CHAPTER 26

1. (*a*) $[1 - (v/c)^2]^{1/2} = \{1 - [(20{,}000 \text{ m/s})/(3.00 \times 10^8 \text{ m/s})]^2\}^{1/2} =$ 1.00.
   (*b*) $[1 - (v/c)^2]^{1/2} = [1 - (0.0100)^2]^{1/2} =$ 0.99995.
   (*c*) $[1 - (v/c)^2]^{1/2} = [1 - (0.100)^2]^{1/2} =$ 0.995.
   (*d*) $[1 - (v/c)^2]^{1/2} = [1 - (0.900)^2]^{1/2} =$ 0.436.
   (*e*) $[1 - (v/c)^2]^{1/2} = [1 - (0.990)^2]^{1/2} =$ 0.141.
   (*f*) $[1 - (v/c)^2]^{1/2} = [1 - (0.999)^2]^{1/2} =$ 0.0447.

2. You measure the contracted length. We find the rest length from
   $L = L_0[1 - (v/c)^2]^{1/2}$;
   $48.2 \text{ m} = L_0[1 - (0.850)^2]^{1/2}$, which gives $L_0 =$ 91.5 m.

3. We find the lifetime at rest from
   $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$;
   $4.76 \times 10^{-6} \text{ s} = ?t_0/\{1 - [(2.70 \times 10^8 \text{ m/s})/(3.00 \times 10^8 \text{ m/s})]^2\}^{1/2}$, which gives $?t_0 =$ 2.07×10^{-6} s.

4. You measure the contracted length:
   $L = L_0[1 - (v/c)^2]^{1/2}$
   $= (100 \text{ ly})\{1 - [(2.60 \times 10^8 \text{ m/s})/(3.00 \times 10^8 \text{ m/s})]^2\}^{1/2} =$ 49.9 ly.

5. The rest length of his car is 6.00 m. For his car you measure the contracted length:
   $L_1 = L_{01}[1 - (v/c)^2]^{1/2}$
   $= (6.00 \text{ m})[1 - (0.37)^2]^{1/2} =$ 5.57 m.
   He measured the contracted length of your car. We find the rest length from
   $L_2 = L_{02}[1 - (v/c)^2]^{1/2}$;
   $6.21 \text{ m} = L_{02}[1 - (0.37)^2]^{1/2}$, which gives $L_{02} =$ 6.68 m.

6. We determine the speed from the time dilation:
   $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$;
   $4.10 \times 10^{-8} \text{ s} = (2.60 \times 10^{-8} \text{ s})/[1 - (v/c)^2]^{1/2}$, which gives $v =$ $0.793c$.

7. We determine the speed from the length contraction:
   $L = L_0[1 - (v/c)^2]^{1/2}$;
   $25 \text{ ly} = (90 \text{ ly})[1 - (v/c)^2]^{1/2}$, which gives $v =$ $0.96c$.

8. For a 1.00 per cent change, the factor in the expressions for time dilation and length contraction must equal $1 - 0.0100 = 0.9900$:
   $[1 - (v/c)^2]^{1/2} = 0.9900$, which gives $v =$ $0.141c$.

9. In the Earth frame, the clock on the Enterprise will run slower.
   (*a*) We find the elapsed time on the ship from
   $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$;
   $5.0 \text{ yr} = ?t_0/[1 - (0.89)^2]^{1/2}$, which gives $?t_0 =$ 2.3 yr.
   (*b*) We find the elapsed time on the Earth from
   $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$
   $= (5.0 \text{ yr})/[1 - (0.89)^2]^{1/2} =$ 11 yr.

10. (*a*) To an observer on Earth, 75.0 ly is the rest length, so the time will be
    $t_{Earth} = L_0/v = (75.0 \text{ ly})/0.950c =$ 78.9 yr.
    (*b*) We find the dilated time on the spacecraft from
    $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$;
    $78.9 \text{ yr} = ?t_0/[1 - (0.950)^2]^{1/2}$, which gives $?t_0 =$ 24.6 yr.
    (*c*) To the spacecraft observer, the distance to the star is contracted:
    $L = L_0[1 - (v/c)^2]^{1/2} = (75.0 \text{ ly})[1 - (0.950)^2]^{1/2} =$ 23.4 ly.
    (*d*) To the spacecraft observer, the speed of the spacecraft is
    $v = L/?t = (23.4 \text{ ly})/24.6 \text{ yr} =$ $0.950c$, as expected.

