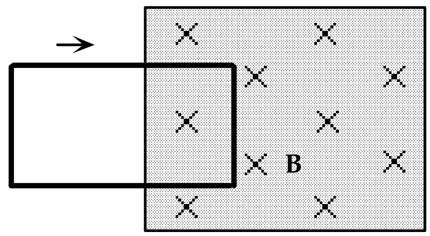
CHAPTER 21

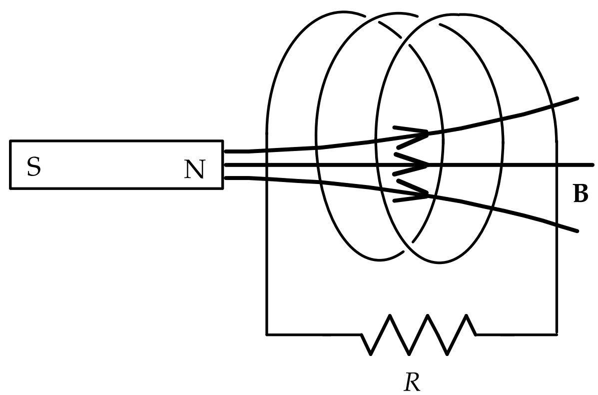
1. The magnitude of the average induced emf is
   å = – ®Φ_B/®*t* = – *A* ®*B*/®*t* = – ?(0.046 m)^2(0 – 1.5 T)/(0.20 s) = 0.050 V.

2. We assume the plane of the coil is perpendicular to the magnetic field. The magnitude of the average induced emf is
   å = – ®Φ_B/®*t* = – *A* ®*B*/®*t* = – ?(0.080 m)^2(0 – 1.10 T)/(0.15 s) = 0.15 V.

3. As the coil is pushed into the field, the magnetic flux increases into the page. To oppose this increase, the flux produced by the induced current must be out of the page, so the induced current is counterclockwise.



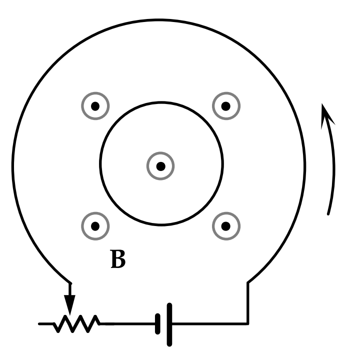
4. As the magnet is pushed into the coil, the magnetic flux increases to the right. To oppose this increase, flux produced by the induced current must be to the left, so the induced current in the resistor will be from right to left.

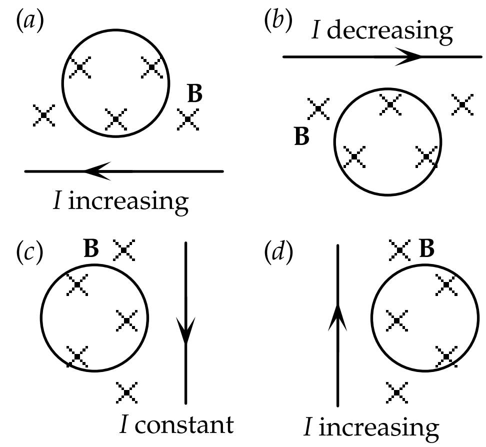


5. The average induced emf is
   å = – *N* ®Φ_B/®*t* = – (2)[(+38 Wb) – (– 30 Wb)]/(0.42 s) = -3.2×10^2 V.

6. We choose up as the positive direction. The average induced emf is
   å = – ®Φ_B/®*t* = – *A* ®*B*/®*t* = – ?(0.036 m)^2[(– 0.25 T) – (+ 0.63 T)]/(0.15 s) = 2.4×10^{-2} V = 24 mV.

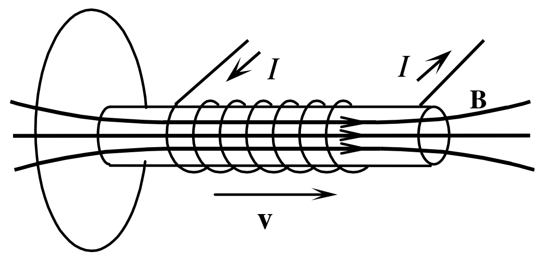
7. (*a*) As the resistance is increased, the current in the outer loop will decrease. Thus the flux through the inner loop, which is out of the page, will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux out of the page, so the direction of the induced current will be counterclockwise.
   (*b*) If the small loop is placed to the left, the flux through the small loop will be into the page and will decrease. To oppose this decrease, the induced current in the inner loop will produce a flux into the page, so the direction of the induced current will be clockwise.





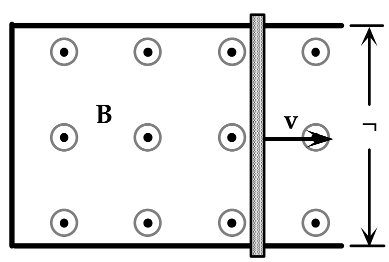
8. (*a*) The increasing current in the wire will cause an increasing field into the page through the loop. To oppose this increase, the induced current in the loop will produce a flux out of the page, so the direction of the induced current will be counterclockwise.
   (*b*) The decreasing current in the wire will cause a decreasing field into the page through the loop. To oppose this decrease, the induced current in the loop will produce a flux into the page, so the direction of the induced current will be clockwise.
   (*c*) Because the current is constant, there will be no change in flux, so the induced current will be zero.
   (*d*) The increasing current in the wire will cause an increasing field into the page through the loop. To oppose this increase, the induced current in the loop will produce a flux out of the page, so the direction of the induced current will be counterclockwise.

9. As the solenoid is pulled away from the loop, the magnetic flux to the right through the loop decreases. To oppose this decrease, the flux produced by the induced must be to the right, so the induced current is counterclockwise as viewed from the solenoid.



10. (*a*) The average induced emf is
    $\varepsilon = -\Delta\Phi_B/\Delta t = -A\,\Delta B/\Delta t = -\pi(0.10\text{ m})^2[(-0.45\text{ T}) - (+0.52\text{ T})]/(0.180\text{ s}) =$ 0.17 V.
    (*b*) The positive result for the induced emf means the induced field is away from the observer, so the induced current is clockwise.

11. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from
    $\varepsilon = BLv$
    $= (0.800\text{ T})(0.120\text{ m})(0.150\text{ m/s})$
    $= 1.44 \times 10^{-2}\text{ V} =$ 14.4 mV.
    (*b*) Because the upward flux is increasing, the induced flux will be into the page, so the induced current is clockwise. Thus the induced emf in the rod is down, which means that the electric field will be down. We find its magnitude from
    $E = \varepsilon/\ell = (1.44 \times 10^{-2}\text{ V})/(0.120\text{ m}) =$ 0.120 V/m down.



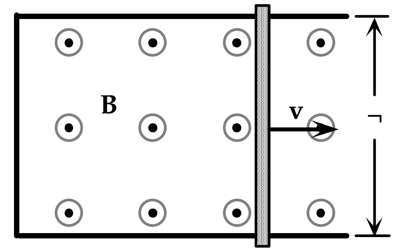
12. (*a*) The magnetic flux through the loop is into the paper and decreasing, because the area is decreasing. To oppose this decrease, the induced current in the loop will produce a flux into the paper, so the direction of the induced current will be clockwise.
    (*b*) We choose into the paper as the positive direction. The average induced emf is
    $\varepsilon = -\Delta\Phi_B/\Delta t = -B\,\Delta A/\Delta t = -\pi B\,\Delta(D^2)/\Delta t$
    $= -\pi(0.75\text{ T})[(0.030\text{ m})^2 - (0.100\text{ m})^2]/(0.50\text{ s}) = 4.3 \times 10^{-2}\text{ V} =$ 43 mV.
    (*c*) We find the average induced current from
    $I = \varepsilon/R = (43\text{ mV})/(2.5\ \Omega) =$ 17 mA.

13. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the speed from

$\mathcal{E} = BLv$;

$100 \times 10^{-3}\ \text{V} = (0.90\ \text{T})(0.132\ \text{m})v$,

which gives $v =$ 0.84 m/s.



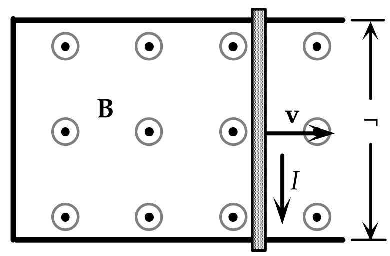
(*b*) Because the outward flux is increasing, the induced flux will be into the page, so the induced current is clockwise. Thus the induced emf in the rod is down, which means that the electric field will be down. We find its magnitude from

$E = \mathcal{E}/\ell = (100 \times 10^{-3}\ \text{V})/(0.132\ \text{m}) =$ 0.758 V/m down.

14. (*a*) Because the velocity is perpendicular to the magnetic field and the rod, we find the induced emf from

$\mathcal{E} = BLv$

$= (0.75\ \text{T})(0.300\ \text{m})(1.9\ \text{m/s}) =$ 0.43 V.



