7;$
$R_y = B_y - A_y$
$= 40.0 \sin 56.0° - 66.0 \sin 28.0° = 2.2.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(-80.7)^2 + (2.2)^2]^{1/2} =$ 80.7;
$\tan\theta = R_y/R_x = (2.2)/(80.7) = 0.0273$, which gives
$\theta = 1.56°$ above – x-axis.

Note that we have used the magnitude of R_x for the angle indicated on the diagram.

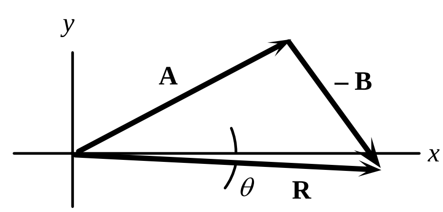


(*b*) For the components we have

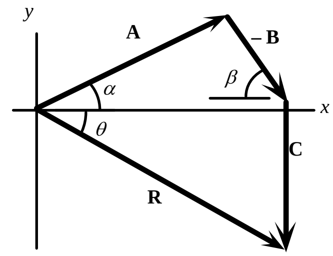
$R_x = A_x - B_x$
$= 66.0 \cos 28.0° - (-40.0 \cos 56.0°) = 80.7;$
$R_y = A_y - B_y$
$= 66.0 \sin 28.0° - 40.0 \sin 56.0° = -2.2.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(80.7)^2 + (-2.2)^2]^{1/2} =$ 80.7;
$\tan\theta = R_y/R_x = (2.2)/(80.7) = 0.0273$, which gives
$\theta = 1.56°$ below + x-axis, which is opposite to the result from (*a*).



14. (*a*) For the components we have



$R_x = A_x - B_x + C_x$
$= 66.0 \cos 28.0° - (- 40.0 \cos 56.0°) + 0$
$= 80.7;$
$R_y = A_y - B_y + C_y$
$= 66.0 \sin 28.0° - 40.0 \sin 56.0° - 46.8$
$= - 49.0.$

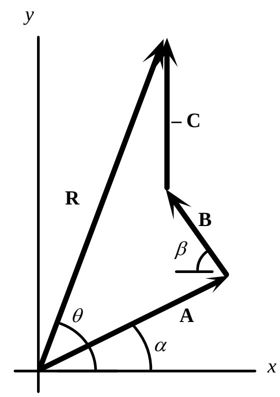
We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(80.7)^2 + (- 49.0)^2]^{1/2}$
$= 94.4;$

$\tan \theta = R_y/R_x = (49.0)/(80.7) = 0.607$, which gives
$\theta = 31.3°$ below $+ x$-axis.

Note that we have used the magnitude of R_y for the angle indicated on the diagram.

(*b*) For the components we have



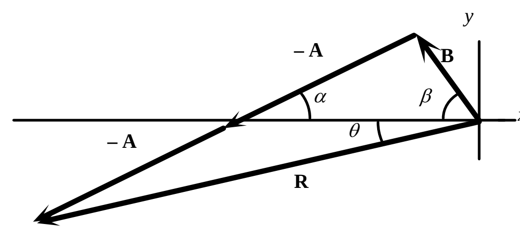
$R_x = A_x + B_x - C_x$
$= 66.0 \cos 28.0° + (- 40.0 \cos 56.0°) - 0$
$= 35.9;$
$R_y = A_y + B_y - C_y$
$= 66.0 \sin 28.0° + 40.0 \sin 56.0° - (- 46.8)$
$= 111.0.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(35.9)^2 + (111.0)^2]^{1/2}$
$= 117;$

$\tan \theta = R_y/R_x = (111.0)/(35.9) = 3.09$, which gives
$\theta = 72.1°$ above $+ x$-axis.

(*c*) For the components we have



$R_x = B_x - 2A_x$
$= - 40.0 \cos 56.0° - 2(66.0 \cos 28.0°)$
$= - 139.0;$
$R_y = B_y - 2A_y$
$= 40.0 \sin 56.0° - 2(66.0 \sin 28.0°)$
$= - 28.8.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(- 139.0)^2 + (- 28.8)^2]^{1/2}$
$= 142;$

$\tan \theta = R_y/R_x = (28.8)/(139.0) = 0.207$, which gives
$\theta = 11.7°$ below $- x$-axis.

15. (*a*) For the components we have

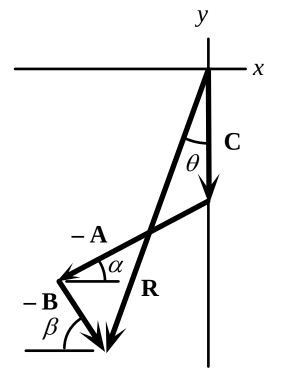
$R_x = C_x - A_x - B_x$
$= 0 - 66.0 \cos 28.0° - (-40.0 \cos 56.0°)$
$= -35.9;$
$R_y = C_y - A_y - B_y$
$= -46.8 - 66.0 \sin 28.0° - 40.0 \sin 56.0°$
$= -111.0.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(-35.9)^2 + (-111.0)^2]^{1/2}$
$=$ 117;
$\tan \theta = R_y/R_x = (111.0)/(35.9) = 3.09,$

which gives

$\theta = 72.1°$ below $-x$-axis.



(*b*) For the components we have

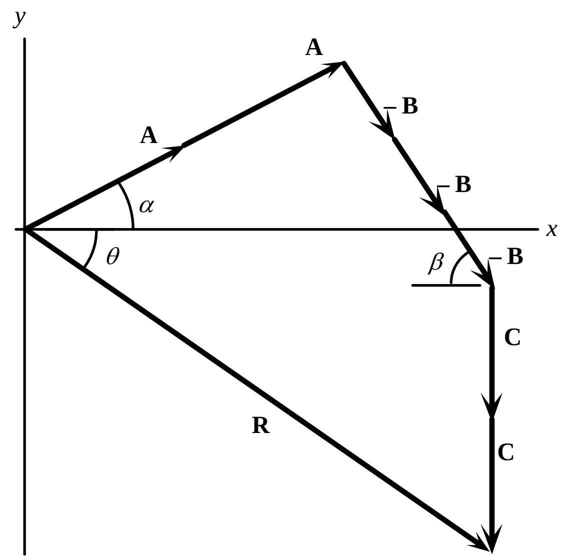
$R_x = 2A_x - 3B_x + 2C_x$
$= 2(66.0 \cos 28.0°) - 3(-40.0 \cos 56.0°) + 2(0)$
$= 183.8;$
$R_y = 2A_y - 3B_y + 2C_y$
$= 2(66.0 \sin 28.0°) - 3(40.0 \sin 56.0°) + 2(-46.8)$
$= -131.2.$

We find the resultant from

$R = (R_x^2 + R_y^2)^{1/2} = [(183.8)^2 + (-131.2)^2]^{1/2}$
$=$ 226;
$\tan \theta = R_y/R_x = (131.2)/(183.8) = 0.714,$

which gives

$\theta = 35.5°$ below $+x$-axis.



16. (*a*) For the vertical component we have

$a_V = (3.80 \text{ m/s}^2) \sin 30.0° =$ 1.90 m/s^2 down.

(*b*) Because the elevation change is the vertical displacement, we find the time from the vertical motion, taking down as the positive direction:

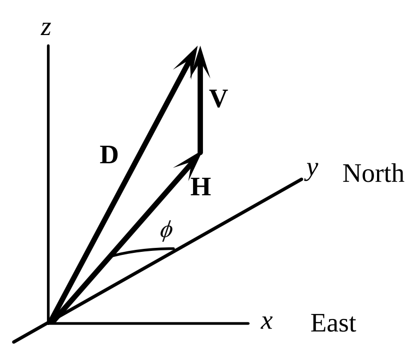
$y = v_{0y}t + !a_V t^2;$
$335 \text{ m} = 0 + !\,(1.90 \text{ m/s}^2)t^2$, which gives $t = 18.8$ s.

17. For the components we have

$D_x = H_x = -4580 \sin 32.4° =$ -2454 m;
$D_y = H_y = +4580 \cos 32.4° =$ $+3867$ m;
$D_z = V =$ $+2085$ m.