11. (*a*) You measure the contracted length. We find the rest length from
    $L = L_0[1 - (v/c)^2]^{1/2}$;
    $5.80 \text{ m} = L_0[1 - (0.580)^2]^{1/2}$, which gives $L_0 =$ 7.12 m.
    Distances perpendicular to the motion do not change, so the rest height is 1.20 m.
    (*b*) We find the dilated time in the sports vehicle from
    $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$;
    $20.0 \text{ s} = ?t_0/[1 - (0.580)^2]^{1/2}$, which gives $?t_0 =$ 16.3 s.
    (*c*) To your friend, you moved at the same relative speed: $0.580c$.
    (*d*) She would measure the same time dilation: 16.3 s.

12. In the Earth frame, the average lifetime of the pion will be dilated:
    $?t = ?t_0/[1 - (v^2/c^2)]^{1/2}$.
    The speed as a fraction of the speed of light is
    $v/c = d/c\ ?t = d[1 - (v^2/c^2)]^{1/2}/c\ ?t_0$;
    $v/c = (10.0 \text{ m})[1 - (v^2/c^2)]^{1/2}/(3.00 \times 10^8 \text{ m/s})(2.60 \times 10^{-8} \text{ s})$,
    which gives $v =$ $0.789c = 2.37 \times 10^8 \text{ m/s}$.

13. The mass of the proton is
    $m = m_0/[1 - (v^2/c^2)]^{1/2} = (1.67 \times 10^{-27} \text{ kg})/[1 - (0.90)^2]^{1/2} =$ 3.8×10^{-27} kg.

14. We find the speed from
    $m = m_0/[1 - (v^2/c^2)]^{1/2}$;
    $2m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.866c$.

15. We find the speed from
    $m = m_0/[1 - (v^2/c^2)]^{1/2}$;
    $1.10m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.417c$.

16. We convert the speed: (40,000 km/h)/(3.6 ks/h) = 1.11×10^4 m/s. Because this is much smaller than c, we can simplify the factor in the mass equation:

 $1/[1 - (v^2/c^2)]^{1/2} \approx 1/[1 - !(v/c)^2] \approx 1 + !(v/c)^2$.

 For the fractional change in mass, we have

 $(m - m_0)/m_0 = \{1/[1 - (v^2/c^2)]^{1/2}\} - 1 \approx 1 + !(v/c)^2 - 1 = !(v/c)^2$;

 $(m - m_0)/m_0 = ![(1.11 \times 10^4 \text{ m/s})/(3.00 \times 10^8 \text{ m/s})]^2 = 6.8 \times 10^{-10} =$ $6.8 \times 10^{-8}\%$.

17. (*a*) We find the speed from

 $m = m_0/[1 - (v^2/c^2)]^{1/2}$;

 $10{,}000 m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives

 $[1 - (v^2/c^2)]^{1/2} = 1.00 \times 10^{-4}$, or $(v/c)^2 = 1 - 1.00 \times 10^{-8}$.

 When we take the square root, we get

 $v/c = (1 - 1.00 \times 10^{-8})^{1/2} \approx 1 - !(1.00 \times 10^{-8}) = 1 - 0.50 \times 10^{-8}$.

 Thus the speed is 1.5 m/s less than c.

 (*b*) The contracted length of the tube is

 $L = L_0[1 - (v/c)^2]^{1/2} = (3.0 \text{ km})(1.00 \times 10^{-4}) = 3.0 \times 10^{-4} \text{ km} =$ 30 cm.

18. The kinetic energy is

 $\text{KE} = (m - m_0)c^2 = (3 - 1)m_0c^2 = 2m_0c^2 = 2(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 =$ 1.6×10^{-13} J (1.0 MeV).

19. We find the increase in mass from

 $?m = ?E/c^2 = (4.82 \times 10^4 \text{ J})/(3.00 \times 10^8 \text{ m/s})^2 =$ 5.36×10^{-13} kg.

 Note that this is so small, most chemical reactions are considered to have mass conserved.

20. We find the loss in mass from

 $?m = ?E/c^2 = (200 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})/(3.00 \times 10^8 \text{ m/s})^2 =$ 3.56×10^{-28} kg.

21. The rest energy of the electron is

 $E = m_0c^2 = (9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 =$ 8.20×10^{-14} J

 $= (8.20 \times 10^{-14} \text{ J})/(1.60 \times 10^{-13} \text{ J/MeV}) =$ 0.511 MeV.

22. The rest mass of the proton is

 $m_0 = E/c^2 = (1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2/(1.60 \times 10^{-13} \text{ J/MeV})c^2 =$ 939 MeV/c^2.

23. We find the necessary mass conversion from

 $?m = ?E/c^2 = (8 \times 10^{19} \text{ J})/(3.00 \times 10^8 \text{ m/s})^2 =$ 9×10^2 kg.

24. We find the energy equivalent of the mass from

 $E = mc^2 = (1.0 \times 10^{-3} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 =$ 9.0×10^{13} J.