(*b*) We find the induced current from

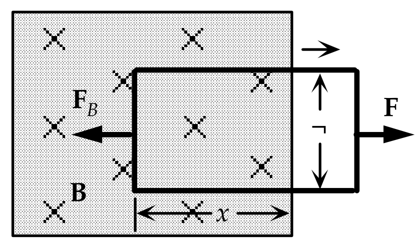
$I = \mathcal{E}/R = (0.43\ \text{V})/(2.5\ \Omega + 25.0\ \Omega) =$ 0.016 A.

(*c*) The induced current in the rod will be down. Because this current is in an upward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, which we find from

$F = ILB = (0.016\ \text{A})(0.300\ \text{m})(0.75\ \text{T}) =$ 3.5×10^{-3} N.

15. As the loop is pulled from the field, the flux through the loop will decrease. We find the induced emf from

$\mathcal{E} = -\Delta\Phi_B/\Delta t = -B\,\Delta A/\Delta t = -B\ell\,\Delta x/\Delta t = -B\ell(-v) = B\ell v$.



Because the inward flux is decreasing, the induced flux will be into the page, so the induced current is clockwise, given by

$I = \mathcal{E}/R$.

Because this current in the left hand side of the loop is in a downward magnetic field, there will be a magnetic force to the left. To keep the rod moving, there must be an equal external force to the right, which we find from

$F = I\ell B = (\mathcal{E}/R)\ell B = B^2\ell^2 v/R = (0.450\ \text{T})^2(0.350\ \text{m})^2(3.40\ \text{m/s})/(0.230\ \Omega) =$ 0.367 N.

16. (*a*) For the resistance of the loop, we have

$R = \rho L/A = (1.68 \times 10^{-8}\ \Omega\cdot\text{m})(20)\pi(0.310\ \text{m})/\pi(1.3 \times 10^{-3}\ \text{m})^2 = 0.0616\ \Omega$.

The induced emf is

$\mathcal{E} = -\Delta\Phi_B/\Delta t = -A\,\Delta B/\Delta t = -(20)\,\pi(0.155\ \text{m})^2(8.65 \times 10^{-3}\ \text{T/s}) = -0.0131\ \text{V}$.

Thus the induced current is

$I = \mathcal{E}/R = (0.0131\ \text{V})/(0.0616\ \Omega) =$ 0.21 A.

(*b*) Thermal energy is produced in the wire at the rate of

$P = I^2R = (0.21\ \text{A})^2(0.0616\ \Omega) = 2.8 \times 10^{-3}\ \text{W} =$ 2.8 mW.

17. If we assume that the movable rod starts at the bottom of the **U**, in a time t it will have moved a distance $x = vt$. For the resistance of the **U**, we have

    $R = \rho L/A = \rho(2vt + ¬)/A$.

    The induced emf is

    $å = BLv$;

    so the induced current is

    $I = å/R = B¬v/[\rho(2vt + ¬)/A] =$ $B¬vA/\rho(2vt + ¬)$.

18. For the resistance of the loop, we have

    $R = \rho L/A = \rho ?D/(?d^2 = 4\rho D/d^2$.

    The induced emf is

    $å = -®\Phi_B/®t = -(?D^2\, ®B/®t$;

    so the induced current is

    $I = å/R = -(?Dd^2/16\rho)\, ®B/®t$.

    In the time $®t$ the amount of charge that will pass a point is

    $Q = I\, ®t = -(?Dd^2/\rho)\, ®B = -[?(0.132 \text{ m})(2.25 \times 10^{-3} \text{ m})^2/16(1.68 \times 10^{-8}\ \Omega á \text{m})](0 - 0.750 \text{ T}) =$ 5.86 C.

19. The maximum induced emf is

    $å = NBA\omega$.

    If the only change is in the rotation speed, for the two conditions we have

    $å_2/å_1 = \omega_2/\omega_1$;

    $å_2/(12.4 \text{ V}) = (2500 \text{ rpm})/(1000 \text{ rpm})$ which gives $å_2 =$ 31.0 V.

20. We find the number of turns from

    $å_{peak} = NBA\omega$,

    $24.0 \text{ V} = N(0.420 \text{ T})(0.050 \text{ m})^2(60 \text{ rev/s})(2? \text{ rad/rev})$, which gives $N =$ 61 turns.

21. The induced emf is

    $å = NBA\omega \sin \omega t$.

    For the rms value of the output, we have

    $V_{rms} = [(å^2)_{av}]^{1/2} = [(NBA\omega)^2(\sin^2 \omega t)_{av}]^{1/2} = NBA\omega[(\sin^2 \omega t)_{av}]^{1/2}$.

    The $\sin^2 \omega t$ function varies from 0 to 1, with an average value of 1/2, so we get

    $V_{rms} = NBA\omega(1/Ì2) = NBA\omega/Ì2$.

22. We find the rotation speed from

    $å_{peak} = NBA\omega$,

    $120 \text{ V} = (720 \text{ turns})(0.550 \text{ T})(0.210 \text{ m})^2\omega$, which gives $\omega = 6.87 \text{ rad/s} =$ 1.09 rev/s.

23. We find the peak emf from

    $å_{peak} = NBA\omega = (450 \text{ turns})(0.75 \text{ T})?(0.050 \text{ m})^2(60 \text{ rev/s})(2? \text{ rad/rev}) = 999 \text{ V}$.

    The rms voltage output is

    $V_{rms} = å_{peak}/Ì2 = (999 \text{ V})/Ì2 = 707 \text{ V} =$ 0.71 kV.

    If only the rotation frequency changes, to double the rms voltage output, we must double the rotation speed, so we have

    $f_2 = 2f_1 = 2(60 \text{ rev/s}) =$ 120 rev/s.

24. We find the counter emf from

$\mathring{a} - \mathring{a}_{back} = IR$;

$120\ V - \mathring{a}_{back} = (9.20\ A)(3.75\ \Omega)$, which gives $\mathring{a}_{back} =$ 86 V.

25. Because the counter emf is proportional to the rotation speed, for the two conditions we have

$\mathring{a}_{back2}/\mathring{a}_{back1} = \omega_2/\omega_1$;

$\mathring{a}_{back2}/(72\ V) = (2500\ rpm)/(1800\ rpm)$, which gives $\mathring{a}_{back2} =$ 100 V.

26. Because the counter emf is proportional to both the rotation speed and the magnetic field, for the two conditions we have

$\mathring{a}_{back2}/\mathring{a}_{back1} = (\omega_2/\omega_1)(B_2/B_1)$;

$(65\ V)/(100\ V) = [(2500\ rpm)/(1000\ rpm)](B_2/B_1)$, which gives $B_2/B_1 = 0.26$.

27. Because the counter emf is proportional to the rotation speed, we find the new value from

$\mathring{a}_{back2}/\mathring{a}_{back1} = \omega_2/\omega_1$;

$\mathring{a}_{back2}/(108\ V) = (1/2)$, which gives $\mathring{a}_{back2} = 54\ V$.

We find the new current from

$\mathring{a} - \mathring{a}_{back} = IR$;

$120\ V - 54\ V = I(5.0\ \Omega)$, which gives $I =$ 13 A.

28. (*b*) At start up there will be no induced emf in the armature. Because the line voltage is across each resistor, we find the currents from

$I_{field0} = \mathring{a}/R_{field} = (115\ V)/(66.0\ \Omega) = 1.74\ A$;

$I_{armature0} = \mathring{a}/R_{armature} = (115\ V)/(5.00\ \Omega) = 23.0\ A$.

We use the junction condition to find the total current:

$I_0 = I_{field0} + I_{armature0} = 1.74\ A + 23.0\ A =$ 24.7 A.

(*c*) At full speed, the back emf is maximum. Because the line voltage is across the field resistor, we find the field current from

$I_{field0} = \mathring{a}/R_{field} = (115\ V)/(66.0\ \Omega) = 1.74\ A$.

We find the armature current from

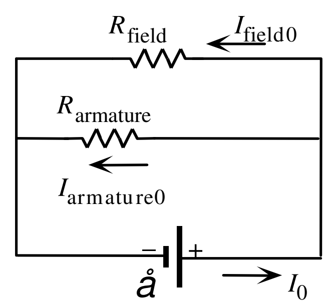
$\mathring{a} - \mathring{a}_{back} = I_{armature}R_{armature}$;

$115\ V - 105\ V = I_{armature}(5.00\ \Omega)$,

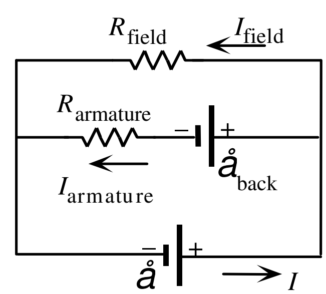
which gives $I_{armature} = 2.0\ A$.

Thus the total current is

$I_0 = I_{field} + I_{armature} = 1.74\ A + 2.0\ A =$ 3.7 A.