By extending the Pythagorean theorem, we find the length from

$D = (D_x^2 + D_y^2 + D_z^2)^{1/2}$
$= [(-2454 \text{ m})^2 + (3867 \text{ m})^2 + (2085 \text{ m})^2]^{1/2}$
$=$ 5032 m.



18. (*a*) We find the x-component from

$A^2 = A_x^2 + A_y^2$;

$(90.0)^2 = A_x^2 + (-55.0)^2$; which gives $A_x = \pm 71.2$.

(*b*) If we call the new vector **B**, we have

$R_x = A_x + B_x$;

$-80.0 = +71.2 + B_x$, which gives $B_x = -151.2$;

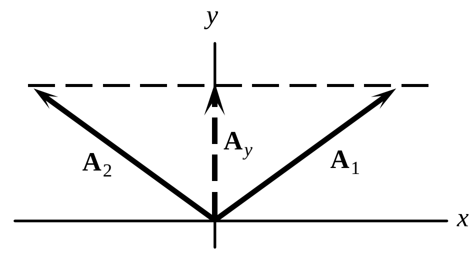
$R_y = A_y + B_y$;

$0 = -55.0 + B_y$, which gives $B_y = +55.0$.

We find the resultant from

$B = (B_x^2 + B_y^2)^{1/2} = [(-151.2)^2 + (+55.0)^2]^{1/2} =$ 161;

$\tan\theta = B_y/B_x = (55.0)/(151.2) = 0.364$, which gives $\theta = 20°$ above $-x$-axis.



19. We choose a coordinate system with the origin at the takeoff point, with x horizontal and y vertical, with the positive direction down. We find the time for the tiger to reach the ground from its vertical motion:

$y = y_0 + v_{0y}t + !a_yt^2$;

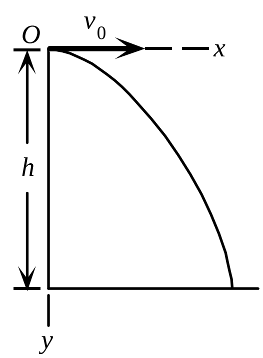
$7.5 \text{ m} = 0 + 0 + !(9.80 \text{ m/s}^2)t^2$, which gives $t = 1.24$ s.

The horizontal motion will have constant velocity.

We find the distance from the base of the rock from

$x = x_0 + v_{0x}t$;

$x = 0 + (4.5 \text{ m/s})(1.24 \text{ s}) =$ 5.6 m.



20. We choose a coordinate system with the origin at the takeoff point, with x horizontal and y vertical, with the positive direction down. We find the height of the cliff from the vertical displacement:

$y = y_0 + v_{0y}t + !a_yt^2$;

$y = 0 + 0 + !(9.80 \text{ m/s}^2)(3.0 \text{ s})^2 =$ 44 m.

The horizontal motion will have constant velocity.

We find the distance from the base of the cliff from

$x = x_0 + v_{0x}t$;

$x = 0 + (1.6 \text{ m/s})(3.0 \text{ s}) =$ 4.8 m.

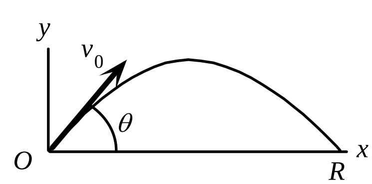
21. Because the water returns to the same level, we can use the expression for the horizontal range:

$R = v_0^2 \sin(2\theta)/g$;

$2.0 \text{ m} = (6.5 \text{ m/s})^2 \sin(2\theta)/(9.80 \text{ m/s}^2)$, which gives

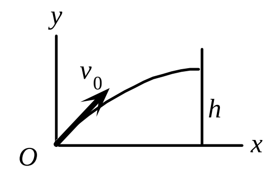
$\sin(2\theta) = 0.463$, or $2\theta = 28°$ and $152°$,

so the angles are 14° and 76°.



At the larger angle the water has a smaller horizontal velocity but spends more time in the air, because of the larger initial vertical velocity. Thus the horizontal displacement is the same for the two angles.

22. The horizontal velocity is constant, and the vertical velocity will be zero when the pebbles hit the window. Using the coordinate system shown, we find the vertical component of the initial velocity from



$v_y{}^2 = v_{0y}{}^2 + 2a_y(h - y_0)$;

$0 = v_{0y}{}^2 + 2(-9.80\ \text{m/s}^2)(8.0\ \text{m} - 0)$, which gives $v_{0y} = 12.5$ m/s.

(We choose the positive square root because we know that the pebbles are thrown upward.)
We find the time for the pebbles to hit the window from the vertical motion:

$v_y = v_{0y} + a_y t$;

$0 = 12.5\ \text{m/s} + (9.80\ \text{m/s}^2)t$, which gives $t = 1.28$ s.

For the horizontal motion we have

$x = x_0 + v_{0x}t$;

$9.0\ \text{m} = 0 + v_{0x}(1.28\ \text{s})$, which gives $v_{0x} = 7.0$ m/s.

Because the pebbles are traveling horizontally when they hit the window, this is their speed.

23. The ball passes the goal posts when it has traveled the horizontal distance of 36.0 m. From this we can find the time when it passes the goal posts:

$x = v_{0x}t$; $36.0\ \text{m} = (20.0\ \text{m/s}) \cos 37.0° \ t$, which gives $t = 2.25$ s.

To see if the kick is successful, we must find the height of the ball at this time:

$y = y_0 + v_{0y}t + !a_y t^2 = 0 + (20.0\ \text{m/s}) \sin 37.0° \ (2.25\ \text{s}) + !(-9.80\ \text{m/s}^2)(2.25\ \text{s})^2 = 2.24$ m.

Thus the kick is unsuccessful because it passes 0.76 m below the bar.
To have a successful kick, the ball must pass the goal posts with an elevation of at least 3.00 m. We find the time when the ball has this height from

$y = y_0 + v_{0y}t + !a_y t^2$;

$3.00\ \text{m} = 0 + (20.0\ \text{m/s}) \sin 37.0° \ t + !(-9.80\ \text{m/s}^2)t^2$.

The two solutions of this quadratic equation are $t = 0.28$ s, 2.17 s.
The horizontal distance traveled by the ball is found from

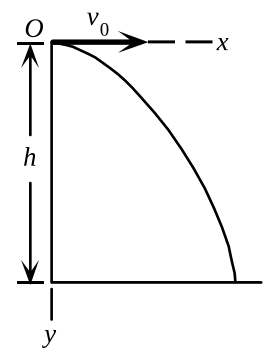
$x = v_{0x}t = (20.0\ \text{m/s}) \cos 37.0° \ t$.

For the two times, we get $x = 4.5$ m, 34.7 m.
Thus the kick must be made no farther than 34.7 m from the goal posts (and no nearer than 4.5 m).
If the vertical velocity is found at these two times from

$v_y = v_{0y} + a_y t = (20.0\ \text{m/s}) \sin 37.0° + (-9.80\ \text{m/s}^2)t = +9.3$ m/s, -9.3 m/s,

we see that the ball is falling at the goal posts for a kick from 34.7 m and rising at the goal posts for a kick from 4.5 m.

24. We choose a coordinate system with the origin at the release point, with x horizontal and y vertical, with the positive direction down. We find the time of fall from the vertical displacement:



$y = y_0 + v_{0y}t + !a_y t^2$;

$56\ \text{m} = 0 + 0 + !(9.80\ \text{m/s}^2)t^2$, which gives $t = 3.38$ s.

The horizontal motion will have constant velocity.
We find the initial speed from

$x = x_0 + v_{0x}t$;

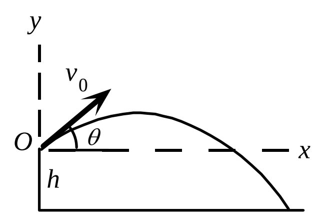
$45\ \text{m} = 0 + v_0(3.38\ \text{s})$, which gives $v_0 = 13$ m/s.

25. The horizontal motion will have constant velocity, v_{0x}.
Because the projectile lands at the same elevation, we find the vertical velocity on impact from
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 0 = v_{0y}^2$, so $v_y = -v_{0y}$.
(We choose the negative square root because we know that the projectile is coming down.)
The speed at impact is
$v = (v_x^2 + v_y^2)^{1/2} = [v_{0x}^2 + (-v_{0y})^2]^{1/2} = (v_{0x}^2 + v_{0y}^2)^{1/2}$, which is the initial speed.