 If this energy increases the gravitational energy, we have

 $E = mgh$;

 $9.0 \times 10^{13} \text{ J} = m(9.80 \text{ m/s}^2)(100 \text{ m})$, which gives $m =$ 9.2×10^{10} kg.

25. If the kinetic energy is equal to the rest energy, we have

$\text{KE} = (m - m_0)c^2 = m_0c^2$, or $m = 2m_0$.

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2}$;

$2m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives $v = 0.866c$.

26. (*a*) We find the work required from

$W = \Delta\text{KE} = (m - m_0)c^2 = m_0c^2(\{1/[1 - (v/c)^2]^{1/2}\} - 1)$

$= (939\text{ MeV})(\{1/[1 - (0.998)^2]^{1/2}\} - 1) = 13.9 \times 10^3\text{ MeV} =$ 13.9 GeV (2.23×10^{-9} J).

(*b*) The momentum of the proton is

$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$

$= (1.67 \times 10^{-27}\text{ kg})(0.998)(3.00 \times 10^8\text{ m/s})/[1 - (0.998)^2]^{1/2} =$ 7.91×10^{-18} kg · m/s.

27. (*a*) The radiation falls on a circle with the Earth's radius. We find the increase in mass from

$?m = ?E/c^2 = (1400\text{ W/m}^2)\text{p}(6.38 \times 10^6\text{ m})^2(3.16 \times 10^7\text{ s})/(3.00 \times 10^8\text{ m/s})^2 =$ 6.3×10^7 kg.

Note that there will be mass loss from re-radiation into space.

(*b*) If the Sun radiates uniformly, the rate reaching the Earth is the rate through a sphere with a radius equal to the distance from the Sun to the Earth. We find the loss in mass from

$?m = ?E/c^2 = (1400\text{ W/m}^2)4\text{p}(1.50 \times 10^{11}\text{ m})^2(3.16 \times 10^7\text{ s})/(3.00 \times 10^8\text{ m/s})^2 =$ 1.4×10^{17} kg.

28. The speed of the proton is

$v = (2.50 \times 10^8\text{ m/s})/(3.00 \times 10^8\text{ m/s}) = 0.833c$.

The kinetic energy is

$\text{KE} = (m - m_0)c^2 = m_0c^2(\{1/[1 - (v/c)^2]^{1/2}\} - 1)$

$= (939\text{ MeV})(\{1/[1 - (0.833)^2]^{1/2}\} - 1) =$ 760 MeV (1.22×10^{-10} J).

The momentum of the proton is

$p = mv = m_0v\{1/[1 - (v/c)^2]^{1/2}\}$

$= (1.67 \times 10^{-27}\text{ kg})(2.50 \times 10^8\text{ m/s})\{1/[1 - (0.833)^2]^{1/2}\} =$ 7.55×10^{-19} kg · m/s.

29. The total energy of the proton is

$E = \text{KE} + m_0c^2 = 750\text{ MeV} + 939\text{ MeV} = 1689\text{ MeV}$.

The relation between the momentum and energy is

$(pc)^2 = E^2 - (m_0c^2)^2$;

$p^2(3.00 \times 10^8\text{ m/s})^2 = [(1689\text{ MeV})^2 - (939\text{ MeV})^2](1.60 \times 10^{-13}\text{ J/MeV})^2$,

which gives $p =$ 7.49×10^{-19} kg · m/s.

30. The kinetic energy acquired by the proton is

$\text{KE} = qV = (1\text{ e})(75\text{ MV}) = 75\text{ MeV}$.

The mass of the proton is

$m = m_0 + \text{KE}/c^2 = 939\text{ MeV}/c^2 + (75\text{ MeV})/c^2 = 1014\text{ MeV}/c^2$.

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2}$;

$1014\text{ MeV}/c^2 = 939\text{ MeV}/c^2/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.377c$.

31. The mass of the electron is

$m = m_0 + \mathrm{KE}/c^2 = 0.511\ \mathrm{MeV}/c^2 + (1.00\ \mathrm{MeV})/c^2 = 1.51\ \mathrm{MeV}/c^2.$

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2};$

$1.51\ \mathrm{MeV}/c^2 = 0.511\ \mathrm{MeV}/c^2/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.941c$.

32. The kinetic energy acquired by the electron is

$\mathrm{KE} = qV = (1\ \mathrm{e})(0.025\ \mathrm{MV}) = 0.025\ \mathrm{MeV}.$

The mass of the electron is

$m = m_0 + \mathrm{KE}/c^2 = 0.511\ \mathrm{MeV}/c^2 + (0.025\ \mathrm{MeV})/c^2 = 0.536\ \mathrm{MeV}/c^2 =$ $1.05m_0$.