Starting



Full Speed

29. (*a*) We find the emf of the generator from the load conditions:

$V = \mathring{a} - I_{\text{armature}}R_{\text{armature}}$;

$200\text{ V} = \mathring{a} - (50\text{ A})(0.40\ \Omega)$, which gives $\mathring{a} = 220\text{ V}$.

When there is no load on the generator, the current is zero, so the voltage will be the emf: 220 V.

Note that no-load means little torque required to turn the generator.

(*b*) Because the generator emf is proportional to the rotation speed, we find the new value from

$\mathring{a}_2/\mathring{a}_1 = \omega_2/\omega_1$;

$\mathring{a}_2/(220\text{ V}) = (800\text{ rpm}/1000\text{ rpm})$, which gives $\mathring{a}_2 = 176\text{ V}$.

We find the new load voltage from

$V_2 = \mathring{a}_2 - I_{\text{armature}}R_{\text{armature}} = 176\text{ V} - (50\text{ A})(0.40\ \Omega) =$ 156 V.

Note that the power output is reduced to 7.8 kW.

30. We find the number of turns in the secondary from

$V_S/V_P = N_S/N_P$;

$(10{,}000\text{V})/(120\text{ V}) = N_S/(5000\text{ turns})$, which gives $N_S =$ 4.17×10^5 turns.

31. Because $N_S < N_P$, this is a step-down transformer.

We find the ratio of the voltages from

$V_S/V_P = N_S/N_P = (120\text{ turns})/(420\text{ turns}) =$ 0.285.

For the ratio of currents, we have

$I_S/I_P = N_P/N_S = (420\text{ turns})/(120\text{ turns}) =$ 3.50.

32. With 100% efficiency, the power on each side of the transformer is the same:

$I_PV_P = I_SV_S$, so we have

$I_S/I_P = V_P/V_S = (16\text{ V})/(120) =$ 0.13.

33. We find the ratio of the number of turns from

$N_S/N_P = V_S/V_P = (12 \times 10^3\text{ V})/(220\text{ V}) =$ 55.

If the transformer is connected backward, the role of the turns will be reversed:

$V_S/V_P = N_S/N_P$;

$V_S/(220\text{ V}) = 1/55$, which gives $V_S =$ 4.0 V.

34. (*a*) We assume 100% efficiency, so we have

$I_PV_P = I_SV_S$;

$(15\text{ A})/(0.75\text{ A}) = (120\text{ V})/V_S$, which gives $V_S =$ 6.0 V.

(*b*) Because $V_S < V_P$, this is a step-down transformer.

35. (*a*) We assume 100% efficiency, so we find the input voltage from

$P = I_PV_P$;

$100\text{ W} = (20\text{ A})V_P$, which gives $V_P = 5.0\text{ V}$.

Because $V_S > V_P$, this is a step-up transformer.

(*b*) For the voltage ratio we have

$V_S/V_P = (12\text{ V})/(5.0\text{ V}) =$ 2.4.

36. (*a*) Because $V_S < V_P$, this is a step-down transformer.
(*b*) We assume 100% efficiency, so we find the current in the secondary from
$P = I_S V_S$;
40 W = I_S(12 V), which gives I_S = 3.3 A.
(*c*) We find the current in the primary from
$P = I_P V_P$;
40 W = I_P(120 V), which gives I_S = 0.33 A.
(*d*) We find the resistance of the bulb from
$V_S = I_S R_S$;
120 V = (3.33 A)R_S, which gives R_S = 3.6 Ω.

37. We find the output voltage from
$V_S/V_P = N_S/N_P$;
V_S/(120 V) = (1240 turns)/(330 turns), which gives V_S = 451 V.
We find the input current from
$I_S/I_P = N_P/N_S$;
(15.0 A)/I_P = (330 turns)/(1240 turns), which gives I_P = 56.4 A.

38. (*a*) We can find the current in the transmission lines from the output emf:
$P_{out} = IV_{out}$;
30×10^6 W = $I(45 \times 10^3$ V), which gives I = 667 A.
We find the input emf from
$\mathcal{E}_{in} = V_{out} + IR_{lines} = 45 \times 10^3$ V + (667 A)(4.0 Ω) = 48×10^3 = 48 kV.
(*b*) The power loss in the lines is
$P_{loss} = I^2 R_{lines}$ = (667 A)^2(4.0 Ω) = 1.78×10^6 W = 1.78 MW.
The total power is
$P_{total} = P_{out} + P_{loss}$ = 30 MW + 1.78 MW = 31.8 MW,
so the fraction lost is
(1.78 MW)/(31.8 MW) = 0.056 (5.6%).

39. We can find the current in the transmission lines from the power transmitted to the user:
$P_T = IV$, or $I = P_T/V$.
The power loss in the lines is
$P_L = I^2 R_L = (P_T/V)^2 R_L = (P_T)^2 R_L/V^2$.

40. Without the transformers, we can find the delivered current, which is the current in the transmission lines, from the delivered power:

   $P_{out} = IV_{out}$;

   50×10^3 W = I(120V), which gives I = 417 A.

   The power loss in the lines is

   $P_{L0} = I^2R_{lines}$ = (417 A)^2(2)(0.100 Ω) = 3.48×10^4 W = 34.8 kW.

   With the transformers, to deliver the same power at 120 V, the delivered current from the step-down transformer is still 417 A.

   If the step-down transformer is 99% efficient, we have

   $(0.99)I_{P2}V_{P2} = I_{S2}V_{S2}$;

   $(0.99)I_{P2}$(1200 V) = (417 A)(120 V), which gives I_{P2} = 42.1 A.

   Because this is the current in the lines, the power loss in the lines is

   $P_{L2} = I_{P2}{}^2R_{lines}$ = (42.1 A)^2(2)(0.100 Ω) = 3.55×10^2 W = 0.355 kW.

   For the 1% losses in the transformers, we approximate the power in each transformer:

   P_{Lt} = (0.01)(50 kW + 50.36 kW) = 1.00 kW.

   The total power loss using the transformers is

   P_L = 0.355 kW + 1.00 kW = 1.4 kW.

   The power saved by using the transformers is

   $P_{saved} = P_{L0} - P_L$ = 34.8 kW – 1.4 kW = 33.4 kW.

41. We can find the delivered current, which is the current in the transmission lines, from the delivered power:

   $P_{out} = IV_{out}$;

   300×10^6 W = $I(600 \times 10^3$ V), which gives I = 500 A.

   We can approximate the power at the input to the lines as 102% of the delivered power to account for the loss. Thus the power loss in the lines is

   $P_L = (0.02)(1.02)P_{out} = I^2R_{lines}$;

   (0.02)(1.02)(300×10^6 W) = (500 A)$^2R_{lines}$, which gives R_{lines} = 24.5 Ω.

   We find the radius of the 2 lines, each 200 km long, from

   $R = \rho L/A$;

   24.5 Ω = (2.65×10^{-8} Ω á m)(2)(200×10^3 m)/?r^2 , which gives $r = 1.174 \times 10^{-2}$ m.

   Thus the diameter of the lines should be 2.35 cm.

42. We find the induced emf from

   ả = – L ®I/®t = – (0.120 H)(10.0 A – 25.0 A)/(0.350 s) = 5.14 V.

   The emf is in the direction of the current, to oppose the decrease in the current.

43. We use the inductance of a solenoid:

   $L = \mu_0 AN^2$/¬ = (4? × 10^{-7} T á m/A)?(1.45×10^{-2} m)^2(10,000 turns)^2/(0.60 m) = 0.14 H.

44. Because the current in increasing, the emf is negative. We find the self-inductance from

   ả = – L ®I/®t;

   – 8.50 V = – L[31.0 mA – (– 28.0 mA)]/(42.0 ms), which gives L = 6.05 H.

45. We find the number of turns from the inductance of a solenoid:

   $L = \mu_0 AN^2$/¬;

   0.100 H = (4? × 10^{-7} T á m/A)?(2.6×10^{-2} m)$^2N^2$/(0.300 m), which gives N = 3.4×10^3 turns.

46. We use the inductance of a solenoid:

$L = \mu_0 A N^2/¬ = (4? \times 10^{-7}\ \mathrm{T á m/A})?(1.25 \times 10^{-2}\ \mathrm{m})^2(1000\ \mathrm{turns})^2/(0.282\ \mathrm{m}) = 2.2 \times 10^{-3}\ \mathrm{H} =$ 2.2 mH.

If we form the ratio of inductances for the two conditions, we have

$L_2/L_1 = (\mu/\mu_0)(N_2/N_1)^2$;

$1 = (1000)[N_2/(1000\ \mathrm{turns})]^2$, which gives $N_2 =$ 32 turns.

47. We find the induced emf from

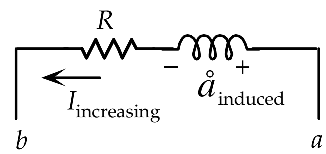
$å_{induced} = - L\ ®I/®t = - (0.440\ \mathrm{H})(3.50\ \mathrm{A/s}) = - 1.54\ \mathrm{V}.$

The negative sign indicates a direction opposite to the current.