26. We find the time of flight from the vertical displacement:
$y = y_0 + v_{0y}t + !a_yt^2$;
$0 = 0 + (20.0\text{ m/s})(\sin 37.0°)t + !(-9.80\text{ m/s}^2)t^2$, which gives $t = 0, 2.46$ s.
The ball is kicked at $t = 0$, so the football hits the ground 2.46 s later.

27. We choose a coordinate system with the origin at the release point, with x horizontal and y vertical, with the positive direction down.
The horizontal motion will have constant velocity. We find the time required for the fall from
$x = x_0 + v_{0x}t$;
$36.0\text{ m} = 0 + (22.2\text{ m/s})t$, which gives $t = 1.62$ s.
We find the height from the vertical motion:
$y = y_0 + v_{0y}t + !a_yt^2$;
$h = 0 + 0 + !(9.80\text{ m/s}^2)(1.62\text{ s})^2 =$ 12.9 m.

28. We choose a coordinate system with the origin at the release point, with x horizontal and y vertical, with the positive direction up.
We find the time required for the fall from the vertical motion:
$y = y_0 + v_{0y}t + !a_yt^2$;
$-2.2\text{ m} = 0 + (14\text{ m/s})(\sin 40°)t + !(-9.80\text{ m/s}^2)t^2$.
The solutions of this quadratic equation are $t = -0.22$ s, 2.06 s.
Because the shot is released at $t = 0$, the physical answer is 2.06 s.
We find the horizontal distance from
$x = x_0 + v_{0x}t$;
$x = 0 + (14\text{ m/s})(\cos 40°)(2.06\text{ s}) =$ 22 m.



29. Because the initial and final locations are at the same level, we can use the expression for the horizontal range. The horizontal range on Earth is given by
$R = v_0^2 \sin(2\theta_0)/g$, whereas on the moon it is
$R_{\text{moon}} = v_0^2 \sin(2\theta_0)/g_{\text{moon}}$.
Because we have the same v_0 and θ_0, when we divide the two equations, we get
$R_{\text{moon}}/R = g/g_{\text{moon}}$, or
$R_{\text{moon}} = (g/g_{\text{moon}})R = [g/(g/6)]R = 6R$, so a person could jump six times as far.

30. (*a*) Because the athlete lands at the same level, we can use the expression for the horizontal range:

$R = v_0^2 \sin(2\theta_0)/g;$

7.80 m = $v_0^2 \sin[2(30°)]/(9.80 \text{ m/s}^2)$, which gives $v_0 = 9.39$ m/s.

(*b*) For an increase of 5%, the initial speed becomes $v_0' = (1 + 0.05)v_0 = (1.05)v_0$, and the new range is

$R' = v_0'^2 \sin(2\theta_0)/g = (1.05)^2 v_0^2 \sin(2\theta_0)/g = 1.10R.$

Thus the increase in the length of the jump is

$R' - R = (1.10 - 1)R = 0.10(7.80 \text{ m}) =$ 0.80 m.

31. We choose a coordinate system with the origin at the release point, with x horizontal and y vertical, with the positive direction down.
Because the horizontal velocity of the package is constant at the horizontal velocity of the airplane, the package is always directly under the airplane. The time interval is the time required for the fall, which we find from the vertical motion:

$y = y_0 + v_{0y}t + !a_y t^2;$

160 m = 0 + 0 + !$(9.80 \text{ m/s}^2)t^2$, which gives $t = 5.71$ s.

32. (*a*) We choose a coordinate system with the origin at the release point, with x horizontal and y vertical, with the positive direction down. We find the time of flight from the horizontal motion:

$x = x_0 + v_{0x}t;$

120 m = 0 + (250 m/s)t, which gives $t = 0.480$ s.

We find the distance the bullet falls from

$y = y_0 + v_{0y}t + !a_y t^2;$

$y = 0 + 0 + !(9.80 \text{ m/s}^2)(0.480 \text{ s})^2 =$ 1.13 m.

(*b*) The bullet will hit the target at the same elevation, so we can use the expression for the horizontal range:

$R = v_0^2 \sin(2\theta_0)/g;$

120 m = $(250 \text{ m/s})^2 \sin(2\theta_0)/(9.80 \text{ m/s}^2)$, which gives

$\sin(2\theta_0) = 0.0188$, or $2\theta_0 = 1.08°$, $\theta_0 = 0.54°$.

The larger angle of 89.5° is unrealistic.

33. We choose a coordinate system with the origin at the release point, with y vertical and the positive direction up. At the highest point the vertical velocity will be zero, so we find the time to reach the highest point from

$v_y = v_{0y} + a_y t_{\text{up}};$

$0 = v_{0y} + (-g)t_{\text{up}}$, which gives $t_{\text{up}} = v_{0y}/g$.

We find the elevation h at the highest point from

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0);$

$0 = v_{0y}^2 + 2(-g)h$, which gives $h = v_{0y}^2/2g$.

We find the time to fall from the highest point from

$y = y_0 + v_{0y}t + !a_y t_{\text{down}}^2;$

$0 = h + 0 + !(-g)t_{\text{down}}^2$, which gives

$t_{\text{down}} = (2h/g)^{1/2} = [2(v_{0y}^2/2g)/g]^{1/2} = v_{0y}/g$, which is the same as t_{up}.

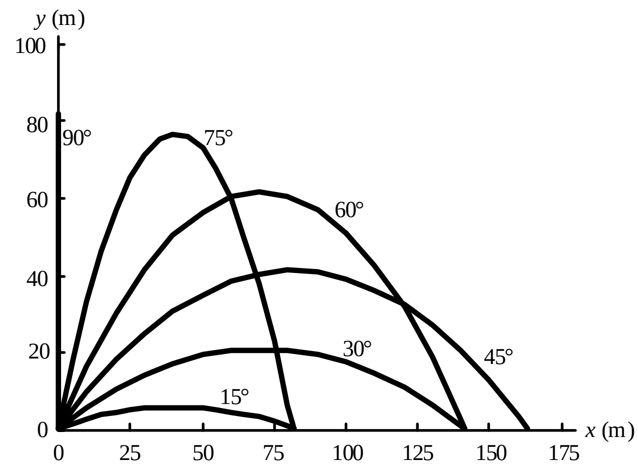
34. To plot the trajectory, we need a relationship between x and y, which can be obtained by eliminating t from the equations for the two components of the motion:

$x = v_{0x}t = v_0 (\cos\theta)t$;

$y = y_0 + v_{0y}t + !a_yt^2 = 0 + v_0 (\sin\theta)t + !(-g)t^2$.

The relationship is

$y = (\tan\theta)x - !g(x/v_0 \cos\theta)^2$.



35. (*a*) At the highest point, the vertical velocity $v_y = 0$. We find the maximum height h from

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$;

$0 = [(75.2 \text{ m/s}) \sin 34.5°]^2 + 2(-9.80 \text{ m/s}^2)(h - 0)$, which gives $h = 92.6$ m.

(*b*) Because the projectile returns to the same elevation, we have

$y = y_0 + v_{0y}t + !a_yt^2$;

$0 = 0 + (75.2 \text{ m/s})(\sin 34.5°)t + !(-9.80 \text{ m/s}^2)t^2$, which gives $t = 0$, and 8.69 s.

Because $t = 0$ was the launch time, the total time in the air was 8.69 s.

(*c*) We find the horizontal distance from

$x = v_{0x}t = (75.2 \text{ m/s})(\cos 34.5°)(8.69 \text{ s}) =$ 539 m.

(*d*) The horizontal velocity will be constant: $v_x = v_{0x} = (75.2 \text{ m/s}) \cos 34.5° = 62.0$ m/s.

We find the vertical velocity from

$v_y = v_{0y} + a_yt = (75.2 \text{ m/s}) \sin 34.5° + (-9.80 \text{ m/s}^2)(1.50 \text{ s}) = 27.9$ m/s.