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2};$

$0.536\ \mathrm{MeV}/c^2 = (0.511\ \mathrm{MeV}/c^2)/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.302c$.

33. If M_0 is the rest mass of the new particle, for conservation of energy we have

$2mc^2 = 2m_0c^2/[1 - (v^2/c^2)]^{1/2} = M_0c^2$, which gives $M_0 =$ $2m_0/[1 - (v^2/c^2)]^{1/2}$.

Because energy is conserved, there was no loss.

The final particle is at rest, so the kinetic energy loss is the initial kinetic energy of the two colliding particles:

$\mathrm{KE}_{\mathrm{loss}} = 2(m - m_0)c^2 =$ $2m_0c^2\left(\{1/[1 - (v^2/c^2)]^{1/2}\} - 1\right)$.

34. The total energy of the proton is

$E = mc^2 = \mathrm{KE} + m_0c^2 = !mc^2 + m_0c^2$, which gives $m = 2m_0 = 2(1.67 \times 10^{-27}\ \mathrm{kg}) =$ $3.34 \times 10^{-27}\ \mathrm{kg}$.

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2};$

$2m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives $v =$ $0.866c$.

35. The total energy of the electron is

$E = mc^2 = \mathrm{KE} + m_0c^2 = m_0c^2 + m_0c^2 = 2m_0c^2$, which gives $m = 2m_0$.

We find the speed from

$m = m_0/[1 - (v^2/c^2)]^{1/2};$

$2m_0 = m_0/[1 - (v^2/c^2)]^{1/2}$, which gives $[1 - (v^2/c^2)]^{1/2} = !$, so $v =$ $0.866c$.

The momentum of the electron is

$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$

$= (9.11 \times 10^{-31}\ \mathrm{kg})(0.866)(3.00 \times 10^8\ \mathrm{m/s})/(!) =$ $4.73 \times 10^{-22}\ \mathrm{kg \cdot m/s}$.

36. (*a*) The kinetic energy is

$\mathrm{KE} = (m - m_0)c^2 = m_0c^2\left(\{1/[1 - (v/c)^2]^{1/2}\} - 1\right)$

$= (37{,}000\ \mathrm{kg})(3.00 \times 10^8\ \mathrm{m/s})^2\left(\{1/[1 - (0.21)^2]^{1/2}\} - 1\right) =$ $7.6 \times 10^{19}\ \mathrm{J}$.

(*b*) When we use the classical expression, we get

$\mathrm{KE_c} = !mv^2 = !(37{,}000\ \mathrm{kg})[(0.21)(3.00 \times 10^8\ \mathrm{m/s})]^2 = 7.34 \times 10^{19}\ \mathrm{J}.$

The error is

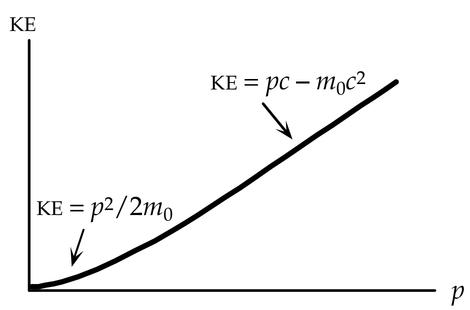
$(7.34 - 7.6)/(7.6) = -0.04 =$ -4%.

37. The speed of the proton is
$v = (9.8 \times 10^7\ \text{m/s})/(3.00 \times 10^8\ \text{m/s}) = 0.327c$.
The kinetic energy is
$\text{KE} = (m - m_0)c^2 = m_0c^2\big(\{1/[1 - (v/c)^2]^{1/2}\} - 1\big)$
$= (939\ \text{MeV})\big(\{1/[1 - (0.327)^2]^{1/2}\} - 1\big) =$ 55 MeV (8.7×10^{-12} J).
The momentum of the proton is
$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$
$= (1.67 \times 10^{-27}\ \text{kg})(9.8 \times 10^7\ \text{m/s})\{1/[1 - (0.327)^2]^{1/2}\} =$ $1.7 \times 10^{-19}\ \text{kg}\cdot\text{m/s}$.
From the classical expressions, we get
$\text{KE}_c = !mv^2 = !(1.67 \times 10^{-27}\ \text{kg})(9.8 \times 10^7\ \text{m/s})^2 = 8.02 \times 10^{-12}$ J, with an error of
$(8.0 - 8.7)/(8.7) = -0.08 =$ – 8%.
$p = mv = (1.67 \times 10^{-27}\ \text{kg})(9.8 \times 10^7\ \text{m/s}) = 1.6 \times 10^{-19}\ \text{kg}\cdot\text{m/s}$, with an error of
$(1.6 - 1.7)/(1.7) = -0.06 =$ – 6%.