If we start at point b and add the potential changes, we get

$V_b + IR + |å_{induced}| = V_a$, or

$V_{ab} = (3.00\ \mathrm{A})(2.25\ \Omega) + 1.54\ \mathrm{V} =$ 8.29 V.



48. If D represents the diameter of the solenoid, the length of the wire is $N(?D)$. Because this is constant, we have

$N_1?D_1 = N_2?D_2$, or $N_2/N_1 = D_1/D_2 = !$.

The solenoid is tightly wound, so the length of the solenoid is $¬ = Nd$, where d is the diameter of the wire. Thus we have

$¬_2/¬_1 = N_2/N_1 = !$.

We use the inductance of a solenoid:

$L = \mu_0 A N^2/¬$, and form the ratio of inductances for the two conditions, so we have

$L_2/L_1 = (D_2/D_1)^2(N_2/N_1)^2/(¬_2/¬_1) = (2)^2(!)^2/(!) =$ 2.

49. (*a*) We know from Problem 20–38 that the magnetic field at the radius R of a torus is

$B = \mu_0 NI/2?R$.

If the current changes by $®I$, the magnetic field change is

$®B = (\mu_0 N/2?R)\ ®I$.

If we assume that the field is uniform inside the torus (because the field varies with the distance from the center of the torus, this is not actually true, but will be a good approximation if R È r), the emf induced by a current change is

$å = - N\ ®\Phi_B/®t = - NA\ ®B/®t = - N(\mu_0 N?r^2/2?R)\ ®I/®t = - (\mu_0 N^2 r^2/2R)\ ®I/®t$.

When we compare this to

$å = - L\ ®I/®t$, we see that $L = \mu_0 N^2 r^2/2R$.

Because the circumference of the torus is $¬ = 2?R$, we can write this as

$L = \mu_0 N^2?r^2/¬$, which is consistent with that of a solenoid.

(*b*) For the given data, we have

$L = (4? \times 10^{-7}\ \mathrm{T á m/A})(1000\ \mathrm{turns})^2(0.75 \times 10^{-2}\ \mathrm{m})^2/2(0.20\ \mathrm{m}) = 1.76 \times 10^{-4}\ \mathrm{H} =$ 0.18 mH.

50. (*a*) For two inductors placed in series, the current through each inductor is the same. This current is also the current through the equivalent inductor, so the total emf is

$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$

$= (-L_1 \Delta I/\Delta t) + (-L_2 \Delta I/\Delta t) = -(L_1 + L_2)\Delta I/\Delta t = -L_{series}\Delta I/\Delta t$, which gives $L_{series} = L_1 + L_2$.

(*b*) For two inductors placed in parallel, the potential difference across each inductor, which is the emf, is the same:

$\mathcal{E} = \mathcal{E}_1 = \mathcal{E}_2 = -L_1 \Delta I_1/\Delta t = -L_2 \Delta I_2/\Delta t = -L_{parallel}\Delta I/\Delta t.$

The total current through the equivalent inductor is

$I = I_1 + I_2$, so we have

$\Delta I/\Delta t = \Delta I_1/\Delta t + \Delta I_2/\Delta t;$

$-\mathcal{E}/L_{parallel} = -\mathcal{E}/L_1 - \mathcal{E}/L_2$, which gives $1/L_{parallel} = (1/L_1) + (1/L_2)$, or $L_{parallel} = L_1L_2/(L_1 + L_2)$.

(*c*) Each inductor will have an additional emf term:

$\mathcal{E}_1 = -L_1 \Delta I_1/\Delta t \pm M\Delta I_2/\Delta t$, and $\mathcal{E}_2 = -L_2 \Delta I_2/\Delta t \pm M\Delta I_1/\Delta t$,

where the sign of the mutual inductance term is determined by the orientation of one winding with respect to the other.

For the series arrangement, if we assume that the windings are such that the fields of each inductor are in the same direction, the induced emf from the mutual inductance will be in the same direction (have the same sign) as the self-induced emfs. Because $I_1 = I_2 = I$, we have

$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2;$

$-L_{series}\Delta I/\Delta t = (-L_1 \Delta I/\Delta t - M\Delta I/\Delta t) + (-L_2 \Delta I/\Delta t - M\Delta I/\Delta t) = -(L_1 + L_2 + 2M)\Delta I/\Delta t$,

which gives $L_{series} = L_1 + L_2 + 2M$.

For the parallel arrangement, if we assume that the windings are such that the internal fields of each inductor are in the same direction, the induced emf from the mutual inductance will be in the opposite direction (have the opposite sign) from the self-induced emfs:

$\mathcal{E} = \mathcal{E}_1 = -L_1 \Delta I_1/\Delta t + M\Delta I_2/\Delta t = \mathcal{E}_2 = -L_2 \Delta I_2/\Delta t + M\Delta I_1/\Delta t.$

When these equations are combined, we get

$\Delta I_1/\Delta t = -(L_2 + M)\mathcal{E}/(L_1L_2 - M^2)$, and $\Delta I_2/\Delta t = -(L_1 + M)\mathcal{E}/(L_1L_2 - M^2)$.

Because $I = I_1 + I_2$, we have

$\Delta I/\Delta t = \Delta I_1/\Delta t + \Delta I_2/\Delta t;$

$-\mathcal{E}/L_{parallel} = -(L_2 + M)\mathcal{E}/(L_1L_2 - M^2) - (L_1 + M)\mathcal{E}/(L_1L_2 - M^2)$,

which gives

$L_{parallel} = (L_1L_2 - M^2)/(L_1 + L_2 + 2M)$.

51. The magnetic field of the solenoid, which passes through the coil is

$B = \mu_0 N_1 I_1/\ell$.

When the current in the solenoid changes, the induced emf in the coil is

$\mathcal{E} = -N_2 A\Delta B/\Delta t = -N_2(\mu_0 N_1 A/\ell)\Delta I_1/\Delta t = -M\Delta I_1/\Delta t.$

Thus we get

$M = \mu_0 N_1 N_2 A/\ell$.

52. (*a*) We find the magnetic field from the first coil using the expression for a solenoid:

$B = \mu N_1 I_1/\ell = (3000)(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1500\ \text{turns})I_1/(0.30\ \text{m}) = (18.85\ \text{T/A})I_1.$

The induced emf in the second coil is

$\mathcal{E} = -N_2 A\Delta B/\Delta t$

$= -(800\ \text{turns})\pi(0.020\ \text{m})^2(18.85\ \text{T/A})\Delta I_1/\Delta t$

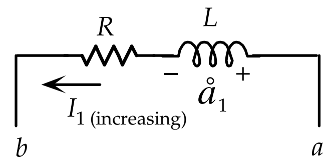
$= -(18.95\ \text{T}\cdot\text{m}^2/\text{A})(0 - 3.0\ \text{A})/(0.0080\ \text{s}) = 7.1 \times 10^3\ \text{V}.$

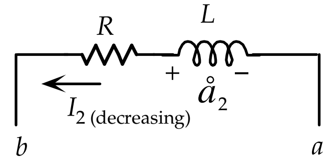
(*b*) We find the mutual inductance from

$\mathcal{E} = -M\Delta I_1/\Delta t;$

7.1×10^3 V = – M(0 – 3.0 A)/0.0080 s), which gives M = 19.0 H.

53. The direction of the induced emf will oppose the change in the current, as shown in the diagrams.





If we start at point b and add the potential changes, we get

$V_b + IR \pm |\varepsilon_{\text{induced}}| = V_a$, or

$V_{ab} = IR \pm |\varepsilon_{\text{induced}}|$;

$22.5\text{ V} = (0.860\text{ A})R + L\,(3.40\text{ A/s})$;

$16.2\text{ V} = (0.700\text{ A})R + L\,(-1.80\text{ A/s})$.

We have two equations for two unknowns, with the results:

$R = 24.3\ \Omega$, and $L = 0.463\text{ H}$.

54. The magnetic energy in the field is

$U = u_B V = \frac{1}{2}(B^2/\mu_0)L\pi r^2$

$= \frac{1}{2}[(0.80\text{ T})^2/(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})](0.36\text{ m})\pi(1.0 \times 10^{-2}\text{ m})^2 = 29\text{ J}$.

55. The initial energy stored in the inductor is

$U_0 = \frac{1}{2}LI_0^2 = \frac{1}{2}(60.0 \times 10^{-3}\text{ H})(50.0 \times 10^{-3}\text{ A})^2 = 7.5 \times 10^{-5}\text{ J}$.

For the increase in energy, we have

$U/U_0 = (I/I_0)^2$;

$10 = (I/50.0\text{ mA})^2$, which gives $I = 158\text{ mA}$.

We find the time from

$\Delta I/\Delta t = 100\text{ mA/s} = (158\text{ mA} - 50.0\text{ mA})/\Delta t$, which gives $\Delta t = 1.08\text{ s}$.