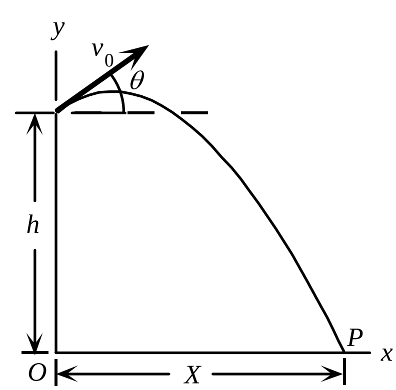
The magnitude of the velocity is

$v = (v_x^2 + v_y^2)^{1/2} = [(62.0 \text{ m/s})^2 + (27.9 \text{ m/s})^2]^{1/2} =$ 68.0 m/s.

We find the angle from

$\tan\theta = v_y/v_x = (27.9 \text{ m/s})/(62.0 \text{ m/s}) = 0.450$, which gives $\theta = 24.2°$ above the horizontal.

36. (*a*) We choose a coordinate system with the origin at the base of the cliff, with x horizontal and y vertical, with the positive direction up.



We find the time required for the fall from the vertical motion:

$y = y_0 + v_{0y}t + !a_yt^2$;

$0 = 125 \text{ m} + (105 \text{ m/s})(\sin 37.0°)t + !(-9.80 \text{ m/s}^2)t^2$,

which gives $t = -1.74, 14.6$ s.

Because the projectile starts at $t = 0$, we have $t = 14.6$ s.

(*b*) We find the range from the horizontal motion:

$X = v_{0x}t = (105 \text{ m/s})(\cos 37.0°)(14.6 \text{ s})$

$= 1.22 \times 10^3 \text{ m} =$ 1.22 km.

(*c*) For the velocity components, we have

$v_x = v_{0x} = (105 \text{ m/s}) \cos 37.0° =$ 83.9 m/s.

$v_y = v_{0y} + a_yt = (105 \text{ m/s}) \sin 37.0° + (-9.80 \text{ m/s}^2)(14.6 \text{ s}) =$ – 79.9 m/s.

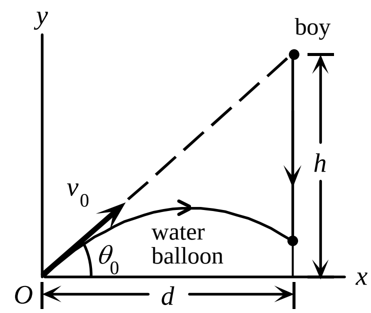
(*d*) When we combine these components, we get

$v = (v_x^2 + v_y^2)^{1/2} = [(83.9 \text{ m/s})^2 + (-79.9 \text{ m/s})^2]^{1/2} =$ 116 m/s.

(*e*) We find the angle from

$\tan \theta = v_y/v_x = (79.9 \text{ m/s})/(83.9 \text{ m/s}) = 0.952$, which gives $\theta = 43.6°$ below the horizontal.

37. We use the coordinate system shown in the diagram. To see if the water balloon hits the boy, we will find the location of the water balloon and the boy when the water balloon passes the vertical line that the boy follows. We find the time at which this occurs from the horizontal motion of the water balloon:



$d = v_{0x}t = (v_0 \cos \theta_0)t$, which gives $t = d/(v_0 \cos \theta_0)$.

At this time the location of the boy is

$y_{boy} = y_{0boy} + v_{0yboy}t + !a_y t^2$

$= h + 0 + !(- g)[d/(v_0 \cos \theta_0)]^2 = h - !g[d/(v_0 \cos \theta_0)]^2$.

The vertical position of the water balloon will be

$y_{balloon} = y_{0balloon} + v_{0yballoon}t + !a_y t^2$

$= 0 + (v_0 \sin \theta_0)[d/(v_0 \cos \theta_0)] + !(- g)[d/(v_0 \cos \theta_0)]^2$

$= d \sin \theta_0/ \cos \theta_0 - !g[d/(v_0 \cos \theta_0)]^2 = d \tan \theta_0 - !g[d/(v_0 \cos \theta_0)]^2$.

Because $h = d \tan \theta_0$, we have $y_{balloon} = y_{boy}$.

38. (*a*) The time to fall is found from $y = y_0 + v_{0y}t + !a_y t^2$;

$0 = 235 \text{ m} + 0 + !(- 9.80 \text{ m/s}^2)t_1^2$, which gives $t_1 = 6.93$ s.

Because the goods have the same horizontal speed as the airplane, they will always be directly below the airplane and will have a horizontal displacement of

$x_1 = v_{0x}t_1 = (69.4 \text{ m/s})(6.93 \text{ s}) =$ 480 m,

which is the distance from the climbers that the goods must be dropped.

(*b*) If the goods are given a vertical velocity, they will still have the horizontal velocity of the airplane. We find the time to fall from

$x_2 = v_{0x}t_2$;

$425 \text{ m} = (69.4 \text{ m/s})t_2$, which gives $t_2 = 6.12$ s.

We find the necessary vertical velocity from

$y = y_0 + v_{0y}t_2 + !a_y t_2^2$;

$0 = 235 \text{ m} + v_{0y}(6.12 \text{ s}) + !(- 9.80 \text{ m/s}^2)(6.12 \text{ s})^2$, which gives $v_{0y} = - 8.41$ m/s (down).

(*c*) For the velocity components after the fall, we have

$v_x = v_{0x} = 69.4$ m/s.

$v_y = v_{0y} + a_y t_2 = - 8.41 \text{ m/s} + (- 9.80 \text{ m/s}^2)(6.12 \text{ s}) = - 68.4$ m/s.

When we combine these components, we get

$v = (v_x^2 + v_y^2)^{1/2} = [(69.4 \text{ m/s})^2 + (- 68.4 \text{ m/s})^2]^{1/2} =$ 97.5 m/s.

39. We will take down as the positive direction. The direction of motion is the direction of the velocity. For the velocity components, we have

$v_x = v_{0x} = v_0$.

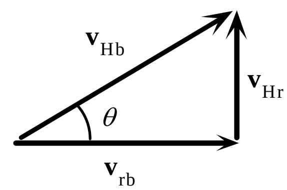
$v_y = v_{0y} + a_y t = 0 + gt = gt$.

We find the angle that the velocity vector makes with the horizontal from

$\tan \theta = v_y/v_x = gt/v_0$, or $\theta = \tan^{-1}(gt/v_0)$ below the horizontal.

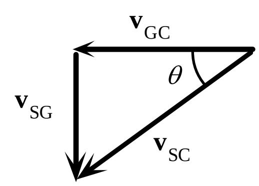
40. If $\mathbf{v}_{sw}$ is the velocity of the ship with respect to the water,
$\mathbf{v}_{js}$ the velocity of the jogger with respect to the ship, and
$\mathbf{v}_{jw}$ the velocity of the jogger with respect to the water, then
$\mathbf{v}_{jw} = \mathbf{v}_{js} + \mathbf{v}_{sw}$.
We choose the direction of the ship as the positive direction. Because all vectors are parallel, in each case the motion is one-dimensional.
When the jogger is moving toward the bow, we have
$v_{jw} = v_{js} + v_{sw} = 2.0$ m/s + 8.5 m/s = 10.5 m/s in the direction of the ship's motion.
When the jogger is moving toward the stern, we have
$v_{jw} = -v_{js} + v_{sw} = -2.0$ m/s + 8.5 m/s = 6.5 m/s in the direction of the ship's motion.

41. If $\mathbf{v}_{Hr}$ is the velocity of Huck with respect to the raft,
$\mathbf{v}_{Hb}$ the velocity of Huck with respect to the bank, and
$\mathbf{v}_{rb}$ the velocity of the raft with respect to the bank, then
$\mathbf{v}_{Hb} = \mathbf{v}_{Hr} + \mathbf{v}_{rb}$, as shown in the diagram.



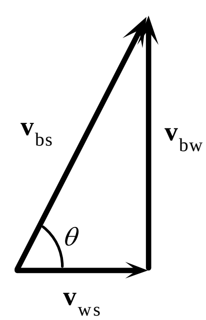
From the diagram we get
$v_{Hb} = (v_{Hr}^2 + v_{rb}^2)^{1/2} = [(1.0 \text{ m/s})^2 + (2.7 \text{ m/s})^2]^{1/2} =$ 2.9 m/s.
We find the angle from
$\tan \theta = v_{Hr}/v_{rb} = (1.0 \text{ m/s})/(2.7 \text{ m/s}) = 0.37$, which gives $\theta = 20°$ from the river bank.