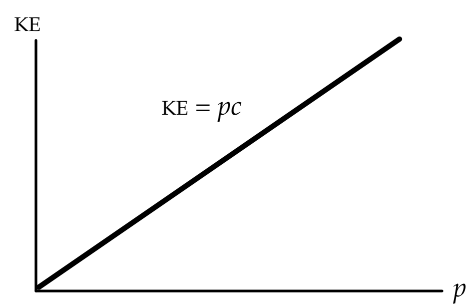
38. If we ignore the recoil of the neptunium nucleus, the increase in kinetic energy is the kinetic energy of the alpha particle;
$\text{KE}_\alpha = [m_{\text{Am}} - (m_{\text{Np}} + m_\alpha)]c^2$;
$5.5\ \text{MeV} = [241.05682\ \text{u} - (m_{\text{Np}} + 4.00260\ \text{u})]c^2(931.5\ \text{MeV/u}c^2)$, which gives $m_{\text{Np}} =$ 237.04832 u.

39. The increase in kinetic energy comes from the decrease in potential energy:
$\text{KE} = (m - m_0)c^2 = m_0c^2\big(\{1/[1 - (v/c)^2]^{1/2}\} - 1\big)$;
$7.60 \times 10^{-14}\ \text{J} = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2\big(\{1/[1 - (v/c)^2]^{1/2}\} - 1\big)$, which gives $v =$ $0.855c$.

40. (*a*)



(*b*)



41. The total energy of the proton is
$E = mc^2 = \text{KE} + m_0c^2 = 900\ \text{GeV} + 0.938\ \text{GeV} = 901\ \text{GeV}$, so the mass is $901\ \text{GeV}/c^2$.
We find the speed from
$m = m_0/[1 - (v^2/c^2)]^{1/2}$;
$901\ \text{GeV}/c^2 = (0.938\ \text{GeV}/c^2)/[1 - (v^2/c^2)]^{1/2}$, which gives $[1 - (v^2/c^2)]^{1/2} = 1.04 \times 10^{-3}$, so $v = 1.00c$.
The magnetic force provides the radial acceleration:
$qvB = mv^2/r$, or
$B = mv/qr = m_0v/qr[1 - (v^2/c^2)]^{1/2}$
$= (1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})/(1.6 \times 10^{-19}\ \text{C})(1.0 \times 10^3\ \text{m})(1.04 \times 10^{-3}) =$ 3.0 T.
Note that the mass is constant during the revolution.

42. Because the total energy of the muons becomes electromagnetic energy, we have

$E = m_1c^2 + m_2c^2 = m_0/[1 - (v_1^2/c^2)]^{1/2} + m_0/[1 - (v_2^2/c^2)]^{1/2}$
$= (105.7\ \text{MeV}/c^2)(c^2)/[1 - (0.33)^2]^{1/2} + (105.7\ \text{MeV}/c^2)(c^2)/[1 - (0.50)^2]^{1/2} =$ 234 MeV.

43. The magnetic force provides the radial acceleration:
$qvB = mv^2/r$, or
$m = qBr/v = E/c^2$.
With $v \approx c$, and $q = 1$ e, we get
$E\ (\text{eV}) = (1)Brc^2/c = Brc$.
Note that the mass is constant during the revolution.

44. If we use the mass-speed relation,
$m = m_0/[1 - (v^2/c^2)]^{1/2}$,
and solve for the speed, we get
$v = c[1 - (m_0^2/m^2)]^{1/2}$.
Thus for the momentum we get
$p = mv = mc[1 - (m_0^2/m^2)]^{1/2} = [(mc)^2 - (m_0c)^2]^{1/2} = [(mc^2)^2 - (m_0c^2)^2]^{1/2}/c$.
When we use $\text{KE} = mc^2 - m_0c^2$, we get
$p = [(\text{KE} + m_0c^2)^2 - (m_0c^2)^2]^{1/2}/c = [(\text{KE})^2 + 2(\text{KE})m_0c^2]^{1/2}/c$.

45. We find the speed in the frame of the Earth from
$u = (v + u')/(1 + vu'/c^2) = (0.50c + 0.50c)/[1 + (0.50)(0.50)] =$ $0.80c$.