56. The magnetic energy in the field is

$U = u_B V = \frac{1}{2}(B^2/\mu_0)h\pi r^2$

$= \frac{1}{2}[(0.50 \times 10^{-4}\text{ T})^2/(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})](10 \times 10^3\text{ m})\pi(6.38 \times 10^6\text{ m})^2 = 5.1 \times 10^{15}\text{ J}$.

57. (*a*) For an LR circuit, we have

$I = I_{\max}(1 - e^{-t/\tau})$;

$0.80I_{\max} = (1 - e^{-t/\tau})$, or $-t/\tau = -(7.20\text{ ms})/\tau = \ln 0.20$, which gives $\tau = 4.47\text{ ms}$.

(*b*) We find the inductance from

$\tau = L/R$;

$4.47 \times 10^{-3}\text{ s} = L/(250\ \Omega)$, which gives $L = 1.12\text{ H}$.

58. At $t = 0$ there is no voltage drop across the resistor, so we have

$V = L(\Delta I/\Delta t)_0$, or $(\Delta I/\Delta t)_0 = V/L$.

The maximum value of the current is reached after a long time, when there is no voltage across the inductor:

$V = I_{\max}R$, or $I_{\max} = V/R$.

We find the time to reach maximum if the initial rate were maintained from

$V/R = (\Delta I/\Delta t)_0 t = (V/L)t$, which gives $t = L/R = \tau$.

59. For an LR circuit, we have

$I = I_{max}(1 - e^{-t/\tau})$, which we can write as

$e^{-t/\tau} = 1 - (I/I_{max})$, or $t/\tau = -\ln[1 - (I/I_{max})] = -\ln[(I_{max} - I)/I_{max}]$.

(*a*) $t_a/\tau = -\ln(0.10)$, which gives $t_a/\tau =$ 2.3.

(*b*) $t_b/\tau = -\ln(0.010)$, which gives $t_b/\tau =$ 4.6.

(*c*) $t_c/\tau = -\ln(0.001)$, which gives $t_c/\tau =$ 6.9.

60. (*a*) We use the inductance of a solenoid:

$L = \mu_0 AN^2/¬$.

Because they are tightly wound, the number of turns is determined by the diameter of the wire:

$N = ¬/d$.

If we form the ratio of inductances for the two conditions, we have

$L_1/L_2 = (N_1/N_2)^2 = (d_2/d_1)^2 = 2^2 =$ 4.

(*b*) The length of wire used for the turns is $¬_{wire} = N(?D)$, where D is the diameter of the solenoid. Thus for the ratio of resistances, we have

$R_1/R_2 = (¬_{wire1}/¬_{wire2})(d_2/d_1)^2 = (N_1/N_2)(d_2/d_1)^2 = (d_2/d_1)^3$.

For the ratio of the time constants, we get

$\tau_1/\tau_2 = (L_1/L_2)(R_2/R_1) = (L_1/L_2)(d_1/d_2)^3 = (4)(!)^3 =$!.

61. We find the frequency from

$X_L = \omega L = 2?fL$;

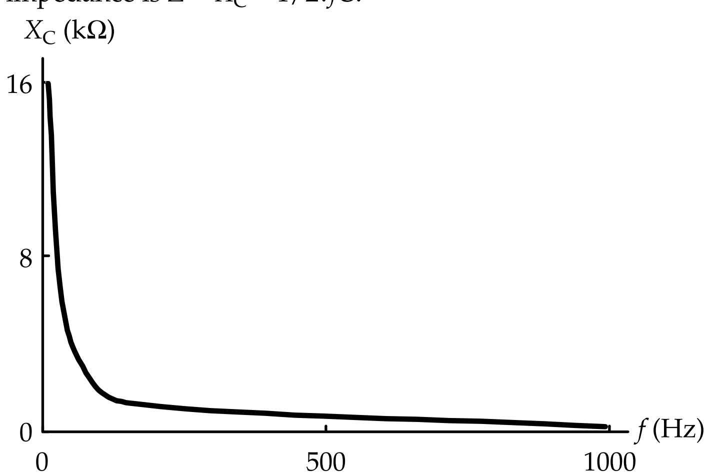
$1.5\ k\Omega = 2?f(0.160\ H)$, which gives $f =$ 1.5 kHz.

62. We find the frequency from

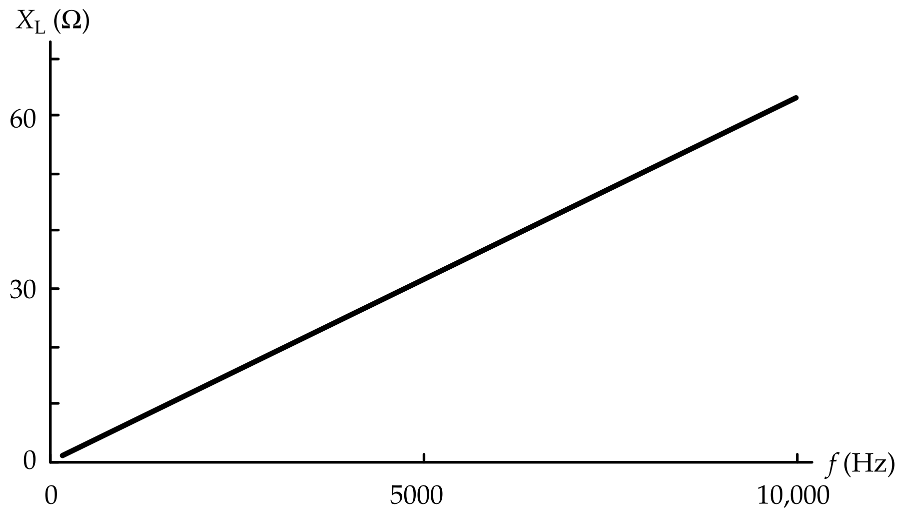
$X_C = 1/2?fC$;

$250\ \Omega = 1/2?f(9.20 \times 10^{-6}\ F)$, which gives $f =$ 69.2 Hz.

63. The impedance is $Z = X_C = 1/2?fC$.



64. The impedance is $Z = X_L = \omega L = 2?fL$.



65. We find the impedance from

$Z = X_L = \omega L = 2?fL = 2?(10.0 \text{ kHz})(0.160 \text{ H}) =$ 10.1 kΩ.

For the rms current we have

$I_{rms} = V_{rms}/X_L = (240 \text{ V})/(10.1 \text{ k}\Omega) =$ 23.9 mA.

66. We find the reactance from

$X_L = V_{rms}/I_{rms} = (240 \text{ V})/(12.8 \text{ A}) = 18.8\ \Omega$.

We find the inductance from

$X_L = 2?fL$;

$18.8\ \Omega = 2?(60 \text{ Hz})L$, which gives $L = 0.050 \text{ H} =$ 50 mH.

67. (*a*) We find the impedance from

$Z = X_C = 1/2?fC = 1/2?(700 \text{ Hz})(0.030 \times 10^{-6} \text{ F}) = 7.6 \times 10^3\ \Omega =$ 7.6 kΩ.

(*b*) We find the peak value of the current from

$I_{peak} = Ì2\, I_{rms} = Ì2(V_{rms}/Z) = Ì2(2.0 \text{ kV})/(7.6 \text{ k}\Omega) =$ 0.37 A.

68. Because the capacitor and resistor are in parallel, their currents are

$I_C = V/X_C$, and $I_R = V/R$.

The total current is $I = I_C + I_R$.

(*a*) The reactance of the capacitor is

$X_{C1} = 1/2?f_1C = 1/2?(60 \text{ Hz})(0.60 \times 10^{-6} \text{ F}) = 4.42 \times 10^3\ \Omega = 4.42 \text{ k}\Omega$.

For the fraction of current that passes through C, we have

fraction1 $= I_{C1}/(I_{C1} + I_R) = (1/X_{C1})/[(1/X_{C1}) + (1/R)]$

$= (1/4.42 \text{ k}\Omega)/[(1/4.42 \text{ k}\Omega) + (1/300 \text{ k}\Omega)] = 0.064 =$ 6.4%.

(*b*) The reactance of the capacitor is

$X_{C2} = 1/2?f_2C = 1/2?(60{,}000 \text{ Hz})(0.60 \times 10^{-6} \text{ F}) = 4.42\ \Omega$.

For the fraction of current that passes through C, we have

fraction2 $= I_{C2}/(I_{C2} + I_R) = (1/X_{C2})/[(1/X_{C2}) + (1/R)]$

$= (1/4.42\ \Omega)/[(1/4.42\ \Omega) + (1/300 \text{ k}\Omega)] = 0.99 =$ 99%.

Thus most of the high-frequency current passes through the capacitor.

69. Because the capacitor and resistor are in series, the impedance of the circuit is

$Z = (R^2 + X_C^2)^{1/2}$,

so the current is

$I = V/Z$,

and the voltage across the resistor is

$V_R = IR$.