42. If $\mathbf{v}_{SG}$ is the velocity of the snow with respect to the ground,
$\mathbf{v}_{CG}$ the velocity of the car with respect to the ground, and
$\mathbf{v}_{SC}$ the velocity of the snow with respect to the car, then
$\mathbf{v}_{SC} = \mathbf{v}_{SG} + \mathbf{v}_{GC} = \mathbf{v}_{SG} - \mathbf{v}_{CG}$, as shown in the diagram.



From the diagram we get
$\cos \theta = v_{GC}/v_{SC}$, or $\cos 30° = (25 \text{ m/s})/v_{SC}$, which gives $v_{SC} = 29$ m/s;
$\tan \theta = v_{SG}/v_{GC}$, or $\tan 30° = v_{SG}/(25 \text{ m/s})$, which gives $v_{SG} = 14$ m/s (down).

43. (*a*) If $\mathbf{v}_{bs}$ is the velocity of the boat with respect to the shore,
$\mathbf{v}_{bw}$ the velocity of the boat with respect to the water, and
$\mathbf{v}_{ws}$ the velocity of the water with respect to the shore, then
$\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$, as shown in the diagram.



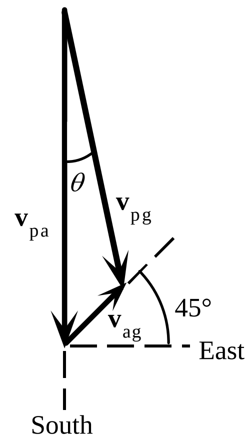
From the diagram we get
$v_{bs} = (v_{bw}^2 + v_{ws}^2)^{1/2} = [(2.30 \text{ m/s})^2 + (1.20 \text{ m/s})^2]^{1/2} =$ 2.59 m/s.
We find the angle from
$\tan \theta = v_{bw}/v_{wr} = (2.30 \text{ m/s})/(1.20 \text{ m/s}) = 1.92$, which gives
$\theta = 62.4°$ from the shore.

(*b*) Because the boat will move with constant velocity, the displacement will be
$d = v_{bs}t = (2.59 \text{ m/s})(3.00 \text{ s}) =$ 7.77 m at 62.4° to the shore.

44. Because the planes are approaching along the same line, for the relative velocity we have
$v = v_1 - v_2 = 835 \text{ km/h} - (-835 \text{ km/h}) = 1670 \text{ km/h}$.
We find the time before they would meet from

$t = d/v = (10.0 \text{ km})/(1670 \text{ km/h}) =$ 0.00600 h = 21.6 s.

45. If $\mathbf{v}_{pa}$ is the velocity of the airplane with respect to the air,
$\mathbf{v}_{pg}$ the velocity of the airplane with respect to the ground, and
$\mathbf{v}_{ag}$ the velocity of the air(wind) with respect to the ground, then
$\mathbf{v}_{pg} = \mathbf{v}_{pa} + \mathbf{v}_{ag}$, as shown in the diagram.



(*a*) From the diagram we find the two components of $\mathbf{v}_{pg}$:

$v_{pgE} = v_{ag} \cos 45° = (100 \text{ km/h}) \cos 45° = 70.7 \text{ km/h};$

$v_{pgS} = v_{pa} - v_{ag} \sin 45° = 500 \text{ km/h} - (100 \text{ km/h}) \cos 45° = 429 \text{ km/h}.$

For the magnitude we have

$v_{pg} = (v_{pgE}{}^2 + v_{pgS}{}^2)^{1/2} = [(70.7 \text{ km/h})^2 + (429 \text{ km/h})^2]^{1/2} =$ 435 km/h.

We find the angle from

$\tan \theta = v_{pgE}/v_{pgS} = (70.7 \text{ km/h})/(429 \text{ km/h}) = 0.165$, which gives

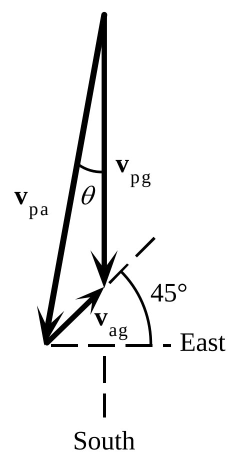
$\theta = 9.36°$ east of south.

(*b*) Because the pilot is expecting to move south, we find the easterly distance from this line from

$d = v_{pgE}t = (70.7 \text{ km/h})(10 \text{ min})/(60 \text{ min/h}) =$ 12 km.

Of course the airplane will also not be as far south as it would be without the wind.

46. If $\mathbf{v}_{pa}$ is the velocity of the airplane with respect to the air,
$\mathbf{v}_{pg}$ the velocity of the airplane with respect to the ground, and
$\mathbf{v}_{ag}$ the velocity of the air(wind) with respect to the ground, then
$\mathbf{v}_{pg} = \mathbf{v}_{pa} + \mathbf{v}_{ag}$, as shown in the diagram.



From the diagram we have

$\sin \theta = (v_{ag} \cos 45°)/v_{pa} = (100 \text{ km/h})(\cos 45°)/(500 \text{ km/h}) = 0.141$,

which gives

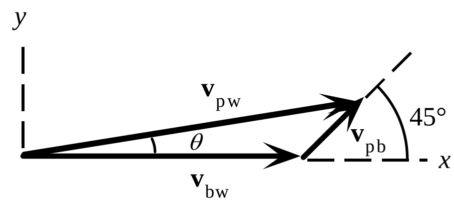
$\theta = 8.13°$ west of south.

47. From the vector diagram of Example 3–10, we have

$v_{BW}{}^2 = v_{BS}{}^2 + v_{WS}{}^2$;

$(1.85 \text{ m/s})^2 = v_{BS}{}^2 + (1.20 \text{ m/s})^2$, which gives $v_{BS} = 1.41 \text{ m/s}$.

48. If $\mathbf{v}_{pw}$ is the velocity of the passenger with respect to the water, $\mathbf{v}_{pb}$ the velocity of the passenger with respect to the boat, and $\mathbf{v}_{bw}$ the velocity of the boat with respect to the water, then $\mathbf{v}_{pw} = \mathbf{v}_{pb} + \mathbf{v}_{bw}$, as shown in the diagram.



From the diagram we find the two components of $\mathbf{v}_{pw}$:

$v_{pwx} = v_{pb} \cos 45° + v_{bw}$
$= (0.50 \text{ m/s}) \cos 45° + 1.50 \text{ m/s} = 1.85 \text{ m/s}.$

$v_{pwy} = v_{pb} \sin 45° = (0.50 \text{ m/s}) \sin 45° = 0.354 \text{ m/s}.$

For the magnitude we have

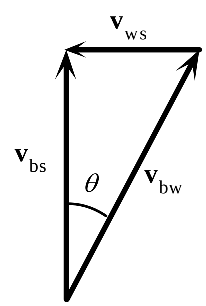
$v_{pw} = (v_{pwx}^2 + v_{pwy}^2)^{1/2} = [(1.85 \text{ m/s})^2 + (0.354 \text{ m/s})^2]^{1/2} =$ 1.9 m/s.

We find the angle from

$\tan \theta = v_{pwy}/v_{pwx} = (0.354 \text{ m/s})/(1.85 \text{ m/s}) = 0.191$, which gives

$\theta = 11°$ above the water.

49. If $\mathbf{v}_{bs}$ is the velocity of the boat with respect to the shore, $\mathbf{v}_{bw}$ the velocity of the boat with respect to the water, and $\mathbf{v}_{ws}$ the velocity of the water with respect to the shore, then $\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$, as shown in the diagram.

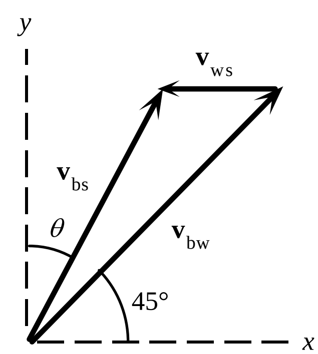


(*a*) From the diagram we have

$v_{ws} = v_{bw} \sin \theta = (3.60 \text{ m/s}) \sin 27.5° =$ 1.66 m/s.