46. (*a*) In the reference frame of the second spaceship, the Earth is moving at $0.50c$, and the first spaceship is moving at $0.50c$ relative to the Earth. Thus the speed of the first spaceship relative to the second is
$u = (v + u')/(1 + vu'/c^2) = (0.50c + 0.50c)/[1 + (0.50)(0.50)] =$ $0.80c$.
(*b*) In the reference frame of the first spaceship, the Earth is moving at $-0.50c$, and the second spaceship is moving at $-0.50c$ relative to the Earth. Thus the speed of the second spaceship relative to the first is
$u = (v + u')/(1 + vu'/c^2) = [-0.50c + (-0.50c)]/[1 + (-0.50)(-0.50)] =$ $-0.80c$, as expected.

47. We take the positive direction in the direction of the *Enterprise*. In the reference frame of the alien vessel, the Earth is moving at $-0.60c$, and the *Enterprise* is moving at $+0.90c$ relative to the Earth. Thus the speed of the *Enterprise* relative to the alien vessel is
$u = (v + u')/(1 + vu'/c^2) = (-0.60c + 0.90c)/[1 + (-0.60)(+0.90)] =$ $0.65c$.
Note that the relative speed of the two vessels as seen on Earth is $0.90c - 0.60c = 0.30c$.

48. We take the positive direction in the direction of the first spaceship.
(*a*) In the reference frame of the Earth, the first spaceship is moving at $+0.65c$, and the second spaceship is moving at $+0.91c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is
$u = (v + u')/(1 + vu'/c^2) = (+0.65c + 0.91c)/[1 + (0.65)(0.91)] =$ $0.98c$.
(*b*) In the reference frame of the Earth, the first spaceship is moving at $+0.65c$, and the second spaceship is moving at $-0.91c$ relative to the first. Thus the speed of the second spaceship relative to the Earth is
$u = (v + u')/(1 + vu'/c^2) = [+0.65c + (-0.91c)]/[1 + (0.65)(-0.91)] =$ $-0.64c$.

49. The electrostatic force provides the radial acceleration:
    $ke^2/r^2 = mv^2/r$.
    Thus we find the speed from
    $v^2 = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.6 \times 10^{-19}\ \mathrm{C})^2/(9.11 \times 10^{-31}\ \mathrm{kg})(0.5 \times 10^{-10}\ \mathrm{m})$,
    which gives $v = 2 \times 10^6$ m/s.
    Because this is less than $0.1c$, the electron is not relativistic.

50. Because the North Pole is has no tangential velocity, the clock there will measure a year (3.16×10^7 s).
    The clock at the equator has the tangential velocity of the equator:
    $v = r\omega = (6.38 \times 10^6\ \mathrm{m})(2p\ \mathrm{rad})/(24\ \mathrm{h})(3600\ \mathrm{s/h}) = 464\ \mathrm{m/s}$.
    The clock at the equator will run slow:
    $t_{\mathrm{equator}} = t_{\mathrm{North}}[1 - (v^2/c^2)]^{1/2} \approx t_{\mathrm{North}}[1 - !(v/c)^2]$.
    Thus the difference in times is
    $t_{\mathrm{North}} - t_{\mathrm{equator}} = t_{\mathrm{North}}!(v/c)^2 = (3.16 \times 10^7\ \mathrm{s})![(464\ \mathrm{m/s})/(3.00 \times 10^8\ \mathrm{m/s})]^2 =$ 3.8×10^{-5} s.

51. (*a*) To travelers on the spacecraft, the distance to the star is contracted:
    $L = L_0[1 - (v/c)^2]^{1/2} = (4.3\ \mathrm{ly})[1 - (v/c)^2]^{1/2}$.
    Because the star is moving toward the spacecraft, to cover this distance in 4.0 yr, the speed of the star must be
    $v = L/t = (4.3\ \mathrm{ly}/4.0\ \mathrm{yr})[1 - (v/c)^2]^{1/2} = (1.075c)[1 - (v/c)^2]^{1/2}$, which gives $v = 0.73c$.
    Thus relative to the Earth-star system, the speed of the spacecraft is $0.73c$.
    (*b*) According to observers on Earth, clocks on the spacecraft run slow:
    $t_{\mathrm{Earth}} = t/[1 - (v^2/c^2)]^{1/2} = (4.0\ \mathrm{yr})/[1 - (0.73)^2]^{1/2} =$ 5.9 yr.
    Note that this agrees with the time found from distance and speed:
    $t_{\mathrm{Earth}} = L_0/v = (4.3\ \mathrm{ly})/(0.73\mathrm{c}) = 5.9$ yr.

52. The dependence of the mass on the speed is
    $m = m_0/[1 - (v^2/c^2)]^{1/2}$.
    If we consider a box with sides x_0, y_0, and z_0, dimensions perpendicular to the motion, which we take to be the x-axis, do not change, but the length in the direction of motion will contract:
    $x = x_0[1 - (v/c)^2]^{1/2}$.
    Thus the density is
    $\rho = m/xyz = m_0/[1 - (v^2/c^2)]^{1/2}x_0[1 - (v^2/c^2)]^{1/2}y_0z_0 =$ $\rho_0/[1 - (v^2/c^2)]$.