(*a*) The reactance of the capacitor is

$X_{C1} = 1/2?f_1C = 1/2?(60\text{ Hz})(2.0 \times 10^{-6}\text{ F}) = 1.33 \times 10^3\ \Omega = 1.33\text{ k}\Omega$.

The impedance of the circuit is

$Z_1 = (R^2 + X_{C1}^2)^{1/2} = [(0.500\text{ k}\Omega)^2 + (1.33\text{ k}\Omega)^2]^{1/2} = 1.42\text{ k}\Omega$.

The current is

$I_1 = V/Z_1 = (50\text{ mV})/(1.42\text{ k}\Omega) = 35.2\ \mu\text{A}$.

The voltage across the resistor is

$V_{R1} = I_1R = (35.2\ \mu\text{A})(0.500\text{ k}\Omega) =$ 18 mV.

(*b*) The reactance of the capacitor is

$X_{C1} = 1/2?f_1C = 1/2?(60{,}000\text{ Hz})(2.0 \times 10^{-6}\text{ F}) = 1.33\ \Omega$.

The impedance of the circuit is

$Z_1 = (R^2 + X_{C1}^2)^{1/2} = [(0.500\text{ k}\Omega)^2 + (0.00133\text{ k}\Omega)^2]^{1/2} = 0.500\text{ k}\Omega$.

The current is

$I_1 = V/Z_1 = (50\text{ mV})/(0.500\text{ k}\Omega) = 100\ \mu\text{A}$.

The voltage across the resistor is

$V_{R1} = I_1R = (100\ \mu\text{A})(0.500\text{ k}\Omega) =$ 50 mV.

Thus the high-frequency signal passes to the resistor.

70. (*a*) The reactance of the inductor is

$X_{L1} = 2?f_1L = 2?(60\text{ Hz})(0.50\text{ H}) = 188\ \Omega$.

The impedance of the circuit is

$Z_1 = (R^2 + X_{L1}^2)^{1/2} = [(30\text{ k}\Omega)^2 + (0.188\text{ k}\Omega)^2]^{1/2} =$ 30 kΩ.

(*b*) The reactance of the inductor is

$X_{L2} = 2?f_2L = 2?(30\text{ kHz})(0.50\text{ H}) = 94.2\text{ k}\Omega$.

The impedance of the circuit is

$Z_2 = (R^2 + X_{L2}^2)^{1/2} = [(30\text{ k}\Omega)^2 + (94.2\text{ k}\Omega)^2]^{1/2} =$ 99 kΩ.

71. (*a*) The reactance of the capacitor is

$X_{C1} = 1/2?f_1C = 1/2?(100\text{ Hz})(4.0 \times 10^{-6}\text{ F}) = 398\ \Omega = 0.398\text{ k}\Omega$.

The impedance of the circuit is

$Z_1 = (R^2 + X_{C1}^2)^{1/2} = [(1.5\text{ k}\Omega)^2 + (0.398\text{ k}\Omega)^2]^{1/2} =$ 1.6 kΩ.

(*b*) The reactance of the capacitor is

$X_{C2} = 1/2?f_2C = 1/2?(10{,}000\text{ Hz})(4.0 \times 10^{-6}\text{ F}) = 3.98\ \Omega = 0.00398\text{ k}\Omega$.

The impedance of the circuit is

$Z_2 = (R^2 + X_{C2}^2)^{1/2} = [(1.5\text{ k}\Omega)^2 + (0.00398\text{ k}\Omega)^2]^{1/2} =$ 1.5 kΩ.

72. We find the impedance from

$Z = V_{rms}/I_{rms} = (120\text{ V})/(70\text{ mA}) =$ 1.7 kΩ.

73. At 60 Hz, the reactance of the inductor is
$X_{L1} = 2?f_1L = 2?(60 \text{ Hz})(0.420 \text{ H}) = 158 \ \Omega.$
The impedance of the circuit is
$Z_1 = (R^2 + X_{L1}{}^2)^{1/2} = [(2.5 \text{ k}\Omega)^2 + (0.158 \text{ k}\Omega)^2]^{1/2} = 2.51 \text{ k}\Omega.$
Thus the impedance at the new frequency is
$Z_2 = 2Z_1 = 2(2.51 \text{ k}\Omega) = 5.02 \text{ k}\Omega.$
We find the new reactance from
$Z_2 = (R^2 + X_{L2}{}^2)^{1/2};$
$5.02 \text{ k}\Omega = [(2.5 \text{ k}\Omega)^2 + X_{L2}{}^2]^{1/2}$, which gives $X_{L2} = 4.34 \text{ k}\Omega.$
We find the new frequency from
$X_{L2} = 2?f_2L;$
$4.34 \text{ k}\Omega = 2?f_2(0.420 \text{ H})$, which gives $f_2 =$ 1.6 kHz.

74. (*a*) The reactance of the capacitor is
$X_C = 1/2?fC = 1/2?(60 \text{ Hz})(0.80 \times 10^{-6} \text{ F}) = 3.32 \times 10^3 \ \Omega = 3.32 \text{ k}\Omega.$
The impedance of the circuit is
$Z = (R^2 + X_C{}^2)^{1/2} = [(28.8 \text{ k}\Omega)^2 + (3.32 \text{ k}\Omega)^2]^{1/2} = 29.0 \text{ k}\Omega.$
The rms current is
$I_{rms} = V_{rms}/Z = (120 \text{ V})/(29.0 \text{ k}\Omega) =$ 4.14 mA.
(*b*) We find the phase angle from
$\cos \phi = R/Z = (28.8 \text{ k}\Omega)/(29.0 \text{ k}\Omega) = 0.993.$
In an *RC* circuit, the current leads the voltage, so $\phi =$ – 6.6¡.
(*c*) The power dissipated is
$P = I_{rms}{}^2R = (4.14 \times 10^{-3} \text{ A})^2(28.8 \times 10^3 \ \Omega) =$ 0.49 W.
(*d*) The rms readings across the elements are
$V_R = I_{rms}R = (4.14 \text{ mA})(28.8 \text{ k}\Omega) =$ 119 V;
$V_C = I_{rms}X_C = (4.14 \text{ mA})(3.32 \text{ k}\Omega) =$ 14 V.
Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

75. (*a*) The reactance of the inductor is
$X_L = 2?fL = 2?(60 \text{ Hz})(0.900 \text{ H}) = 339 \ \Omega = 0.339 \text{ k}\Omega.$
The impedance of the circuit is
$Z = (R^2 + X_L{}^2)^{1/2} = [(1.80 \text{ k}\Omega)^2 + (0.339 \text{ k}\Omega)^2]^{1/2} = 1.83 \text{ k}\Omega.$
The rms current is
$I_{rms} = V_{rms}/Z = (120 \text{ V})/(1.83 \text{ k}\Omega) =$ 65.5 mA.
(*b*) We find the phase angle from
$\cos \phi = R/Z = (1.80 \text{ k}\Omega)/(1.83 \text{ k}\Omega) = 0.984.$
In an *RL* circuit, the current lags the voltage, so $\phi =$ + 10.4¡.
(*c*) The power dissipated is
$P = I_{rms}{}^2R = (65.5 \times 10^{-3} \text{ A})^2(1.80 \times 10^3 \ \Omega) =$ 7.73 W.
(*d*) The rms readings across the elements are
$V_R = I_{rms}R = (65.5 \text{ mA})(1.80 \text{ k}\Omega) =$ 118 V;
$V_L = I_{rms}X_L = (65.5 \text{ mA})(0.339 \text{ k}\Omega) =$ 22 V.
Note that, because the maximum voltages occur at different times, the two readings do not add to the applied voltage of 120 V.

76. The reactance of the capacitor is
$X_C = 1/2?fC = 1/2?(10.0 \times 10^3\ \text{Hz})(5000 \times 10^{-12}\ \text{F}) = 3.18 \times 10^3\ \Omega = 3.18\ \text{k}\Omega.$
The reactance of the inductor is
$X_L = 2?fL = 2?(10.0\ \text{kHz})(0.0220\ \text{H}) = 1.38\ \text{k}\Omega.$
The impedance of the circuit is
$Z = [R^2 + (X_L - X_C)^2]^{1/2} = [(8.70\ \text{k}\Omega)^2 + (1.38\ \text{k}\Omega - 3.18\ \text{k}\Omega)^2]^{1/2} =$ 8.88 kΩ.
We find the phase angle from
$\tan \phi = (X_L - X_L)/R = (1.38\ \text{k}\Omega - 3.18\ \text{k}\Omega)/(8.70\ \text{k}\Omega) = -0.207$, so $\phi =$ – 11.7¡.
The rms current is
$I_{rms} = V_{rms}/Z = (300\ \text{V})/(8.70\ \text{k}\Omega) =$ 34.5 mA.

77. We find the reactance from
$Z = [R^2 + X_L^2]^{1/2}$;
$35\ \Omega = [R^2 + (30\ \Omega)^2]^{1/2}$, which gives $R =$ 18 Ω.