(*b*) $v_{bs} = v_{bw} \cos \theta = (3.60 \text{ m/s}) \cos 27.5° =$ 3.19 m/s.

50. If $\mathbf{v}_{bs}$ is the velocity of the boat with respect to the shore, $\mathbf{v}_{bw}$ the velocity of the boat with respect to the water, and $\mathbf{v}_{ws}$ the velocity of the water with respect to the shore, then $\mathbf{v}_{bs} = \mathbf{v}_{bw} + \mathbf{v}_{ws}$, as shown in the diagram.



We find the angle of the boat's motion from the distances:

$\tan \theta = d_{shore}/d_{river} = (110 \text{ m})/(260 \text{ m}) = 0.423$, which gives $\theta = 22.9°$.

The y-component of $\mathbf{v}_{bs}$ is also the y-component of $\mathbf{v}_{bw}$:

$v_{bsy} = v_{bwy} = (2.20 \text{ m/s}) \sin 45° = 1.56 \text{ m/s}.$

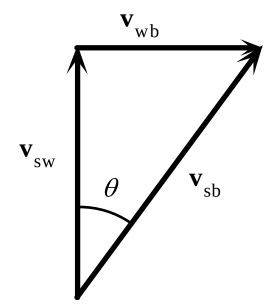
We find the x-component from

$v_{bsx} = v_{bsy} \tan \theta = (1.56 \text{ m/s}) \tan 22.9° = 0.657 \text{ m/s}.$

For the x-component of the relative velocity, we use the diagram to get

$|v_{ws}| = v_{bwx} - v_{bsx} = (2.20 \text{ m/s}) \cos 45° - 0.657 \text{ m/s} =$ 0.90 m/s.

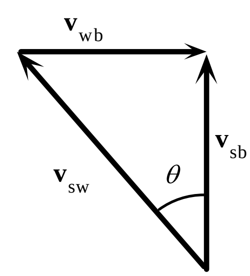
51. If $\mathbf{v}_{sb}$ is the velocity of the swimmer with respect to the bank,
$\mathbf{v}_{sw}$ the velocity of the swimmer with respect to the water, and
$\mathbf{v}_{wb}$ the velocity of the water with respect to the bank, then
$\mathbf{v}_{sb} = \mathbf{v}_{sw} + \mathbf{v}_{wb}$, as shown in the diagram.



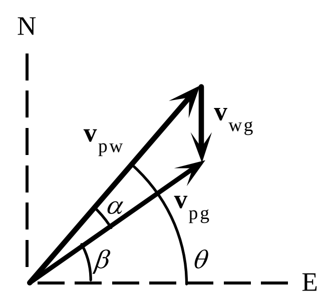
(*a*) We find the angle from
$\tan \theta = v_{wb}/v_{sw} = (0.80 \text{ m/s})/(1.00 \text{ m/s}) = 0.80$, which gives $\theta = 38.7°$.
Because the swimmer travels in a straight line, we have
$\tan \theta = d_{shore}/d_{river}$; $0.80 = d_{shore}/(150 \text{ m})$, which gives $d_{shore} =$ 120 m.

(*b*) We can find how long it takes by using the components across the river:
$t = d_{river}/v_{sw} = (150 \text{ m})/(1.00 \text{ m/s}) =$ 150 s (2.5 min).

52. We have a new diagram, as shown. From the diagram, we have
$\sin \theta = v_{wb}/v_{sw} = (0.80 \text{ m/s})/(1.00 \text{ m/s}) = 0.80$,
which gives $\theta = 53°$.



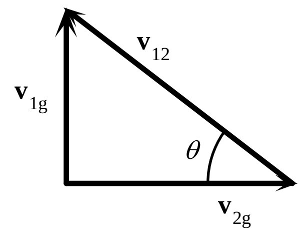
53. If $\mathbf{v}_{pw}$ is the velocity of the airplane with respect to the wind,
$\mathbf{v}_{pg}$ the velocity of the airplane with respect to the ground, and
$\mathbf{v}_{wg}$ the velocity of the wind with respect to the ground, then
$\mathbf{v}_{pg} = \mathbf{v}_{pw} + \mathbf{v}_{wg}$, as shown in the diagram.



We have two unknowns: $\mathbf{v}_{pg}$ and θ (or α).
If we use the law of sines for the vector triangle, we have
$v_{pw}/\sin (90° + \beta) = v_{wg}/\sin \alpha$, or
$\sin \alpha = (v_{wg}/v_{pw})\sin (90° + \beta)$
$= [(100 \text{ km/h})/(600 \text{ km/h})] \sin 125.0° = 0.137$, or $\alpha = 7.85°$.
Thus we have $\theta = \alpha + \beta = 7.85° + 35.0° =$ 42.9° N of E.

54. If $\mathbf{v}_{cg}$ is the velocity of the car with respect to the ground,
$\mathbf{v}_{mg}$ the velocity of the motorcycle with respect to the ground, and
$\mathbf{v}_{mc}$ the velocity of the motorcycle with respect to the car, then
$\mathbf{v}_{mc} = \mathbf{v}_{mg} - \mathbf{v}_{cg}$.
Because the motion is in one dimension, for the initial relative velocity we have
$v_{mc} = v_{mg} - v_{cg} = (90.0 \text{ km/h} - 75.0 \text{ km/h})/(3.6 \text{ ks/h}) = 4.17 \text{ m/s}$.
For the linear motion, in the reference frame of the car we have
$x = x_0 + v_0 t + !at^2$;
$60.0 \text{ m} = 0 + (4.17 \text{ m/s})(10.0 \text{ s}) + !a(10.0 \text{ s})^2$, which gives $a = 0.366 \text{ m/s}^2$.
Note that this is also the acceleration in the reference frame of the ground.

55. The velocities are shown in the diagram.



For the relative velocity of car 1 with respect to car 2, we have

$\mathbf{v}_{12} = \mathbf{v}_{1g} - \mathbf{v}_{2g}$.

For the magnitude we have

$v_{12} = (v_{1g}^2 + v_{2g}^2)^{1/2}$

$= [(30\ \text{km/h})^2 + (50\ \text{km/h})^2]^{1/2} =$ 58 km/h.

We find the angle from

$\tan\theta = v_{1g}/v_{2g} = (30\ \text{km/h})/(50\ \text{km/h}) = 0.60$, which gives $\theta = 31°$.

For the relative velocity of car 2 with respect to car 1, we have

$\mathbf{v}_{21} = \mathbf{v}_{2g} - \mathbf{v}_{1g} = -(\mathbf{v}_{1g} - \mathbf{v}_{2g}) = -\mathbf{v}_{12} =$ 58 km/h opposite to $\mathbf{v}_{12}$.

56. If $\mathbf{v}_{sg}$ is the velocity of the speeder with respect to the ground,

$\mathbf{v}_{pg}$ the velocity of the police car with respect to the ground, and

$\mathbf{v}_{ps}$ the velocity of the police car with respect to the speeder, then

$\mathbf{v}_{ps} = \mathbf{v}_{pg} - \mathbf{v}_{sg}$.

Because the motion is in one dimension, for the initial relative velocity we have

$v_{0ps} = v_{0pg} - v_{sg} = (90\ \text{km/h} - 140\ \text{km/h})/(3.6\ \text{ks/h}) = -13.9\ \text{m/s}$.

If we use a coordinate system in the reference frame of the speeder with the origin at the speeder, the distance that the police car travels before its acceleration is

$x_0 = v_{0ps}t_1 = (-13.9\ \text{m/s})(1.00\ \text{s}) = -13.9\ \text{m}$.

The negative sign indicates that the police car is falling behind.

For the linear motion of the police car after accelerating, in the reference frame of the car we have

$x = x_0 + v_{0ps}t_2 + !at_2^2$;

$0 = -13.9\ \text{m} + (-13.9\ \text{m/s})t_2 + !(2.00\ \text{m/s}^2)t_2^2$, which gives $t_2 = -0.94\ \text{s}, 14.8\ \text{s}$.

Because the police car starts accelerating at $t_2 = 0$, the physical answer is 14.8 s, so the total time is

$t = t_1 + t_2 = 1.00\ \text{s} + 14.8\ \text{s} =$ 15.8 s.