53. We convert the speed: (1500 km/h)/(3.6 ks/h) = 417 m/s.
    The flight time as observed on Earth is
    $t_{\mathrm{Earth}} = 2pr/v = 2p(6.38 \times 10^6\ \mathrm{m})/(417\ \mathrm{m/s}) = 9.62 \times 10^4\ \mathrm{s}$.
    The clock on the plane will run slow:
    $t_{\mathrm{plane}} = t_{\mathrm{Earth}}[1 - (v^2/c^2)]^{1/2} \approx t_{\mathrm{Earth}}[1 - !(v/c)^2]$.
    Thus the difference in times is
    $t_{\mathrm{Earth}} - t_{\mathrm{plane}} = t_{\mathrm{Earth}}!(v/c)^2 = (9.62 \times 10^4\ \mathrm{s})![(417\ \mathrm{m/s})/(3.00 \times 10^8\ \mathrm{m/s})]^2 =$ 9.3×10^{-8} s.

54. We find the mass change from the required energy:
    $E = Pt = m_0c^2$;
    $(100\ \mathrm{W})(3.16 \times 10^7\ \mathrm{s}) = m_0(3.00 \times 10^8\ \mathrm{m/s})^2$, which gives $m_0 =$ 3.5×10^{-8} kg.

55. The minimum energy is required to produce the pair at rest:
$E = 2m_0c^2 = 2(0.511\text{ MeV}) =$ 1.02 MeV (1.64×10^{13} J).

56. (*a*) Because the spring is at rest on the spaceship, its period is
$T= 2p(m/k)^{1/2} = 2p[(1.68\text{ kg})/(48.7\text{ N/m})]^{1/2} =$ 1.17 s.
(*b*) The oscillating mass is a clock. According to observers on Earth, clocks on the spacecraft run slow:
$T_{\text{Earth}} = T/[1 - (v^2/c^2)]^{1/2} = (1.17\text{ s})/[1 - (0.900)^2]^{1/2} =$ 2.68 s.

57. The magnetic force provides the radial acceleration:
$qvB = mv^2/r$, or
$r = mv/qB = m_0v/qB[1 - (v^2/c^2)]^{1/2}$
$= (9.11 \times 10^{-31}\text{ kg})(0.92)(3.00 \times 10^8\text{ m/s})/(1.6 \times 10^{-19}\text{ C})(1.8\text{ T})[1 - (0.92)^2]^{1/2}$
$= 2.2 \times 10^{-3}\text{ m} =$ 2.2 mm.

58. The kinetic energy comes from the decrease in rest mass:
$\text{KE} = [m_n - (m_p + m_e + m_\nu)]c^2$
$= [1.008665\text{ u} - (1.00728\text{ u} + 0.000549\text{ u} + 0)]c^2(931.5\text{ MeV/u}c^2) =$ 0.78 MeV.

59. (*a*) We find the rate of mass loss from
$\Delta m/\Delta t = (\Delta E/\Delta t)/c^2$
$= (4 \times 10^{26}\text{ W})/(3 \times 10^8\text{ m/s})^2 =$ 4.4×10^9 kg/s.
(*b*) We find the time from
$?t = ?m/\text{rate} = (5.98 \times 10^{24}\text{ kg})/(4.4 \times 10^9\text{ kg/s})(3.16 \times 10^7\text{ s/yr}) =$ 4.3×10^7 yr.
(*c*) We find the time for the Sun to lose all of its mass at this rate from
$?t = ?m/\text{rate} = (2.0 \times 10^{30}\text{ kg})/(4.4 \times 10^9\text{ kg/s})(3.16 \times 10^7\text{ s/yr}) =$ 1.4×10^{13} yr.

60. The speed of the particle is
$v = (2.24 \times 10^7\text{ m/s})/(3.00 \times 10^8\text{ m/s}) = 0.747c$.
We use the momentum to find the rest mass:
$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$;
$3.07 \times 10^{-22}\text{ kg}\cdot\text{m/s} = m_0(0.747)(3.00 \times 10^8\text{ m/s})/[1 - (0.747)^2]^{1/2}$,
which gives $m_0 = 9.11 \times 10^{-31}$ kg.
Because the particle has a negative charge, it is an electron.

61. The binding energy is the energy required to provide the increase in rest mass:
$\text{KE} = [(2m_p + 2m_e) - m_{\text{He}}]c^2$
$= [2(1.00783\text{ u}) + 2(1.00867\text{ u}) - 4.00260\text{ u}]c^2(931.5\text{ MeV/u}c^2) =$ 28.3 MeV.