78. From the expression for V, we see that $V_0 = 4.8$ V, and $2?f = 754\ \text{s}^{-1}$.
For the reactances, we have
$X_C = 1/2?fC = 1/(754\ \text{s}^{-1})(3.0 \times 10^{-6}\ \text{F}) = 442\ \Omega.$
$X_L = 2?fL = (754\ \text{s}^{-1})(0.0030\ \text{H}) = 2.26\ \Omega.$
The impedance of the circuit is
$Z = [R^2 + (X_L - X_C)^2]^{1/2} = [(1.40\ \text{k}\Omega)^2 + (0.00226\ \text{k}\Omega - 0.442\ \text{k}\Omega)^2]^{1/2} = 1.47\ \text{k}\Omega.$
The power dissipated is
$P = I_{rms}{}^2R = (V_{rms}/Z)^2R = (V_0/Z$Ì$2)^2R =$!$(V_0/Z)^2R$
$=$!$(4.8\ \text{V}/1.47 \times 10^3\ \Omega)^2(1.40 \times 10^3\ \Omega) = 7.5 \times 10^{-3}\ \text{W} =$ 7.5 mW.

79. We find the rms current from
$P = I_{rms}{}^2R;$
$9.50\ \text{W} = I_{rms}{}^2(250\ \Omega)$, which gives $I_{rms} = 0.195$ A.
We find the impedance from
$Z = V_{rms}/I_{rms} = (50.0\ \text{V})/(0.195\ \text{A}) = 256\ \Omega.$
From this we can find the reactance of the coil:
$Z^2 = R^2 + X_L{}^2;$
$(256\ \Omega)^2 = (250\ \Omega)^2 + X_L{}^2$, which gives $X_L = 57.4\ \Omega$.
We find the frequency that will produce this reactance from
$X_L = 2?fL$;
$57.4\ \Omega = 2?f(0.040\ \text{H})$, which gives $f =$ 228 Hz.

80. The voltage across the resistor is in phase with the current:
$V_R = IR = (I_0 \cos \omega t) = I_0R \cos \omega t.$
The voltage across the inductor leads the current by 90¡ or ?/2:
$V_L = I_0 \cos (\omega t + ?/2)X_L = I_0\omega L \cos (\omega t + ?/2).$
The voltage across the capacitor lags the current by 90¡ or ?/2:
$V_C = I_0 \cos (\omega t - ?/2)X_C = (I_0/\omega C) \cos (\omega t - ?/2).$

81. The resonant frequency is
$f_0 = (1/2?)(1/LC)^{1/2} = [1/(50 \times 10^{-6}\ \text{H})(3500 \times 10^{-12}\ \text{F})]^{1/2} =$ 3.8×10^5 Hz.

82. (*a*) The resonant frequency is given by

$f_0^2 = (1/2?)^2(1/LC)$.

When we form the ratio for the two stations, we get

$(f_{02}/f_{01})^2 = C_1/C_2$;

$(1600 \text{ kHz}/580 \text{ kHz})^2 = (2800 \text{ pF})/C_2$, which gives $C_2 =$ 368 pF.

(*b*) We find the inductance from the first frequency:

$f_{01} = (1/2?)(1/LC_1)^{1/2}$;

$580 \times 10^3 \text{ Hz} = (1/2?)[1/L(2800 \times 10^{-12} \text{ F})]^{1/2}$, which gives $L = 2.68 \times 10^{-5} \text{ H} =$ 26.8 μH.

83. (*a*) We find the capacitance from the frequency:

$f_0 = (1/2?)(1/LC)^{1/2}$;

$3600 \text{ Hz} = (1/2?)[1/(4.8 \times 10^{-3} \text{ H})C]^{1/2}$, which gives $C = 4.07 \times 10^{-7} \text{ F} =$ 0.41 μF.

(*b*) At resonance the impedance is the resistance, so the current is

$I_0 = V_0/R = (50 \text{ V})/(4.4\ \Omega) =$ 11A.

84. (*a*) The frequency of oscillation is the resonance frequency. We find the inductance from

$f_0 = (1/2?)(1/LC)^{1/2}$;

$20 \times 10^3 \text{ Hz} = (1/2?)[1/L(3000 \times 10^{-12} \text{ F})]^{1/2}$, which gives $L = 2.11 \times 10^{-2} \text{ H} =$ 21 mH.

(*b*) The energy initially stored in the capacitor will oscillate between the capacitor and the inductor. We find the maximum current, when all of the energy is stored in the inductor, by equating the maximum energies:

$!CV_0^2 = !LI_{max}^2$;

$(3000 \times 10^{-12} \text{ F})(120 \text{ V})^2 = (2.11 \times 10^{-2} \text{ H})I_{max}^2$, which gives $I_{max} = 4.5 \times 10^{-2} \text{ A} =$ 45 mA.

(*b*) The maximum energy stored in the inductor is

$U_{Lmax} = !LI_{max}^2 = !(2.11 \times 10^{-2} \text{ H})(4.5 \times 10^{-2} \text{ A})^2 =$ 2.2×10^{-5} J.

85. We find the equivalent resistance of the two speakers in parallel from

$1/R_{eq} = (1/R_1) + (1/R_2) = (1/8\ \Omega) + (1/8\ \Omega)$, which gives $R_{eq} = 4\ \Omega$.

Thus the speakers should be connected to the 4 Ω terminals.

86. Because the loudspeaker is connected to the secondary side of the transformer, we have

$Z_P/Z_S = (N_P/N_S)^2$;

$(30 \times 10^3\ \Omega)/(8.0\ \Omega) = (N_P/N_S)^2$, which gives $N_P/N_S =$ 61.

87. (*a*) Because the increasing current in the first loop produces an increasing magnetic field in the second loop, yes, there will be an induced current in the second loop.

(*b*) Because the current in the first loop increases immediately, the induced current will increase immediately.

(*c*) The induced current will stop when the current in the first loop reaches the steady state.

(*d*) To oppose the increase in the magnetic field away from you from the first loop, the induced current will produce a magnetic field toward you. Thus the induced current will be counterclockwise.

(*e*) While there is an induced current, the loops will have parallel opposite currents, so, yes, there will be a force.

(*f*) Because we have opposite currents, the force is repulsive, so the force on each loop is away from the other loop.

88. (*a*) The clockwise current in the left-hand loop produces a magnetic field which is into the page within the loop and out of the page outside the loop. Thus the right-hand loop is in a magnetic field out of the page. Before the current in the left-hand loop reaches its steady state, there will be an induced current in the right-hand loop that will produce a magnetic field into the page to oppose the increase of the field from the left-hand loop. Thus the induced current will be clockwise.

(*b*) After a long time, the current in the left-hand loop will be constant, so there will be no induced current.

(*c*) If the second loop is pulled to the right, the magnetic field out of the page from the left-hand loop through the second loop will decrease. During the motion, there will be an induced current in the right-hand loop that will produce a magnetic field out of the page to oppose the decrease of the field from the left-hand loop. Thus the induced current will be counterclockwise.

89. The average induced emf is

$\varepsilon = -\Delta\Phi_B/\Delta t = -A\,\Delta B/\Delta t = -(0.240\text{ m})^2(0 - 0.755\text{ T})/(0.0400\text{ s}) = 1.09\text{ V}.$

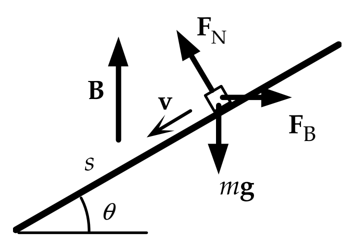
The average current is

$I = \varepsilon/R = (1.09\text{ V})/(6.50\ \Omega) = 0.168\text{ A}.$

The energy dissipated is

$E = I^2R\,\Delta t = (0.168\text{ A})^2(6.50\ \Omega)(0.0400\text{ s}) = 7.33 \times 10^{-3}\text{ J}.$

90. A side view of the rail and bar is shown in the figure. The component of the velocity of the bar that is perpendicular to the magnetic field is $v \cos \theta$, so the induced emf is



$\varepsilon = BLv \cos \theta.$

This produces a current in the wire

$I = \varepsilon/R = (BLv \cos \theta)/R$ into the page.

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field will be horizontal, as shown, with magnitude

$F_B = I\ell B = (B^2\ell^2 v \cos \theta)/R.$

For the wire to slide down at a steady speed, the net force must be zero. If we consider the components along the rail, we have

$F_B \cos \theta - mg \sin \theta = 0$, or

$[(B^2\ell^2 v \cos \theta)/R] \cos \theta = (B^2\ell^2 v \cos^2 \theta)/R = mg \sin \theta;$

$(0.55\text{ T})^2(0.30\text{ m})^2 v(\cos^2 5.0^\circ)/(0.60\ \Omega) = (0.040\text{ kg})(9.80\text{ m/s}^2) \sin 5.0^\circ$, which gives $v = 0.76\text{ m/s}.$

91. The average induced emf is

$\varepsilon = -\Delta\Phi_B/\Delta t = -NA\,\Delta B/\Delta t = -NA[(-B) - (+B)]/\Delta t = 2NAB/\Delta t.$

The average current is

$I = \varepsilon/R = 2NAB/R\,\Delta t,$

so the total charge that passes through the galvanometer is

$Q = I\,\Delta t = (2NAB/R\,\Delta t)\,\Delta t = 2NAB/R$, or $B = QR/2NA.$

92. (*a*) The velocity of the bar is perpendicular to the magnetic field, so the induced emf is

$å = BLv.$

This produces a current in the wire

$I = å/R = BLv/R$ downward.