57. With an initial relative velocity of v_{0ps} in m/s, if we use a coordinate system with the origin at the speeder, the distance that the police car travels before its acceleration is

$x_0 = v_{0ps}t_1 = v_{0ps}\ (1.00\ \text{s})$.

For the 6.00 s of the linear motion of the police car after accelerating, in the reference frame of the car we have

$x = x_0 + v_0t_2 + !at_2^2$;

$0 = v_{ps}(1.00\ \text{s}) + v_{0ps}(6.00\ \text{s}) + !(2.00\ \text{m/s}^2)(6.00\ \text{s})^2$, which gives $v_{0ps} = -5.14\ \text{m/s}$.

Because the motion is in one dimension, for the initial relative velocity we have

$v_{0ps} = v_{0pg} - v_{sg}$;

$(-5.14\ \text{m/s})(3.6\ \text{ks/h}) = (90\ \text{km/h}) - v_{sg}$, which gives $v_{sg} = 109\ \text{km/h}$.

58. The arrow will hit the apple at the same elevation, so we can use the expression for the horizontal range:

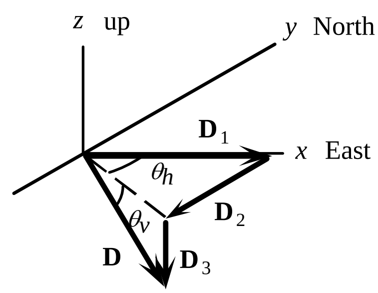
$R = v_0^2 \sin(2\theta_0)/g$;

$27\ \text{m} = (35\ \text{m/s})^2 \sin(2\theta_0)/(9.80\ \text{m/s}^2)$, which gives

$\sin(2\theta_0) = 0.216$, or $2\theta_0 = 12.4°$, $\theta_0 = 6.2°$.

59. (*a*) For the magnitude of the resultant to be equal to the sum of the magnitudes, the two vectors must be parallel.
(*b*) The expression is the one we use when we find the magnitude of a vector from its rectangular components. Thus the two vectors must be perpendicular.
(*c*) The only way to have the sum and difference of two magnitudes be equal is for $V_2 = -V_2$, or $V_2 = 0$. Only a zero vector has zero magnitude: $\mathbf{V}_2 = 0$.

60. The displacement is shown in the diagram.



For the components, we have

$D_x = 50$ m, $D_y = -25$ m, $D_z = -10$ m.

To find the magnitude we extend the process for two dimensions:

$D = (D_x^2 + D_y^2 + D_z^2)^{1/2}$

$= [(50\text{ m})^2 + (-25\text{ m})^2 + (-10\text{ m})^2]^{1/2} =$ 57 m.

The direction is specified by the two angles shown, which we find from

$\tan\theta_h = D_y/D_x = (25\text{ m})/(50\text{ m}) = 0.5,$

which gives $\theta_h = 27°$ from the x-axis toward the $-y$-axis;

$\tan\theta_v = D_z/(D_x^2 + D_y^2)^{1/2}$

$= (10\text{ m})/[(50\text{ m})^2 + (-25\text{ m})^2]^{1/2} = 0.178$, which gives $\theta_v = 10°$ below the horizontal.

61. If we assume constant acceleration along the slope, we have

$v = v_0 + at;$

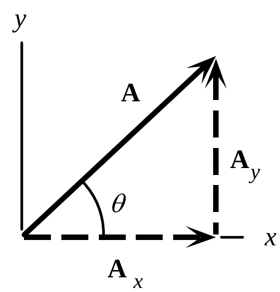
$0 = [(120\text{ km/h})/(3.6\text{ ks/h})] + a(12\text{ s})$, which gives $a = -2.78\text{ m/s}^2$ along the slope.

For the components we have

$a_{\text{horizontal}} = a\cos 30° = (-2.78\text{ m/s}^2)\cos 30° =$ -2.4 m/s^2 (opposite to the truck's motion).

$a_{\text{vertical}} = a\sin 30° = (-2.78\text{ m/s}^2)\sin 30° =$ -1.4 m/s^2 (down).

62. We see from the diagram that



$\cos\theta = A_x/A = (75.4)/(88.5) = 0.852$, or $\theta = \pm 31.6°$.

For the y-component, we have

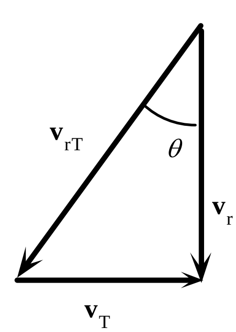
$A_y = A\sin\theta = (88.5)\sin(\pm 31.6°) =$ ± 46.3.

The second vector (below the x-axis) is not shown in the diagram.

63. The velocity of the rain with respect to the train is $\mathbf{v}_{rT} = \mathbf{v}_r - \mathbf{v}_T$, where

$\mathbf{v}_r$ is the velocity of the rain with respect to the ground and

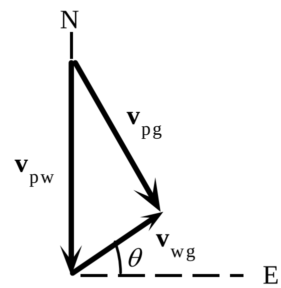
$\mathbf{v}_T$ is the velocity of the train with respect to the ground.



From the diagram we have

$\tan\theta = v_T/v_r$, or $v_r = v_T/\tan\theta$.

64. If $\mathbf{v}_{pw}$ is the velocity of the airplane with respect to the wind,
$\mathbf{v}_{pg}$ the velocity of the airplane with respect to the ground, and
$\mathbf{v}_{wg}$ the velocity of the wind with respect to the ground, then
$\mathbf{v}_{pg} = \mathbf{v}_{pw} + \mathbf{v}_{wg}$, as shown in the diagram.



Because the plane has covered 125 km in 1.00 hour, $v_{pg} = 125$ km/h.
We use the diagram to write the component equations:

$v_{wgE} = v_{pgE} = v_{pg} \sin 45° = (125 \text{ km/h}) \sin 45° = 88.4 \text{ km/h};$

$v_{wgN} = v_{pgN} - v_{pwN} = -v_{pg} \cos 45° - v_{pw}$
$= -(125 \text{ km/h}) \cos 45° - (-155 \text{ km/h}) = 66.6 \text{ km/h}.$

For the magnitude we have

$v_{wg} = (v_{wgE}^2 + v_{wgN}^2)^{1/2} = [(88.4 \text{ km/h})^2 + (66.6 \text{ km/h})^2]^{1/2} =$ 111 km/h.

We find the angle from

$\tan \theta = v_{wgN}/v_{wgE} = (66.6 \text{ km/h})/(88.4 \text{ km/h}) = 0.754$, which gives $\theta = 37.0°$ N of E.

65. If $\mathbf{v}_{ag}$ is the velocity of the automobile with respect to the ground,
$\mathbf{v}_{tg}$ the velocity of the train with respect to the ground, and
$\mathbf{v}_{at}$ the velocity of the automobile with respect to the train, then
$\mathbf{v}_{at} = \mathbf{v}_{ag} - \mathbf{v}_{tg}$.

Because the motion is in one dimension, for the initial relative velocity we have

$v_{at} = v_{ag} - v_{tg} = (95 \text{ km/h} - 75 \text{ km/h})/(3.6 \text{ ks/h}) = 5.56 \text{ m/s}.$

If we use a coordinate system in the reference frame of the train with the origin at the back of the train, we find the time to pass from

$x_1 = v_{at} t_1;$

$1000 \text{ m} = (5.56 \text{ m/s}) t_1$, which gives $t_1 = 1.8 \times 10^2$ s.

With respect to the ground, the automobile will have traveled

$x_{1g} = v_{ag} t_1 = [(95 \text{ km/h})/(3.6 \text{ ks/h})](1.8 \times 10^2 \text{ s}) = 4.8 \times 10^3 \text{ m} =$ 4.8 km.

If they move in opposite directions, we have

$v_{at} = v_{ag} - v_{tg} = [95 \text{ km/h} - (-75 \text{ km/h})]/(3.6 \text{ ks/h}) = 47.2 \text{ m/s}.$

We find the time to pass from

$x_2 = v_{at} t_2;$

$1000 \text{ m} = (47.2 \text{ m/s}) t_2$, which gives $t_2 = 21$ s.