62. We convert the speed: (110 km/h)/(3.6 ks/h) = 30.6 m/s.
Because this is much smaller than c, the mass of the car is
$m = m_0/[1 - (v^2/c^2)]^{1/2} \approx m_0[1 + !(v/c)^2]$.
The fractional change in mass is
$(m - m_0)/m_0 = [1 + !(v/c)^2] - 1 = !(v/c)^2$
$= ![(30.6\text{ m/s})/(3.00 \times 10^8\text{ m/s})]^2 = 5.19 \times 10^{-15} =$ 5.19×10^{-13} %.

63. (*a*) The magnitudes of the momenta are equal:

$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$

$= (1.67 \times 10^{-27}\ \text{kg})(0.935)(3.00 \times 10^8\ \text{m/s})/[1 - (0.935)^2]^{1/2} =$ $1.32 \times 10^{-18}\ \text{kg} \cdot \text{m/s}$.

(*b*) Because the protons are moving in opposite directions, the sum of the momenta is 0.

(*c*) In the reference frame of one proton, the laboratory is moving at $0.935c$. The other proton is moving at $+ 0.935c$ relative to the laboratory. Thus the speed of the other proton relative to the first is

$u = (v + u')/(1 + vu'/c^2) = [+ 0.935c + (+ 0.935c)]/[1 + (+ 0.935)(+ 0.935)] = 0.998c$.

The magnitude of the momentum of the other proton is

$p = mv = m_0v/[1 - (v/c)^2]^{1/2}$

$= (1.67 \times 10^{-27}\ \text{kg})(0.998)(3.00 \times 10^8\ \text{m/s})/[1 - (0.998)^2]^{1/2} =$ $7.45 \times 10^{-18}\ \text{kg} \cdot \text{m/s}$.

64. The neutrino has no rest mass, so we have

$E_\nu = (p_\nu^2c^2 + m_\nu^2c^4)^{1/2} = p_\nu c$.

Because the pi meson decays at rest, momentum conservation tells us that the muon and neutrino have equal and opposite momenta:

$p_\mu = p_\nu = p$.

For energy conservation, we have

$E_\pi = E_\mu + E_\nu$;

$m_\pi c^2 = (p_\mu^2c^2 + m_\mu^2c^4)^{1/2} + p_\nu c = (p^2c^2 + m_\mu^2c^4)^{1/2} + pc$.

If we rearrange and square, we get

$(m_\pi c^2 - pc)^2 = m_\pi^2c^4 - 2m_\pi c^2pc + p^2c^2 = p^2c^2 + m_\mu^2c^4$, or $pc = (m_\pi^2c^2 - m_\mu^2c^2)/2m_\pi$.

The kinetic energy of the muon is

$\text{KE}_\mu = E_\mu - m_\mu c^2 = (m_\pi c^2 - pc) - m_\mu c^2 = m_\pi c^2 - m_\mu c^2 - (m_\pi^2c^2 - m_\mu^2c^2)/2m_\pi$

$= (2m_\pi^2 - 2m_\mu m_\pi - m_\pi^2 + m_\mu^2)c^2/2m_\pi = (m_\pi^2 - 2m_\mu m_\pi + m_\mu^2)c^2/2m_\pi = (m_\pi - m_\mu)^2c^2/2m_\pi$.

65. To an observer in the barn reference frame, if the boy runs fast enough, the measured contracted length will be less than 12.0 m, so the observer can say that the two ends of the pole were simultaneously inside the barn. We find the necessary speed from

$L = L_0[1 - (v/c)^2]^{1/2}$;

$12.0\ \text{m} = (15.0\ \text{m})[1 - (v/c)^2]^{1/2}$, which gives $v = 0.60c$.

To the boy, the barn is moving and thus the length of the barn as he would measure it is less than the length of the pole:

$L = L_0[1 - (v/c)^2]^{1/2} = (12.0\ \text{m})[1 - (0.60)^2]^{1/2} = 7.68\ \text{m}$.

However, simultaneity is relative. Thus when the two ends are simultaneously inside the barn to the barn observer, those two events are not simultaneous to the boy. Thus he would claim that the observer in the barn determined that the ends of the pole were inside the barn at different times, which is also what the boy would say. It is not possible in the boy's frame to have both ends of the pole inside the barn simultaneously.

66. The relation between energy and momentum is

$E = (m_0^2c^4 + p^2c^2)^{1/2} = c(m_0^2c^2 + p^2)^{1/2}$.

For the momentum, we have

$p = mv = Ev/c^2$, or

$v = pc^2/E = pc/(m_0^2c^2 + p^2)^{1/2}$.