Because the current is perpendicular to the magnetic field, the force on the wire from the magnetic field will be horizontal to the left with magnitude

$F_B = I¬B = B^2¬^2v/R.$

An external force of this magnitude must be exerted to the right to maintain the motion. The power expended by this force is

$P = Fv = (B^2¬^2v/R)v = B^2¬^2v^2/R.$

(*b*) The power dissipated in the resistance is

$P = I^2R = (BLv/R)^2R = B^2¬^2v^2/R,$

which is the power expended by the external force. We expect this, since there is no increase in kinetic energy.

93. (*a*) From the efficiency of the transformer, we have $P_S = 0.80P_P$.

For the power input to the transformer, we have

$P_P = I_PV_P;$

(58 W)/0.80 = I_P(110 V), which gives I_P = 0.66 A.

(*b*) We find the secondary voltage from

$P_P = V_S^2/R_S;$

58 W = V_S^2/(2.4 Ω), which gives V_S = 11.8 V.

We find the ratio of the number of turns from

$N_P/N_S = V_P/V_S$ = (110 V)/(11.8 V) = 9.3.

94. (*a*) The voltage drop across the lines is

$®V = 2IR$ = 2(700 A)(0.80 Ω) = 1.12×10^3 V = 1.12 kV.

Thus the voltage at the other end is

$V_{out} = V_{in} - ®V$ = 42 kV – 1.12 kV = 41 kV.

(*b*) The power input is

$P_{in} = IV_{in}$ = (0.700 kA)(42 kV) = 29.4 MW.

(*c*) The power loss in the lines is

$P_{loss} = 2I^2R$ = 2(0.700 kA)^2(0.80 Ω) = 0.78 MW.

(*d*) The power output is

$P_{out} = IV_{out}$ = (0.700 kA)(40.9 kV) = 28.6 MW.

95. We find the peak emf from

$å_{peak} = NBA\omega$ = (125 turns)(0.200 T)(0.066 m)^2(120 rev/s)(2? rad/rev) = 82 V.

96. (*a*) Because we have direct coupling, the torque provided by the motor balances the torque of the friction force:

$NIAB = Fr$;

(300 turns)I(0.10 m)(0.15 m)(0.60 T) = (250 N)(0.25 m), which gives I = 23 A.

(*b*) To maintain the speed, we have a force equal to the friction force, so the power required is

Fv = (250 N)(30 km/h)/(3.6 ks/h) = 2.08×10^3 W.

This must be provided by the net power from the motor, which is

$P_{net} = IV_{in} - I^2R = I(V_{in} - IR) = I$å$_{back} = Fv$;

(23 A)å$_{back}$ = 2.08×10^3 W, which gives å$_{back}$ = 90 V.

(*c*) The power dissipation in the coils is

$P_{loss} = P_{in} - P_{net}$ = (23 A)(10)(12 V) – 2.08×10^3 W = 6.9×10^2 W.

(*d*) The useful power percentage is

$(P_{net}/P_{in})(100) = (I$å$_{back}/IV_{in})(100)$ = (90 V/120 V)(100) = 75%.

97. We find the impedance from

$Z = X_L = V_{rms}/I_{rms}$ = (220 V)/(5.8 A) = 37.9 Ω.

We find the inductance from

X_L = 2?fL;

37.9 Ω = 2?(60 Hz)L, which gives L = 0.10 H.

98. For the current and voltage to be in phase, the net reactance of the capacitor and inductor must be zero, which means that we have resonance. Thus we have

f_0 = (1/2?)$(1/LC)^{1/2}$;

3360 Hz = (1/2?)[1/(0.230 H)C]$^{1/2}$, which gives $C = 9.76 \times 10^{-9}$ F = 9.76 nF.

99. Because the current lags the voltage, this must be an *RL* circuit.

We find the impedance of the circuit from

$Z = V_{rms}/I_{rms}$ = (120 V)/(5.6 A) = 21.4 Ω.

We find the resistance from the phase:

$\cos \phi = R/Z$;

cos 50¡ = R/(21.4 Ω), which gives R = 13.8 Ω.

We find the reactance from the phase:

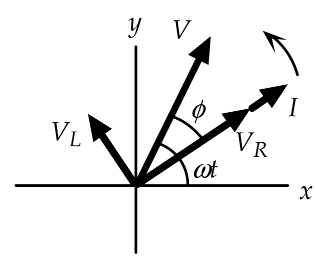
$\sin \phi = X_L/Z$;

sin 50¡ = X_L/(21.4 Ω), which gives X_L = 16.4 Ω.

We find the inductance from

X_L = 2?fL;

16.4 Ω = 2?(60 Hz)L, which gives $L = 4.35 \times 10^{-2}$ H = 43.5 mH.



100. Because the circuit is in resonance, we find the inductance from

f_0 = (1/2?)$(1/LC)^{1/2}$;

48.0×10^6 Hz = (1/2?)[1/L(220×10^{-12} F)]$^{1/2}$, which gives $L = 5.0 \times 10^{-8}$ H.

If r is the radius of the coil, the number of turns is

N = ¬$_{wire}$/2?r.

If d is the diameter of the wire, for closely-wound turns, the length of the coil is

¬ = Nd.

Thus the inductance of the coil is

$L = \mu_0AN^2$/¬ = μ_0?r^2(¬$_{wire}$/2?r)^2/Nd = μ_0¬$_{wire}{}^2$/4?Nd;

5.0×10^{-8} H = (4? × 10^{-7} T á m/A)(14.0 m)^2/4?N(1.1×10^{-3} m), which gives N = 3.56×10^5 turns.

101. We find the resistance of the coil from the dc current:

$R = V_{dc}/I_{dc} = (36 \text{ V})/(2.5 \text{ A}) =$ 14 Ω.

We find the impedance from the ac current:

$Z = V_{rms}/I_{rms} = (120 \text{ V})/(3.8 \text{ A}) = 31.6\ \Omega$.

We find the reactance from

$Z = [R^2 + X_L{}^2]^{1/2}$;

$31.6\ \Omega = [(14.4\ \Omega)^2 + X_L{}^2]^{1/2}$, which gives $X_L = 28.1\ \Omega$.

We find the inductance from

$X_L = 2?fL$;

$28.1\ \Omega = 2?(60 \text{ Hz})L$, which gives $L = 0.075 \text{ H} =$ 75 mH.

102. (*a*) At resonance we have

$2?f_0 = (1/LC)^{1/2}$.

The Q factor is

$Q = V_C/V_R = I_{rms}X_C/I_{rms}R = 1/2?f_0CR = (LC)^{1/2}/CR = (1/R)(L/C)^{1/2}$.

(*b*) We find the inductance from the resonance frequency:

$2?f_0 = (1/LC)^{1/2}$;

$2?(1.0 \times 10^6 \text{ Hz}) = [1/L(0.010 \times 10^{-6} \text{ F})]$, which gives $L = 2.5 \times 10^{-6} \text{ H} =$ 2.5 μH.

We find the resistance required from

$Q = (1/R)(L/C)^{1/2}$;

$1000 = (1/R)[(2.5 \times 10^{-6} \text{ H})/(0.010 \times 10^{-6} \text{ F})]$, which gives $R =$ $1.6 \times 10^{-2}\ \Omega$.

103. We find the impedance from the power factor:

$\cos \phi = R/Z$;

$0.17 = (200\ \Omega)/Z$, which gives $Z = 1.18 \times 10^3\ \Omega$.

We get an expression for the reactances from

$Z^2 = R^2 + (X_L - X_C)^2$;

$(1.18 \times 10^3\ \Omega)^2 = (200\ \Omega)^2 + (X_L - X_C)^2$, which gives $X_L - X_C = \pm\ 1.16 \times 10^3\ \Omega$.

When we express this in terms of the inductance and capacitance, we get

$2?fL - (1/2?fC) = \pm\ 1.16 \times 10^3\ \Omega$;

$2?f(0.020 \text{ H}) - [1/2?f(50 \times 10^{-9} \text{ F})] = \pm\ 1.16 \times 10^3\ \Omega$, which reduces to two quadratic equations:

$0.126f^2 \pm (1.16 \times 10^3 \text{ Hz})f - 3.183 \times 10^6 \text{ Hz}^2$,

which have positive solutions of $f =$ 2.2×10^3 Hz, 1.1×10^4 Hz.