With respect to the ground, the automobile will have traveled

$x_{2g} = v_{ag} t_2 = [(95 \text{ km/h})/(3.6 \text{ ks/h})](21 \text{ s}) = 5.6 \times 10^2 \text{ m} =$ 0.56 km.

66. We can find the time for the jump from the horizontal motion:

$x = v_x t; \quad t = x/v_x = (8.0 \text{ m})/(9.1 \text{ m/s}) =$ 0.88 s.

It takes half of this time for the jumper to reach the maximum height or to fall from the maximum height. If we consider the latter, we have

$y = y_0 + v_{0y} t + !a_y t^2$, or $0 = h_{max} + 0 + !(-9.80 \text{ m/s}^2)(0.44 \text{ s})^2$, which gives $h_{max} = 0.95$ m.

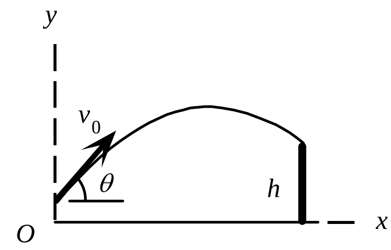
67. Because the golf ball returns to the same elevation, we can use the expression for the horizontal range. The horizontal range on earth is given by $R = v_0^2 \sin(2\theta_0)/g$, whereas on the moon it is

$R_{moon} = v_0^2 \sin(2\theta_0)/g_{moon}.$

Because we assume the same v_0 and θ_0, when we divide the two equations, we get

$R_{\text{moon}}/R = g/g_{\text{moon}}$, or
$g_{\text{moon}} = (R/R_{\text{moon}})g$ = [(30 m)/(180 m)](9.80 m/s^2) = 1.6 m/s^2.

68. We choose a coordinate system with the origin at home plate, x horizontal and y up, as shown in the diagram.



The minimum speed of the ball is that which will have the ball just clear the fence. The horizontal motion is

$x = v_{0x}t$;

$95 \text{ m} = v_0 \cos 40° \, t$, which gives $v_0 t = 124 \text{ m}$.

The vertical motion is

$y = y_0 + v_{0y}t + !a_y t^2$;

$12 \text{ m} = 1.0 \text{ m} + v_0 \sin 40° \, t + !(-9.80 \text{ m/s}^2)t^2$.

We can use the first equation to eliminate $v_0 t$ from the second and solve for t, which gives $t = 3.74$ s.

When this value is used in the first equation, we get $v_0 =$ 33 m/s.

69. We choose a coordinate system with the origin at the takeoff point, with x horizontal and y vertical, with the positive direction down.

We find the time for the diver to reach the water from the vertical motion:

$y = y_0 + v_{0y}t + !a_y t^2$;

$35 \text{ m} = 0 + 0 + !(9.80 \text{ m/s}^2)t^2$, which gives $t = 2.7$ s.

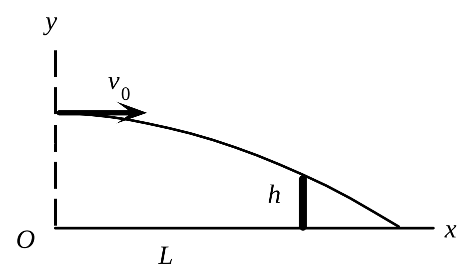
The horizontal motion will have constant velocity.

We find the minimum horizontal initial velocity needed to land beyond the rocky outcrop from

$x = x_0 + v_{0x}t$;

$5 \text{ m} = 0 + v_0(2.7 \text{ s})$, which gives $v_0 = 1.9$ m/s.

70. We use the coordinate system shown in the diagram.



We find the time for the ball to reach the net from the vertical motion:

$y = y_0 + v_{0y}t + !a_y t^2$;

$0.90 \text{ m} = 2.50 \text{ m} + 0 + !(-9.80 \text{ m/s}^2)t^2$, which gives $t = 0.571$ s.

We find the initial velocity from the horizontal motion:

$x = v_{0x}t$;

$15.0 \text{ m} = v_0 \, (0.571 \text{ s})$, which gives $v_0 = 26.3$ m/s.

We find the time for the ball to reach the ground from the vertical motion:

$y = y_0 + v_{0y}t_2 + !a_y t_2^2$;

$0 = 2.50 \text{ m} + 0 + !(-9.80 \text{ m/s}^2)t_2^2$, which gives $t_2 = 0.714$ s.

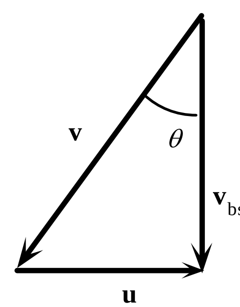
We find where it lands from the horizontal motion:

$x_2 = v_0 t_2 = (26.3 \text{ m/s})(0.714 \text{ s}) = 18.8 \text{ m}$.

Because this is 18.8 m – 15.0 m = 3.8 m beyond the net, which is less than 7.0 m, the serve is good.

71. If $\mathbf{v}_{ag}$ is the velocity of the automobile with respect to the ground,
$\mathbf{v}_{hg}$ the velocity of the helicopter with respect to the ground, and
$\mathbf{v}_{ha}$ the velocity of the helicopter with respect to the automobile, then
$\mathbf{v}_{ha} = \mathbf{v}_{hg} - \mathbf{v}_{ag}$.
For the horizontal relative velocity we have
$v_{ha} = v_{hg} - v_{ag}$ = (185 km/h – 145 km/h)/(3.6 ks/h) = 11.1 m/s.
This is the initial (horizontal) velocity of the explosive, so we can find the time of fall from
$y = y_0 + v_{0y}t + !a_y t^2$;
88.0 m = 0 + 0 + !(9.80 m/s²)t^2, which gives t = 4.24 s.
During this time, we find the horizontal distance the explosive travels with respect to the car from
$x = v_{ha}t$ = (11.1 m/s)(4.24 s) = 47.1 m.
Because the helicopter is always directly above the explosive, this is how far behind the automobile the helicopter must be when the explosive is dropped. Thus we find the angle from
$\tan \theta = y/x$ = (88.0 m)/(47.1 m) = 1.87, which gives θ = 61.8° below the horizontal.

72. (*a*) When the boat moves upstream the speed with respect to the bank is $v - u$. When the boat moves downstream the speed with respect to the bank is $v + u$. For each leg the distance traveled is $!D$, so the total time is
$t = [!D/(v - u)] + [!D/(v + u)] = !D(v + u + v - u)/(v - u)(v + u) =$ $Dv/(v^2 - u^2)$.
(*b*) To move directly across the river the boat must head at an angle θ, as shown in the diagram. From the diagram we see that the speed with respect to the shore is
$v_{bs} = (v^2 - u^2)^{1/2}$,
during both legs of the trip. Thus the total time is
$t = [!D/(v^2 - u^2)^{1/2}] + [!D/(v^2 - u^2)^{1/2}] =$ $D/(v^2 - u^2)^{1/2}$.
We must assume that $u < v$, otherwise the boat will be swept downstream and never make it across the river. This appears in our answers as a negative time in (*a*) and the square root of a negative number in (*b*).



73. We use the coordinate system shown in the diagram, with up positive. For the horizontal motion, we have
$x = v_{0x}t$;
$L = (v_0 \cos \theta)t$;
195 m = ($v_0 \cos \theta$)(7.6 s), which gives $v_0 \cos \theta$ = 25.7 m/s.
For the vertical motion, we have
$y = y_0 + v_{0y}t + !a_y t^2$;
$H = 0 + (v_0 \sin \theta)t + !(-g)t^2$;
155 m = ($v_0 \sin \theta$)(7.6 s) + + !(– 9.80 m/s²)(7.6 s)², which gives $v_0 \sin \theta$ = 57.6 m/s.
We can find the initial angle θ by dividing the two results:
$\tan \theta = (v_0 \sin \theta)/(v_0 \cos \theta)$ = (57.6 m/s)/(25.7 m/s) = 2.274, which gives θ = 66.0°.
Now we can use one of the previous results to find the initial velocity:
v_0 = (25.7 m/s)/cos θ = (25.7 m/s)/cos 66.0° = 63 m/s.
Thus the initial velocity is 63 m/s, 66° above the horizontal.